Landau Phonon-Roton Theory Revisited for Superfluid ^{4}He and Fermi Gases

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Liquid helium and spin-1/2 cold-atom Fermi gases both exhibit in their superfluid phase two distinct types of excitations, gapless phonons and gapped rotons or fermionic pair-breaking excitations. In the long wavelength limit, revising and extending the theory of Landau and Khalatnikov initially developed for helium [Zh. Exp. Teor. Fiz. **19**, 637 (1949)], we obtain universal expressions for three- and four-body couplings among these two types of excitations. We calculate the corresponding phonon damping rates at low temperature and compare them to those of a pure phononic origin in high-pressure liquid helium and in strongly interacting Fermi gases, paving the way to experimental observations.

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Introduction.—Homogeneous superfluids with short-range interactions exhibit, at sufficiently low temperature, phononic excitations ϕ as the only microscopic degrees of freedom. In this universal limit, all superfluids of this type reduce to a weakly interacting phonon gas with a quasi-linear dispersion relation, irrespective of the statistics of the underlying particles and of their interaction strength. Phonon damping then only depends on the dispersion relation close to zero wave number (namely, its slope and third derivative) and on the phonon nonlinear coupling, deduced solely from the system equation of state through Landau-Khalatnikov quantum hydrodynamics [1].

In experiments, however, temperatures are not always low enough to make the dynamics purely phononic. Other elementary excitations can enrich the problem, such as spinless bosonic rotons in liquid helium 4 and spinful fermionic BCS-type pair-breaking excitations in spin-1/2 cold-atom Fermi gases. These excitations, denoted here as γ quasiparticles, exhibit in both cases an energy gap $\Delta > 0$. Remarkably, as shown by Landau and Khalatnikov [1], the phonon-roton coupling, and more generally phonon coupling to all gapped excitations as we shall see, depend to leading order in temperature only on a few parameters of the dispersion relation of the γ quasiparticles, namely the value of the minimum Δ and its location k_0 in wave number space, their derivatives with respect to density, and the effective mass m_* close to $k = k_0$. We have discovered however that the ϕ-γ coupling of Ref. [1] is not exact, a fact apparently unnoticed in the literature. Our goal here is to complete the result of Ref. [1], and to quantitatively obtain phonon damping rates due to the ϕ-γ coupling as functions of temperature, a nontrivial task in the considered strongly interacting systems. We restrict to the collisionless regime $\omega_{\mathbf{q}}\tau_\gamma \gg 1$ and $\omega_{\mathbf{q}}\tau_\phi \gg 1$, where $\omega_{\mathbf{q}}$ is the angular eigenfrequency of the considered phonon mode of wave vector $\mathbf{q}$, and τ_γ (τ_ϕ) is a typical collision time of thermal γ quasiparticles (thermal phonons). An extension to the hydrodynamic regime $\omega_{\mathbf{q}}\tau_\gamma \lesssim 1$ or $\omega_{\mathbf{q}}\tau_\phi \lesssim 1$ may be obtained from kinetic equations [2]. An experimental test of our results seems nowadays at hand, either in liquid helium 4, extending the recent work of Ref. [3], or in homogeneous cold Fermi gases, which the breakthrough of flat-bottom traps [4] allows one to prepare [5] and to acoustically excite by spatiotemporally modulated laser-induced optical potentials [6,7].

