Second Sound in Systems of One-Dimensional Fermions

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We study sound in Galilean invariant systems of one-dimensional fermions. At low temperatures, we find a broad range of frequencies in which in addition to the waves of density there is a second sound corresponding to the ballistic propagation of heat in the system. The damping of the second sound mode is weak, provided the frequency is large compared to a relaxation rate that is exponentially small at low temperatures. At lower frequencies, the second sound mode is damped, and the propagation of heat is diffusive.

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The low-energy properties of systems of one-dimensional interacting fermions are usually described in the framework of the Tomonaga-Luttinger liquid theory [1–4]. Its main feature is that the elementary excitations of the system are treated as noninteracting bosons with linear dispersion. The advantage of this approach is that it adequately describes the low-energy properties of the system at any strength of interaction between the fermions. This theory provided the foundation for understanding the basic properties of one-dimensional electron systems, such as the power law renormalizations of the impurity scattering and tunneling density of states [5,6], observed in subsequent experiments [7–10].

Much of the recent work on the theory of one-dimensional systems focused on the properties not captured by the Luttinger liquid picture, such as the nature and lifetimes of elementary excitations in these systems. When the interactions between the bosonic excitations are taken into account, the excitations in spinless Luttinger liquids become fermions [11] with finite decay rate $\tau_{\rm ex}^{-1} \propto T^{\gamma}$, with the exponent $\gamma = 7$ [12–14] or 6 [15], depending on the details of the interaction between the physical particles forming the Luttinger liquid. For weakly interacting spin-$\frac{1}{2}$ fermions, $\tau_{\rm ex}^{-1} \propto T$ [16]. Importantly, the scattering processes giving rise to the decay of elementary excitations do not involve the backscattering of fermions; i.e., each quasiparticle remains in the vicinity of the nearest Fermi point. The backscattering processes involve hole states near the bottom of the band, and their rate is exponentially small, $\tau^{-1} \propto e^{-D/T}$ [17–21], where D is the energy scale of the order of Fermi energy.

In this Letter, we consider the dynamics of a system of one-dimensional fermions in the absence of disorder at low temperatures $T \ll D$. Such a system possesses three conserved quantities: the total number of particles N, energy E, and momentum P. At very low frequencies $\omega \ll \tau^{-1}$, the system is close to equilibrium and can be described by classical hydrodynamics. We will be primarily interested in the regime

$$\tau^{-1} \ll \omega \ll \tau_{\rm ex}^{-1}. \tag{1}$$

In this case, the gas of elementary excitations is in thermal equilibrium but can move with velocity $u_{\rm ex}$ not equal to the velocity u of the center of mass of the fluid [20]. At such frequencies, the system possesses a fourth conserved quantity: the difference between the numbers of the right- and left-moving fermions $J = N^R - N^L$. Because the relaxation of J involves the backscattering of fermions, it is negligible at $\omega \gg \tau^{-1}$.

The detachment of the gas of elementary excitations from the rest of the fluid is a well-known feature of superfluid ^{4}He [22,23]. The appropriate theoretical description of the motion of this system is in terms of two-fluid hydrodynamics that predicts the existence of two sound modes. The first sound is the usual wave of particle density, whereas the second sound is a wave of entropy that propagates at a different velocity. Our goal is to develop a similar two-fluid hydrodynamics of the system of one-dimensional fermions in the frequency range (1) and to demonstrate the existence of the second sound in this system.

We will focus on the system of one-dimensional spin-$\frac{1}{2}$ fermions of mass m with repulsive interactions and assume spin rotation symmetry and Galilean invariance. To leading order in $T/D \ll 1$, the dynamics of the system is described by the conventional Luttinger liquid theory with a linear excitation spectrum [24]. Our system supports two branches of bosonic excitations, corresponding to the charge and spin sectors of the Hamiltonian and propagating at different velocities, v_ρ and v_σ. The momentum of the system is [3]

$$P = \frac{h}{4L} NJ + \sum_k k(N_k^\rho + N_k^\sigma), \tag{2}$$

where N is the total number of fermions in a system of size L with periodic boundary conditions and h is the Planck constant, while N_k^ρ and N_k^σ are the occupation numbers of the bosonic excitations with momentum k in the charge and spin channels, respectively. The first term in Eq. (2) accounts for the fact that at $N^R \neq N^L$ the ground state of the system has a nonvanishing momentum $p_F J$, where the Fermi momentum $p_F = hN/4L$. Similarly, the energy of the system is given by

