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MICROWAVE SURFACE IMPENDANCE OF SUPERCONDUCTORS OF THE SECOND KIND: In-Bi ALLOYS

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A superconductor of the second kind allows field penetration at a field H_L while the superconductivity is not destroyed until a field $H_U > H_L$ is reached. The state of the material when $H_L < H < H_U$, the "mixed" state, is thought to consist of a microscopic array of current vortices through which the magnetic field penetrates.[1] Superconductors of the second kind must have small coherence distances which usually result from a small electron mean free path in the normal state. Therefore, pure elements are usually superconductors of the first kind, while alloys are usually superconductors of the second kind.

Microwave studies of superconductors of the first kind are done under extreme anomalous conditions, the microwave skin depth being much smaller than the mean free path in the normal state. When the microwave photon energy becomes comparable to the superconductor energy gap (at high frequencies or at temperatures near the critical temperature), quantum effects due to the excitation of quasiparticles are observed.[2] At lower frequencies and temperatures, extremely complicated magnetic field and temperature dependences have been observed which have yet to be explained.[3] In the present materials the zero-field temperature dependence fits a local two-fluid model quite well. Quantum effects due to the excitation of quasiparticles are masked by the large "free carrier absorption" by the quasiparticles due to the short mean free path. In the mixed state the surface impedance shows a dependence on the angle between the microwave current and $\vec{H}$ which appears to be characteristic of the superconductors of the second kind.

We have measured the dependence on $\vec{H}$ and T of the real part R of the surface impedance of In-Bi alloys at 24 kMc/sec. The specimens were polycrystalline plates of either a 2 or 3 at. % Bi alloy approximately 0.5 in. ×0.5 in. ×0.01 in. The samples were prepared by melting the alloy between CaF_2 plates, or by pressing the material between glass plates. The samples were then annealed for one to three months at 80°C. During this annealing the samples recrystallized leaving the surface slightly irregular. We did not use chemical or electrochemical polishing techniques since they could change the impurity concentration of the surface. The surface impedance of alloys should be affected much less by surface damage than the surface impendance of pure metals, since the mean free path of the alloy is intrinsically very small. The samples formed one wall or part of a wall of a TE_{011} cavity which terminated one arm of a microwave bridge. The change in the power reflected from the cavity was proportional to the change in R.

Temperature dependence. —In Fig. 1 we plot R/R_n, where R_n is R in the normal state, versus the temperature T for samples of In + 2 at. % Bi. We have assumed that $R(T<1.6, H=0)=0$, which seems reasonable on the scale of Fig. 1. The points are experimental. The theoretical curve is calculated for a two-fluid model where we assume a local relationship between the current and electric field. In this case[4]

$$\frac{R}{R_n} = \left[\frac{[(\sigma_1/\sigma_n)^2 + (\sigma_2/\sigma_n)^2]^{1/2} - \sigma_2/\sigma_n}{(\sigma_1/\sigma_n)^2 + (\sigma_2/\sigma_n)^2}\right]^{1/2}, \qquad (1)$$

where σ_1 and σ_2 are the real and the imaginary parts of the conductivity, respectively, and σ_n is the conductivity in the normal state. For σ_1/σ_n we use the Gorter-Casimir relation $\sigma_1/\sigma_n = t^4$

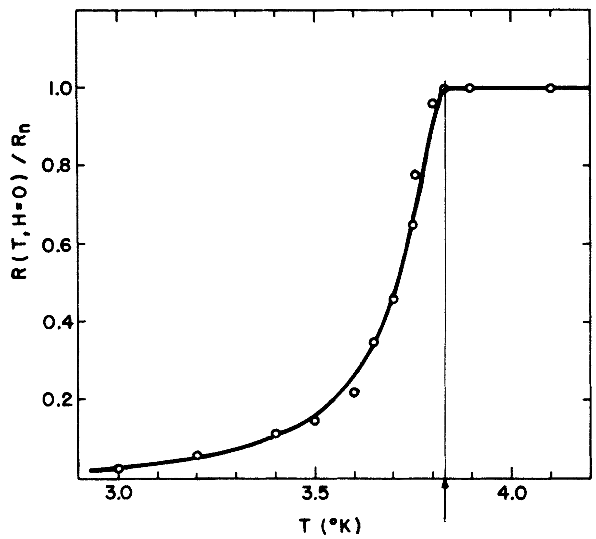

FIG. 1. Temperature dependence of the real part of the microwave surface impedance of In+2 at. % Bi. $H=0$. The curve is theoretical and the points experimental. The arrow indicates the critical temperature, 3.83°K.