Landau-Khalatnikov theory revisited.—We recall the reasoning of Ref. [1] to get the phonon-roton coupling in liquid helium 4, extending it to the phonon-fermionic quasiparticle coupling in unpolarized spin-1/2 Fermi gases. We first treat in first quantization the case of a single roton or fermionic excitation, considered as a γ quasiparticle of position $\mathbf{r}$, momentum $\mathbf{p}$, and spin $s = 0$ or $s = 1/2$. In a homogeneous superfluid of density ρ, its Hamiltonian is given by $\epsilon(\mathbf{p}, \rho)$, an isotropic function of $\mathbf{p}$ such that $p \mapsto \epsilon(\mathbf{p}, \rho)$ is the γ-quasiparticle dispersion relation. In the presence of acoustic waves (phonons), the superfluid acquires position-dependent density $\rho(\mathbf{r})$ and velocity $\mathbf{v}(\mathbf{r})$. For a phonon wavelength large compared to the γ-quasiparticle coherence length [8], here its thermal wavelength $(2\pi\hbar^2/m_* k_B T)^{1/2}$ [9], and for a phonon angular frequency small compared to the γ-quasiparticle "internal" energy Δ, we can write the γ-quasiparticle Hamiltonian in the local density approximation [10,11]:

$$\mathcal{H} = \epsilon(\mathbf{p}, \rho(\mathbf{r})) + \mathbf{p} \cdot \mathbf{v}(\mathbf{r}). \tag{1}$$

The last term is a Doppler effect reflecting the energy difference in the lab frame and in the frame moving with the superfluid. For a weak phononic perturbation of the superfluid, we expand the Hamiltonian to second order in density fluctuations $\delta\rho(\mathbf{r}) = \rho(\mathbf{r}) - \rho$:

$$\mathcal{H} \simeq \epsilon(\mathbf{p},\rho) + \partial_\rho\epsilon(\mathbf{p},\rho)\delta\rho(\mathbf{r}) + \mathbf{p}\cdot\mathbf{v}(\mathbf{r}) + \frac{1}{2}\partial_\rho^2\epsilon(\mathbf{p},\rho)\delta\rho^2(\mathbf{r}), \tag{2}$$

not paying attention yet to the noncommutation of $\mathbf{r}$ and $\mathbf{p}$. Phonons are bosonic quasiparticles connected to the expansion of $\delta\rho(\mathbf{r})$ and $\mathbf{v}(\mathbf{r})$ on eigenmodes of the quantum-hydrodynamic equations linearized around the homogeneous solution at rest in the quantization volume $\mathcal{V}$:

$$\begin{pmatrix}\delta\rho(\mathbf{r})\\ \mathbf{v}(\mathbf{r})\end{pmatrix} = \frac{1}{\mathcal{V}^{1/2}}\sum_{\mathbf{q}\neq\mathbf{0}}\left[\begin{pmatrix}\rho_q\\ \mathbf{v}_\mathbf{q}\end{pmatrix}\hat{b}_\mathbf{q} + \begin{pmatrix}\rho_q\\ -\mathbf{v}_\mathbf{q}\end{pmatrix}\hat{b}^\dagger_{-\mathbf{q}}\right]e^{i\mathbf{q}\cdot\mathbf{r}}, \tag{3}$$

with modal amplitudes $\rho_q = [\hbar\rho q/(2mc)]^{1/2}$ and $\mathbf{v}_\mathbf{q} = [\hbar c/(2m\rho q)]^{1/2}\mathbf{q}$, m being the mass of a superfluid particle and c the sound velocity. The annihilation and creation operators $\hat{b}_\mathbf{q}$ and $\hat{b}^\dagger_\mathbf{q}$ of a phonon of wave vector $\mathbf{q}$ and energy $\hbar\omega_\mathbf{q} = \hbar c q$ obey usual commutation relations $[\hat{b}_\mathbf{q}, \hat{b}^\dagger_{\mathbf{q}'}] = \delta_{\mathbf{q},\mathbf{q}'}$.