$$E = \frac{mv_\rho^2}{2N_0}(N - N_0)^2 + \frac{h^2 N J^2}{32mL^2} + \sum_k [\epsilon_\rho(k) N_k^\rho + \epsilon_\sigma(k) N_k^\sigma]; \quad (3)$$

cf. Ref. [3]. In the first term, N_0 is some reference value of the particle number, and we have used the usual relation between the ground state compressibility and v_ρ. Bosonic excitations in the Luttinger liquid are superpositions of small momentum particle-hole pairs near each Fermi point. At $N^R = N^L$, the energies are $\epsilon_{\rho,\sigma}(k) = v_{\rho,\sigma}|k|$. At $N^R \neq N^L$, the quasiparticle ground state is moving with velocity

$$u_0 = \frac{hJ}{4mL}. \quad (4)$$

The dependence of the quasiparticle energies on u_0,

$$\epsilon_{\rho,\sigma}(k) = v_{\rho,\sigma}|k| + u_0 k, \quad (5)$$

is obtained by performing a Galilean transformation to the stationary frame.

At frequencies below $\tau_{\rm ex}^{-1}$, collisions between the bosonic excitations occur very quickly compared with the typical time scale ω^{-1}, and to a first approximation one can assume that the gas of excitations is in an equilibrium state described by the Bose distribution

$$N_k^{\rho,\sigma} = \left[\exp\left(\frac{\epsilon_{\rho,\sigma}(k) - u_{\rm ex} k}{T}\right) - 1\right]^{-1}. \quad (6)$$

Since the collisions between excitations conserve their total momentum, the equilibrium is characterized by the velocity $u_{\rm ex}$, which is not necessarily equal to the velocity u_0 associated with the Fermi surface.

As discussed above, in the absence of backscattering there are four conserved macroscopic characteristics of the fluid: the number of particles, energy, momentum, and J. The hydrodynamic description of the fluid is obtained by writing these conservation laws in the form of continuity equations on the respective densities:

$$\partial_t n + \partial_x j = 0, \quad (7a)$$

$$\partial_t \varepsilon + \partial_x j_\varepsilon = 0, \quad (7b)$$

$$\partial_t p + \partial_x j_p = 0, \quad (7c)$$

$$\partial_t u_0 + \partial_x j_{u_0} = 0. \quad (7d)$$

Here n, ε, and p are the densities of particles, energy, and momentum of the system, respectively. Instead of density J/L, we use the velocity u_0 defined by Eq. (4). The corresponding currents j, j_ε, j_p, and j_{u_0} are yet to be determined.

Below, we consider only the regime of a small deviation of the system from thermal equilibrium, which will be described by two velocities u_0 and $u_{\rm ex}$, and the deviations of densities n and s of particles and entropy from mean values, $n - n_0$ and $s - s_0$. We start by evaluating ε and p in the leading order in these small parameters. At a finite temperature, the dominant contribution to the energy density ε is due to the quasiparticle excitations. Substituting the occupation numbers (6) into the last term in Eq. (3), we obtain

$$\varepsilon = \frac{\pi T^2}{6\hbar\tilde{v}} = \frac{3\hbar}{2\pi}\tilde{v}s^2, \qquad \tilde{v} = \left(\frac{1}{v_\rho} + \frac{1}{v_\sigma}\right)^{-1}. \quad (8)$$

Here we applied the relation $\partial\varepsilon/\partial s = T$ to find the entropy density $s = \pi T/3\hbar\tilde{v}$ and expressed ε in terms of s. Combining Eqs. (4)–(6) with (2), we find the momentum density

$$p = mnu_0 + \frac{2\varepsilon}{v_2^2}(u_{\rm ex} - u_0), \quad v_2 = \left(\frac{v_\rho^{-1} + v_\sigma^{-1}}{v_\rho^{-3} + v_\sigma^{-3}}\right)^{1/2}. \quad (9)$$