where t is the reduced temperature. To calculate σ_2/σ_n we assume that the mean free path l is sufficiently small that we can use the local limit of the Pippard equation[4] and that only superconducting carriers contribute to σ_2:

$$\vec{J}_s = i\sigma_2 \vec{E} = i\frac{n_s e^2}{m\omega}\frac{\alpha l}{\xi_0}\vec{E}, \qquad (2)$$

where $\vec{J}_s$ is the supercurrent density, n_s is the density of superconducting carriers, ξ_0 the Pippard coherence length, $\vec{E}$ the electric field, ω the frequency, and α a constant of the order of unity. We take[4] $\xi_0 = a\hbar v_F/kT_c$ where v_F is the Fermi velocity, k Boltzmann's constant, T_c is the critical temperature, and a is a constant. We then obtain

$$\frac{\sigma_2}{\sigma_n} = \frac{\alpha}{a}\frac{kT_c(1-t^4)}{\hbar\omega}, \qquad (3)$$

where α/a is the only adjustable parameter. We now substitute these expressions for the conductivities into Eq. (1) to obtain $R(T)/R_n$. The curve in Fig. 1 is calculated for $\alpha/a = 2.1$ and fits the experimental data rather well. If we choose $\alpha = 1$, and a equal to 0.18 as given by the BCS theory, we obtain $\alpha/a = 5.5$. The value of α/a required to fit the experimental data seems to decrease with increasing Landau-Ginsburg parameter κ. For an InBi alloy with 3 at. % Bi ($\kappa \sim 2$), α/a is found to be 1.2 and to be even smaller for Nb_3Sn ($\kappa \sim 10$).

Magnetic field dependence. —Figure 2 shows the magnetic field dependence of R for the sample of Fig. 1. When a magnetic field $\vec{H}$ was applied in the plane of the sample, R increased slowly until a value which we identify with H_L. At this field R began to increase more rapidly but became independent of field at H_U. This rapid rise seems due to the increase in the number of normal electrons with magnetic field penetration. The

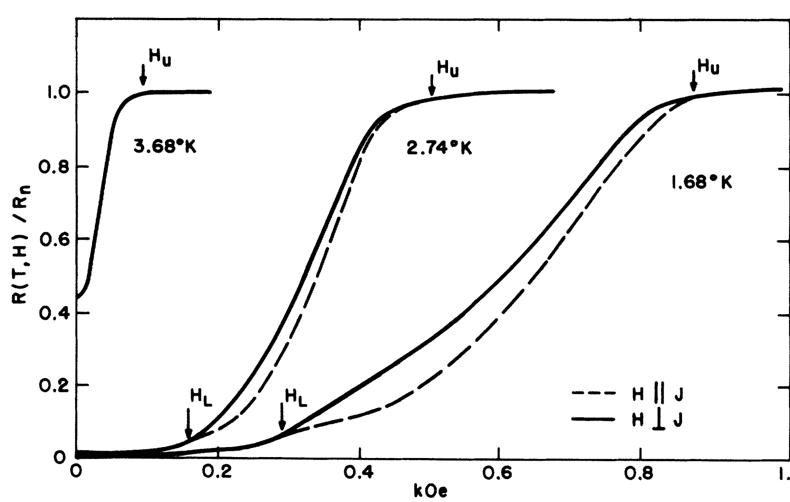

FIG. 2. Magnetic field dependence of the real part of the microwave surface impendance of In+2 at. % Bi at several temperatures. The solid curves are for $\vec{H}$ perpendicular to the microwave current and the dashed curve is for $\vec{H}$ parallel. The arrows indicate the fields chosen for H_L and H_U.

temperature dependence of H_U is in excellent agreement with the Gor'kov theory[5] [$H_U = 1150 \times (1 - t^2)(1 - 0.28t^2 + 0.04t^4)$ Oe]. From the values of H_L and H_U near T_c one obtains[6] $\kappa = 1.4$. These values are in approximate agreement with magnetization measurements made on these alloys.[7]

The most striking feature of the magnetic field dependence of R is that in the mixed state, R is greater when $\vec{H}$ is perpendicular to the microwave current ($R_\perp$) than when it is parallel ($R_\parallel$). This anisotropy appears to be characteristic of the mixed state. We have also observed anisotropy in Nb_3Sn, V_3Si, and Nb.[8] The anisotropy appears to be large when κ is large; $R_\perp/R_\parallel \sim 15$ at 20 kOe for Nb_3Sn. We think that this is not the anisotropy suggested by Ginsburg and Landau[9] since there appears to be anisotropy only in the mixed state and not below H_L. At $H > H_L$ the material is permeated by current vortices or tubes of flux and the density of normal carriers, and therefore the conductivity of the material, is spatially modulated in the direction perpendicular to the magnetic field. If the microwave current is flowing in the direction of these tubes of flux, the higher conductivity regions would tend to "short out" the regions of lower conductivity, reducing $R_\parallel$. This would not occur in the $R_\perp$ case. If, however, the extent in the direction perpendicular to the flux tube of the regions of high and low normal carrier concentration were large compared to a microwave penetration depth, there could be no appreciable anisotropy. In this latter case the current in one region could not be affected by the presence of a region of different conductivity. A preliminary calculation of the surface impendance for a microscopically modulated complex conductivity shows anisotropy. The fact that anisotropy is seen is evidence for the transverse dimension of the flux tubes being comparable to or smaller than the microwave penetration depth.

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