For an arbitrary number of γ quasiparticles, we switch to second quantization and rewrite Eq. (2) as

$$\hat{H} = \sum_{\mathbf{k},\sigma}\epsilon_\mathbf{k}\hat{\gamma}^\dagger_{\mathbf{k}\sigma}\hat{\gamma}_{\mathbf{k}\sigma} + \sum_{\mathbf{k},\mathbf{k}',\mathbf{q},\sigma}\frac{\mathcal{A}_1(\mathbf{k},\mathbf{q};\mathbf{k}')}{\mathcal{V}^{1/2}}(\hat{\gamma}^\dagger_{\mathbf{k}'\sigma}\hat{\gamma}_{\mathbf{k}\sigma}\hat{b}_\mathbf{q} + \mathrm{H.c.}) \times\delta_{\mathbf{k}+\mathbf{q},\mathbf{k}'} + \sum_{\mathbf{k},\mathbf{k}',\mathbf{q},\mathbf{q}',\sigma}\frac{\mathcal{A}_2(\mathbf{k},\mathbf{q};\mathbf{k}',\mathbf{q}')}{\mathcal{V}}\hat{\gamma}^\dagger_{\mathbf{k}'\sigma}\hat{\gamma}_{\mathbf{k}\sigma}\delta_{\mathbf{k}+\mathbf{q},\mathbf{k}'+\mathbf{q}'} \times\left[\hat{b}^\dagger_{\mathbf{q}'}\hat{b}_\mathbf{q} + \frac{1}{2}(\hat{b}_{-\mathbf{q}'}\hat{b}_\mathbf{q} + \mathrm{H.c.})\right], \tag{4}$$

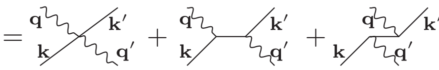
where $\hat{\gamma}_{\mathbf{k}\sigma}$ and $\hat{\gamma}^\dagger_{\mathbf{k}\sigma}$ are bosonic (rotons, $s=0$, $\sigma=0$) or fermionic ($s=1/2$, $\sigma=\uparrow,\downarrow$) annihilation and creation operators of a γ quasiparticle of wave vector $\mathbf{k}=\mathbf{p}/\hbar$ in spin component σ, obeying usual commutation or anticommutation relations. The first sum in the right-hand side of Eq. (4) gives the γ-quasiparticle energy in the unperturbed superfluid, with $\epsilon_\mathbf{k}\equiv\epsilon(\hbar\mathbf{k},\rho)$. The second sum, originating from the Doppler term and the term linear in $\delta\rho$ in Eq. (2), describes absorption or emission of a phonon by a γ quasiparticle, characterized by the amplitude

$$\mathcal{A}_1(\mathbf{k},\mathbf{q};\mathbf{k}') = \rho_q\frac{\partial_\rho\epsilon_\mathbf{k} + \partial_\rho\epsilon_{\mathbf{k}'}}{2} + \mathbf{v}_\mathbf{q}\cdot\frac{\hbar\mathbf{k}+\hbar\mathbf{k}'}{2}, \tag{5}$$

where $\mathbf{q}$, $\mathbf{k}$, and $\mathbf{k}'$ are the wave vectors of the incoming phonon and the incoming and outgoing γ quasiparticles. Equation (5) is invariant under exchange of $\mathbf{k}$ and $\mathbf{k}'$. This results from symmetrization of the various terms, in the form $[f(\mathbf{p})e^{i\mathbf{q}\cdot\mathbf{r}} + e^{i\mathbf{q}\cdot\mathbf{r}}f(\mathbf{p})]/2$ with $\mathbf{r}$ and $\mathbf{p}$ canonically conjugated operators, ensuring that the correct form of Eq. (2) is Hermitian. The third sum in Eq. (4), originating from the terms quadratic in $\delta\rho$ in Eq. (2), describes direct scattering of a phonon on a γ quasiparticle, with the symmetrized amplitude

$$\mathcal{A}_2(\mathbf{k},\mathbf{q};\mathbf{k}',\mathbf{q}') = \rho_q\rho_{q'}\frac{\partial^2_\rho\epsilon_\mathbf{k} + \partial^2_\rho\epsilon_{\mathbf{k}'}}{2}, \tag{6}$$

where the primed wave vectors are the ones of emerging quasiparticles. It also describes negligible two-phonon absorption and emission. The effective amplitude for ϕ-γ scattering is obtained by adding the contributions of the direct process (terms of $\hat{H}$ quadratic in $\hat{b}$) and of the absorption-emission or emission-absorption process (terms linear in $\hat{b}$) treated to second order in perturbation theory [1]:

