NONCONSERVATION OF ENERGY DURING COSMIC EVOLUTION

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In dealing with the problem of Mach's principle the author showed[1] that the motions of stars within a galaxy and of galaxies within a cluster of galaxies as well as the general recession of galaxies are governed in the Newtonian approximation by the Lagrange function

$$L = T - m(\Phi + \Psi), \tag{1}$$

in which $T = \frac{1}{2}m\dot{\vec{r}}^2$ is the kinetic energy of the celestial body considered whose mass is m, Ψ the Newtonian gravitational potential expressing the influence of local inhomogeneities in the distribution of matter, and Φ the cosmic potential composed additively of a scalar potential φ and the scalar product of the velocity $\dot{\vec{r}}$ of this body with a vector potential $\vec{Q}$:

$$\Phi = \varphi(\vec{r}, t) - \dot{\vec{r}}\cdot\vec{Q}(\vec{r}, t),$$
$$\varphi = -\tfrac{1}{2}H^2\vec{r}^2, \quad \vec{Q} = -H\vec{r}. \tag{2}$$

H denotes the Hubble factor of cosmic expansion related to the mean radius G of the curvature of space by the well-known formula

$$H = H(t) = \dot{G}/G. \tag{3}$$

Since the Lagrange function (1) depends explicitly on time, the total energy Λ of our test body, defined, as usual, by the relation

$$\Lambda = \sum_{i=1}^{3} (\partial L/\partial \dot{q}_i)\dot{q}_i - L = T + m(\varphi + \Psi) \tag{4}$$

is not conserved. Its change during the time interval between t_1 and t is

$$\Delta\Lambda = -\int_{t_1}^{t}(\partial L/\partial t)dt = m\int_{t_1}^{t}\dot{H}\vec{r}\cdot(\dot{\vec{r}} - H\vec{r})dt. \tag{5}$$

From Eq. (3) we obtain

$$\dot{H} = -(1+q)H^2, \quad q = -(d^2G/dt^2)/GH^2. \tag{6}$$

Because of the very small value of $\dot{H}$ in the present epoch of cosmic evolution ($\dot{H}_1 \cong 2\times10^{-35}$ sec^{-2}, for astronomical measurements indicate[2] $H_1 \cong 3\times10^{-18}$ sec^{-1}, and $q \cong 1$) the deviations from the exact validity of the law of conservation of energy are immeasurably small in terrestrial physics, but they probably play an important role in cosmology.

The aim of this note is to propose a way for a quantitative examination of the question whether the observed high angular momenta of galaxies (and other related phenomena) are a consequence of the nonconservation of energy during cosmic evolution. We deliberately restrict ourselves to a nonrelativistic treatment of the problem, because there exists no uncertainty in classical analytical dynamics as to how to define the total energy of a particle.

It follows from the equation of motion

$$d^2\vec{r}/dt^2 = -qH^2\vec{r} - \operatorname{grad}\Psi, \tag{7}$$

deduced from the Lagrange function (1), that the Newtonian force $F_N = -m\operatorname{grad}\Psi$ prevails over the cosmic force $\vec{F}_C = -mqH^2\vec{r}$ up to a certain critical distance. If a rich spherical cluster of celestial bodies (i.e., a cluster of galaxies, or a galaxy) with total mass m_0 is concentrated into a relatively small volume of the expanding Friedman universe, the ratio of the absolute value $F_N = \gamma m_0/r^2$ to the absolute value F_C is expressed by the relation

$$F_N/F_C = (\gamma m_0/r^2)/qH^2r = (1/q)(r_0/r)^3, \tag{8}$$

where γ is the Newtonian gravitational constant, and

$$r_0 = (\gamma m_0/H^2)^{1/3}. \tag{9}$$

In the region $r > r_0$ the ratio F_N/F_C drops to zero so fast that the Newtonian force can be completely neglected relative to the cosmic force. A test body here follows with high accuracy the law of general cosmic expansion

$$\dot{\vec{r}} = H\vec{r}, \tag{10}$$

and its total energy, measured from the center of gravity of our cluster, increases, in the first approximation, due to the change of its Newtonian potential energy $m\Psi$.