Then, using Galilean invariance, we immediately obtain the particle current $j = p/m$ in the form

$$j = nu_0 + \frac{2\varepsilon}{mv_2^2}(u_{\rm ex} - u_0). \quad (10a)$$

The remaining three currents can be obtained using the kinetic equation for elementary excitations and accounting for the fact that collisions do not change the number of particles, momentum, energy, and J. The method was developed in the theory of superfluidity [23]. When applied to the Luttinger liquid, the results take the form

$$j_\varepsilon = \sum_{\lambda=\rho,\sigma} \int \frac{dk}{h} N_k^\lambda \left(j\partial_n \epsilon_\lambda(k) + \epsilon_\lambda(k)\frac{\partial \epsilon_\lambda(k)}{\partial k}\right),$$

$$j_p = j_p^{(0)} + \sum_{\lambda=\rho,\sigma} \int \frac{dk}{h} N_k^\lambda \left(n\partial_n \epsilon_\lambda(k) + k\frac{\partial \epsilon_\lambda(k)}{\partial k}\right),$$

$$j_{u_0} = \frac{1}{m}\left(\mu^{(0)} + \sum_{\lambda=\rho,\sigma} \int \frac{dk}{h} N_k^\lambda \partial_n \epsilon_\lambda(k)\right).$$

Here $j_p^{(0)}$ and $\mu^{(0)}$ are the pressure and chemical potential, respectively, of the Luttinger liquid at $T = 0$. Using Eqs. (5) and (6), to leading order in u_0 and $u_{\rm ex}$ we find

$$j_\varepsilon = \varepsilon \frac{\partial_n \tilde{v}}{\tilde{v}} j + 2\varepsilon u_{\rm ex}, \quad (10b)$$

$$j_p = j_p^{(0)} + \varepsilon \frac{\partial_n(n\tilde{v})}{\tilde{v}}, \tag{10c}$$

$$j_{u_0} = \frac{\mu^{(0)}}{m} + \varepsilon \frac{\partial_n \tilde{v}}{m\tilde{v}}. \tag{10d}$$

We are now in a position to transform Eq. (7) into a set of four differential equations on four hydrodynamic parameters of the fluid: n, s, u_0, and $u_{\rm ex}$. Substituting Eq. (10a) into (7a), we find

$$\partial_t n + n\partial_x u_0 + \frac{2\varepsilon}{mv_2^2}(\partial_x u_{\rm ex} - \partial_x u_0) = 0. \tag{11a}$$

When substituting Eq. (8) into (7b), one should use the expression in terms of the entropy density s and keep in mind that $\tilde{v}$ is a function of density n that in turn depends on time. Expressing the resulting $\partial_t n$ with the aid of Eq. (7a) and using the expression (10b) for j_ε, we obtain

$$\partial_t s + s\partial_x u_{\rm ex} = 0. \tag{11b}$$

This result has the form of the continuity equation expressing the conservation of entropy, which holds to the linear order in a deviation from equilibrium. Since the entropy is transported only by the gas of excitations, one expects the entropy current in the form $j_s = su_{\rm ex}$, in agreement with Eq. (11b).

When substituting Eqs. (10c) and (10d) into (7c) and (7d), one must evaluate the derivatives of the ground state chemical potential $\mu^{(0)}$ and pressure $j_p^{(0)}$ with respect to the density. The chemical potential is easily obtained from the first term in Eq. (3), resulting in $\partial_n\mu^{(0)} = mv_\rho^2/n$. The derivative of the pressure is found using the thermodynamic relation $\partial_n j_p^{(0)} = n\partial_n\mu^{(0)} = mv_\rho^2$. Then Eq. (7c) takes the form

$$\partial_t u_0 + \frac{2\varepsilon}{mnv_2^2}(\partial_t u_{\rm ex} - \partial_t u_0) + v_\rho^2\left[1 + \varepsilon\frac{\partial_n^2(n\tilde{v})}{mv_\rho^2\tilde{v}}\right]\frac{\partial_x n}{n} + \frac{2\varepsilon}{mn}\frac{\partial_n(n\tilde{v})}{\tilde{v}}\frac{\partial_x s}{s} = 0. \tag{11c}$$

To leading order at $T \to 0$, substitution of Eq. (10d) into (7d) gives the same result, because in this limit $p = mnu_0$. Taking the difference of these two equations, which accounts for the time dependence of the momentum of the gas of excitations, we arrive at

$$\partial_t u_{\rm ex} - \partial_t u_0 + v_2^2\frac{n\partial_n\tilde{v}}{\tilde{v}}\frac{\partial_x n}{n} + v_2^2\frac{\partial_x s}{s} = 0. \tag{11d}$$