$\mathcal{A}_2^{\mathrm{eff}}(\mathbf{k},\mathbf{q};\mathbf{k}',\mathbf{q}') =$ 

$$= \mathcal{A}_2(\mathbf{k},\mathbf{q};\mathbf{k}',\mathbf{q}') + \frac{\mathcal{A}_1(\mathbf{k},\mathbf{q};\mathbf{k}+\mathbf{q})\mathcal{A}_1(\mathbf{k}',\mathbf{q}';\mathbf{k}'+\mathbf{q}')}{\hbar\omega_\mathbf{q}+\epsilon_\mathbf{k}-\epsilon_{\mathbf{k}+\mathbf{q}}} + \frac{\mathcal{A}_1(\mathbf{k}-\mathbf{q}',\mathbf{q}';\mathbf{k})\mathcal{A}_1(\mathbf{k}-\mathbf{q}',\mathbf{q};\mathbf{k}')}{\epsilon_\mathbf{k}-\hbar\omega_{\mathbf{q}'}-\epsilon_{\mathbf{k}-\mathbf{q}'}} \tag{7}$$

where in the second (third) term the γ quasiparticle first absorbs phonon $\mathbf{q}$ (emits phonon $\mathbf{q}'$) then emits phonon $\mathbf{q}'$ (absorbs phonon $\mathbf{q}$). Up to this point this agrees with Ref. [1], except that the first derivative $\partial_\rho\Delta$ in Eq. (5), thought to be anomalously small in low-pressure helium, was neglected in Ref. [1]. Equation (7), issued from a local density approximation, holds to leading order in a low-energy limit. We then take the $T\to 0$ limit with scaling laws

$$q\approx T, \qquad k-k_0\approx T^{1/2} \tag{8}$$

reflecting the fact that the thermal energy of a phonon is $\hbar c q\approx k_B T$ and the effective kinetic energy of a γ quasiparticle, that admits the expansion

$$\epsilon_\mathbf{k} - \Delta \underset{k\to k_0}{=} \frac{\hbar^2(k-k_0)^2}{2m_*} + O(k-k_0)^3, \tag{9}$$

is also $\approx k_B T$. The coupling amplitudes $\mathcal{A}_1$ and energy denominators in Eq. (7) must be expanded up to relative corrections of order T [12]. On the contrary, it suffices to expand $\mathcal{A}_2$ to leading order T in temperature. We hence get our main result, the effective coupling amplitude of the ϕ-γ scattering to leading order in temperature:

$$\mathcal{A}_2^{\mathrm{eff}}(\mathbf{k},\mathbf{q};\mathbf{k}',\mathbf{q}') \underset{T\to 0}{\sim} \frac{\hbar q}{mc\rho}\left(\frac{1}{2}\rho^2\Delta'' + \frac{(\hbar\rho k_0')^2}{2m_*} + \frac{\hbar^2k_0^2}{2m_*} \times\left\{\left(\frac{\rho\Delta'}{\hbar c k_0}\right)^2 uu' + \frac{\rho\Delta'}{\hbar c k_0}\left[(u+u')\left(uu'-\frac{\rho k_0'}{k_0}\right) + \frac{2m_*c}{\hbar k_0}w\right] + \frac{m_*c}{\hbar k_0}(u+u')w + u^2u'^2 - \frac{\rho k_0'}{k_0}(u^2+u'^2)\right\}\right). \tag{10}$$

Here Δ', k_0', Δ'' are first and second derivatives of Δ and k_0 with respect to ρ; $u = (\mathbf{q}\cdot\mathbf{k}/qk)$, $u' = (\mathbf{q}'\cdot\mathbf{k}/q'k)$, $w = (\mathbf{q}\cdot\mathbf{q}'/qq')$ are cosines of the angles between $\mathbf{k}$, $\mathbf{q}$, and $\mathbf{q}'$; our results hold for $k_0 = 0$ provided the limit $k_0 \to 0$ is taken in Eq. (10). In Eq. (3.17) of Ref. [1], the Δ' terms were neglected as said, but the last term in Eq. (10), with the factor $\rho k_0'/k_0$, was simply forgotten.