Inside the radius r_0, Eq. (8) holds the more accurately, the greater the part of the cluster that is concentrated at the neighborhood of its center of gravity[3]; but in any case the ratio F_N/F_C in this region is so high that it is mainly the Newtonian force that determines here the motion of our test body. During this part of the motion its total energy increases too, but now, in the first approximation, due to the change of its cosmic energy $m\varphi$. If a test particle moves in the local field of a single central body (with $\Psi = -\gamma m_0/r$) along a circular orbit, the cosmic force F_C changes its path into a spiral with increasing radius vector. The increase of its total energy during the time interval Δt is given, in first approximation, by the relation

$$\Delta\Lambda \cong m(\varphi - \varphi_1) \cong -\tfrac{1}{2}mr_1^2(dH^2/dt)_1\Delta t$$
$$= (1+q_1)mr_1^2H_1^3\Delta t = -2(1+q_1)\varphi_1H_1\Delta t > 0, \qquad (11)$$

for $\varphi_1 = -\tfrac{1}{2}mH_1^2r_1^2 < 0$. This domain, with $r < r_0$, thus represents an almost quasiclosed mechanical system whose total energy in the later stages of cosmic evolution increases very slowly.

If q lies near unity, both forces, F_N and F_C, have the same order of magnitude at the distance $r \cong r_0$. This is physically the most interesting region, for here a test body, i.e., a galaxy (or, in the early stages of cosmic expansion, a star), taking part, up to this point, in the general cosmic recession, is captured by the local Newtonian field, and becomes a member of a cluster of galaxies (or of a galaxy, respectively). As is shown in an earlier paper,[4] we obtain very good agreement with the known empirical data about the cluster of galaxies if we identify the radius of a rich spherical cluster of galaxies with the radius r_0 determined by the simple formula (9).

The process of capture is accompanied by a relatively high increase of the total energy of the test body. Quantitatively it can be studied with the help of a high-speed computer. Such investigation, which lies outside the scope of the author's present possibilities, should be carried out in the following three steps:

(1) We consider the radial cosmic recession of a test particle, its capture by the local field of a single central body (with $\Psi = -\gamma m_0/r$), and its free fall in this field. Before and after the capture, the particle moves very approximately according to the relation

$$t = A\{\arcsin(r/r_m)^{1/2} - (r/r_m)^{1/2}(1 - r/r_m)^{1/2}\} + B, \qquad (12)$$

in which the constants A, B, and r_m have different values before and after the capture. During this process the motion is described by Eq. (7). We integrate it numerically for various stages of cosmic evolution in which the capture took place, and compute thereafter the increase of the total energy by Eq. (4).

(2) We consider the motion of a test particle with the mass m in the local field of a single central body (with $\Psi = -\gamma m_0/r$). By Routh's method we deduce from the Lagrange function (1) the following equation of motion:

$$d^2r/dt^2 = -qH^2r - \gamma m_0/r^2 + \alpha^2/m^2r^3, \qquad (13)$$

where

$$\alpha = mr^2(d\psi/dt) = \text{const}. \qquad (14)$$

As the initial value we assume an elliptical orbit around the central body and integrate Eq. (13) in the reversed direction of time (i.e., towards the beginning of expansion) in order to find under which conditions the path of a test body, taking part in the general cosmic expansion, becomes elliptical after the capture.

(3) In order to investigate the formation of the spiral structure of galaxies, we solve the same problem as sub (2), but we replace the single central body by a double star (whose field has no spherical symmetry).

Concluding, let us remark that from the relativistic standpoint the nonconservation of energy is a consequence of the fact that the metric of our space-time continuum,[1]

$$ds^2 = -(dx^2 + dy^2 + dz^2) + (c^2 + 2\Phi)dt^2, \qquad (15)$$

written with neglect of the local gravitational field and with Φ given by Eqs. (2), is not symmetric on reversing the arrow of time.

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[1]J. Pachner, Phys. Rev. 132, 1837 (1963).

[2]M. L. Humason, N. U. Mayall, and A. Sandage, Astron. J. 61, 97 (1956); W. A. Baum, ibid. 62, 6 (1957); A. Sandage, Astrophys. J. 127, 513 (1958); 139, 355 (1961).

[3]The bright and the brightest galaxies are really found to lie in the neighborhood of the center of a cluster of galaxies—see F. Zwicky, Morphological Astronomy (Springer-Verlag, Berlin, Germany, 1957), p. 47, Table IV; p. 60, Table VII; p. 67, Table X.

[4]J. Pachner, Z. Astrophys. 55, 177 (1962).