To study the propagation of collective modes in one-dimensional liquids, we now solve the system of equations (11). In the low-temperature limit, one can set $\varepsilon = 0$ in Eqs. (11a) and (11c). One easily finds two propagating-wave solutions proportional to $e^{-i\omega t + iqx}$. First, Eqs. (11a) and (11c) give rise to a phononlike mode with the spectrum $\omega = v_\rho|q|$. This mode is determined by the dynamics of the variables n and u_0, describing the waves of particle density. Because of the presence of mixing terms in Eq. (11d), the phonon is accompanied by the oscillation of entropy density s and velocity of the gas of excitations $u_{\rm ex}$.

Second, there is a solution with the spectrum $\omega = v_2|q|$ that describes waves of s and $u_{\rm ex}$, whereas $n = n_0$ and $u_0 = \text{const}$. This wave of entropy is fully analogous to the second sound in superfluid ^{4}He. The existence of the second sound in a system of one-dimensional fermions with repulsive interactions is the main result of this Letter.

Our discussion so far assumed that the frequencies of interest are in the range (1). In other words, we set $\tau_{\rm ex} = 0$ and $\tau = \infty$. We shall now relax the latter condition, i.e., assume a large but finite τ and extend our treatment to frequencies $\omega \lesssim \tau^{-1}$. In this regime, one must account for the backscattering processes studied in Refs. [18–21]. Because of the slow rate of these processes, they do not affect the equilibrium form of the distribution function (6). As a result, the state of the system is still described by parameters n, T, u_0, and $u_{\rm ex}$, but because of the backscattering processes the two velocities relax toward each other as

$$\frac{d}{dt}(u_{\rm ex} - u_0) = -\frac{u_{\rm ex} - u_0}{\tau}. \tag{12}$$

It is important to point out that this relaxation does not affect the expressions (10) for the currents and does not violate the conservation laws for the number of particles, energy, and momentum of the system.

In our hydrodynamic description of the one-dimensional system, the first three of the four equations (7) and, respectively, (11) express these three conservation laws and thus remain unchanged. The right-hand side of Eq. (7d) becomes du_0/dt, which is found by applying conservation of momentum condition $dp/dt = 0$ to Eq. (9) and using (12). After that, we recover Eq. (11d) with a simple modification $\partial_t \to \partial_t + \tau^{-1}$.

This modification of the hydrodynamic equations (11) strongly affects the second sound mode at $\omega \lesssim \tau^{-1}$. To a first approximation, we take $\varepsilon/nmv_2^2 \to 0$ and obtain the frequency of the second sound in the form

$$\omega = \sqrt{(v_2 q)^2 - (2\tau)^{-2}} - i(2\tau)^{-1}. \tag{13}$$

At $v_2|q| > (2\tau)^{-1}$, the frequency is reduced, and, more importantly, the second sound decays with the rate $(2\tau)^{-1}$. No wavelike solution exists at $v_2|q| < (2\tau)^{-1}$. Heat propagation over long distances is diffusive: $\omega = -i(v_2^2\tau)q^2$ at $q \to 0$. As a result, the system has a large but finite thermal conductivity κ obtained by multiplying the diffusion coefficient $v_2^2\tau$ by the specific heat $\partial\varepsilon/\partial T$:

$$\kappa = \frac{\pi T v_2^2 \tau}{3\hbar\tilde{v}}. \tag{14}$$

Alternatively, the thermal conductivity can be obtained directly from the modified Eq. (11d). Replacing $\partial_t \to \partial_t + \tau^{-1}$ and considering long time scales gives Eq. (11d) with $\partial_t \to \tau^{-1}$. At $\varepsilon/nmv_2^2 \to 0$, the gas of excitations does not affect particle density n and velocity u_0; see Eqs. (11a) and (11c). Assuming $n = \text{const}$ and $u_0 = 0$ in Eq. (11d), one finds $u_{\text{ex}} = -v_2^2\tau(\partial_x s)/s$. Substituting this result into the expression $j_Q = Tsu_{\text{ex}}$ for the heat current, we obtain $j_Q = -Tv_2^2\tau\partial_x s$. Using our earlier result for the entropy density $s = \pi T/3\hbar\tilde{v}$, we obtain $j_Q = -\kappa\partial_x T$ with κ given by Eq. (14).