Damping rates.—A straightforward application of Eq. (10) is a Fermi-golden-rule calculation of the damping rate $\Gamma_{\mathbf{q}}^{\rm scat}$ of phonons $\mathbf{q}$ due to scattering on γ quasiparticles. The γ quasiparticles are in thermal equilibrium with Bose or Fermi mean occupation numbers $\bar{n}_{\gamma,\mathbf{k}} = [\exp(\epsilon_{\mathbf{k}}/k_BT) - (-1)^{2s}]^{-1}$. So are phonons in modes $\mathbf{q}' \neq \mathbf{q}$, with Bose occupation numbers $\bar{n}_{b,\mathbf{q}'} = [\exp(\hbar\omega_{\mathbf{q}'}/k_BT) - 1]^{-1}$; mode $\mathbf{q}$ is initially excited (e.g., by a sound wave) with an arbitrary number $n_{b,\mathbf{q}}$ of phonons. By including both loss $\mathbf{q}+\mathbf{k}\to\mathbf{q}'+\mathbf{k}'$ and gain $\mathbf{q}'+\mathbf{k}'\to\mathbf{q}+\mathbf{k}$ processes [13] and summing over σ, one finds that $(d/dt)n_{b,\mathbf{q}} = -\Gamma_{\mathbf{q}}^{\rm scat}(n_{b,\mathbf{q}} - \bar{n}_{b,\mathbf{q}})$, with

$$\Gamma_{\mathbf{q}}^{\rm scat} = \frac{2\pi}{\hbar}(2s+1)\int\frac{d^3k d^3q'}{(2\pi)^6}[\mathcal{A}_2^{\rm eff}(\mathbf{k},\mathbf{q};\mathbf{k}',\mathbf{q}')]^2 \times\delta(\epsilon_{\mathbf{k}}+\hbar\omega_{\mathbf{q}}-\epsilon_{\mathbf{k}'}-\hbar\omega_{\mathbf{q}'})\frac{\bar{n}_{b,\mathbf{q}'}\bar{n}_{\gamma,\mathbf{k}'}[1+(-1)^{2s}\bar{n}_{\gamma,\mathbf{k}}]}{\bar{n}_{b,\mathbf{q}}} \tag{11}$$

and $\mathbf{k}' = \mathbf{k}+\mathbf{q}-\mathbf{q}'$. As our low-energy theory only holds for $k_BT \ll \Delta$, the gas of γ quasiparticles is nondegenerate, and $\bar{n}_{\gamma,\mathbf{k}} \simeq \exp(-\epsilon_{\mathbf{k}}/k_BT) \ll 1$ in Eq. (11). By taking the $T\to 0$ limit at fixed $\hbar cq/k_BT$ and setting $\mathcal{A}_2^{\rm eff} = (\hbar\omega_{\mathbf{q}}/\rho)f$, where the dimensionless quantity f only depends on angle cosines, we obtain the equivalent

$$\hbar\Gamma_{\mathbf{q}}^{\rm scat} \underset{T\to 0}{\sim} (2s+1)\frac{e^{-\Delta/k_BT}}{(2\pi)^{9/2}}\frac{k_0^2q^4c}{\rho^2}(m_*k_BT)^{1/2}I, \tag{12}$$

with $I = \int d^2\Omega_{\mathbf{k}}\int d^2\Omega_{\mathbf{q}'}f^2(u,u',w)$ an integral over solid angles of direction $\mathbf{k}$ and $\mathbf{q}'$ [14].