To find the effect of a finite backscattering rate on the first sound, one should solve the set of equations (11) in first order in the small parameter ε/nmv_2^2. At small q, we find

$$\omega = \left[v_\rho + \frac{\pi T^2 \partial_n^2(n^2\tilde{v})}{12\hbar m n v_\rho \tilde{v}^2}\right] q - i\frac{\kappa T}{2mnv_\rho^2}\left[\frac{\partial_n(n\tilde{v})}{\tilde{v}}\right]^2 q^2. \tag{15}$$

This result demonstrates that at $\omega\tau \to 0$ the first sound mode becomes the ordinary thermodynamic sound. In particular, the first term in Eq. (15) contains a correction to the sound velocity, which simply accounts for the temperature dependence of the adiabatic compressibility of the one-dimensional quantum liquid. The second term is imaginary and thus describes attenuation of the sound mode. Indeed, in any medium, thermal conductivity gives rise to the absorption of sound. We have verified that the resulting absorption rate [25] is consistent with the second term in Eq. (15).

In summary, we have studied collective excitations of a system of one-dimensional spin-$\frac{1}{2}$ fermions at a low temperature based on a two-fluid hydrodynamic description of the system. In contrast to liquid ^{4}He, there is no superfluid condensate in our case. The two-fluid nature of the system can be understood as follows. We apply the Luttinger liquid theory to small sections of the one-dimensional system. The state of each section is described by two sets of variables: the occupation numbers of the elementary excitations and the zero modes N and J. In addition, we keep in mind that the excitations equilibrate with each other at the rather short time scale τ_{ex}, whereas their equilibration with the zero modes happens at the much longer scale τ. Thus, in the frequency range (1) the system consists of two components. The excitations form a gas, analogous to the normal component of superfluid ^{4}He, whereas the position- and time-dependent values of densities of N and J describe a second liquid, similar to the superfluid component of ^{4}He.

Our main result is that, in addition to the well-understood acoustic charge and spin excitation modes propagating at velocities v_ρ and v_σ, there is a second sound mode propagating at velocity v_2 given by Eq. (9). This mode describes the waves of entropy; its decay is small for frequencies in the range (1). In contrast to superfluid ^{4}He, at $\omega \ll \tau^{-1}$ the second sound disappears, and the heat transport becomes diffusive. Another system where the second sound exists in a finite frequency range is dielectric crystal [26,27].

Our treatment can be applied to other one-dimensional systems at low temperatures, such as a system of bosons or spin-polarized fermions. The absence of spin excitations in these systems can be accounted for by taking the limit $v_\sigma \to \infty$ in our formulas. In this case, the velocities of the first and second sound modes are both equal to v_ρ in the limit $T \to 0$. It is worth mentioning that at $v_\sigma \to \infty$ our expression (14) for thermal conductivity recovers the result for the spinless one-dimensional system obtained in Ref. [28]. Another important example is that of spin-$\frac{1}{2}$ fermions with attractive interactions. In this case, the energy spectrum of the spin excitations has a finite gap Δ at $T = 0$ [4]. Our two-mode description still applies at $\Delta \ll T$. In the opposite limit, $\Delta \gg T$, the spin excitations are frozen out, and the adaptation of the theory to the spinless case, as described above, should be made.

The existence of the second sound mode means that the heat propagation in the one-dimensional system is ballistic at sufficiently high frequencies $\omega \gg \tau^{-1}$, whereas the usual diffusive heat transport is restored at $\omega \ll \tau^{-1}$. Experimentally, such a frequency dependence of thermal transport may be observed in long ballistic quantum wires, such as those obtained by the cleaved-edge overgrowth technique [29,30]. A time-dependent temperature difference across the wire can be achieved by driving ac current through one of the leads; cf. Ref. [31].

A direct observation of both the first and second sound was recently reported in a system of ^{6}Li atoms in an elongated trap [32]. In this experiment, the system was three-dimensional, and superfluidity was achieved by tuning interactions to resonance by a magnetic field. In order to observe the second sound discussed in this Letter, one can replace the trap in Ref. [32] with an array of narrow traps that are in the one-dimensional regime [33,34].

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