One proceeds similarly for the calculation of the damping rate $\Gamma_{\mathbf{q}}^{a\,{\rm or}\,e}$ of phonons $\mathbf{q}$ due to absorption $\mathbf{q}+\mathbf{k}\to\mathbf{k}'$ or emission $\mathbf{k}'\to\mathbf{q}+\mathbf{k}$ processes by thermal equilibrium γ quasiparticles. We obtain

$$\Gamma_{\mathbf{q}}^{a\,{\rm or}\,e} = \frac{2\pi}{\hbar}(2s+1)\int\frac{d^3k}{(2\pi)^3}[\mathcal{A}_1(\mathbf{k},\mathbf{q};\mathbf{k}')]^2 \times\delta(\hbar\omega_{\mathbf{q}}+\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}'})(\bar{n}_{\gamma,\mathbf{k}}-\bar{n}_{\gamma,\mathbf{k}'}), \tag{13}$$

with $\mathbf{k}' = \mathbf{k}+\mathbf{q}$. Low degeneracy of the γ quasiparticles and energy conservation allow us to write $\bar{n}_{\gamma,\mathbf{k}} - \bar{n}_{\gamma,\mathbf{k}'} \simeq \exp(-\epsilon_{\mathbf{k}}/k_BT)/(1+\bar{n}_{b,\mathbf{q}})$. Energy conservation leads here to a scaling on k different from Eq. (8) as it forces k to be at a nonzero distance from k_0, even in the low-phonon-energy limit: When $q\to 0$ at fixed $\mathbf{k}$, the Dirac delta in Eq. (13) becomes

$$\delta(\hbar\omega_{\mathbf{q}}+\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}'}) \underset{q\to 0}{\sim} (\hbar cq)^{-1}\delta\left(1-u\frac{\frac{d\epsilon_k}{dk}}{\hbar c}\right) \tag{14}$$

and imposes that the group velocity $(1/\hbar)(d\epsilon_k/dk)$ of the incoming γ quasiparticle is larger in absolute value than that, c, of the phonons. This condition, reminiscent of Landau's criterion, restricts wave number k to a domain D not containing k_0. In the low-q limit, that is for q much smaller than the k significantly contributing to Eq. (13), but with no constraint on the ratio $\hbar cq/k_BT$, we write $\mathcal{A}_1$ in Eq. (5) to leading order $q^{1/2}$ in q, and integrate over the direction of $\mathbf{k}$, to obtain

$$\Gamma_{\mathbf{q}}^{a\,{\rm or}\,e} \simeq \frac{(2s+1)\rho}{4\pi mc}\int_D\frac{dkk^2}{|\frac{d\epsilon_k}{dk}|}\frac{e^{-\epsilon_{\mathbf{k}}/k_BT}}{1+\bar{n}_{b,\mathbf{q}}}\left|\partial_\rho\epsilon_k+\frac{\hbar^2c^2k}{\rho\frac{d\epsilon_k}{dk}}\right|^2 \tag{15}$$

$$\underset{T\to 0}{\sim}\frac{(2s+1)\rho k_*^2}{4\pi\hbar^2mc^3}\left|\partial_\rho\epsilon_{k_*}+\frac{\hbar ck_*}{\rho\eta_*}\right|^2\frac{k_BTe^{-\epsilon_{k_*}/k_BT}}{1+\bar{n}_{b,\mathbf{q}}}. \tag{16}$$

Equation (16) is an equivalent when $T\to 0$ at fixed $\hbar cq/k_BT$; k_* is the element of the border of D $((d\epsilon_k/dk)|_{k=k_*} = \eta_*\hbar c,\ \eta_* = \pm)$ with minimal energy ϵ_k (when more than one of such k_* exists, one has to sum their contributions). As $\epsilon_{k_*} > \Delta$, the damping rate due to scattering dominates the one due to absorption-emission in the mathematical limit $T\to 0$; we shall see, however, that this is not always so for typical temperatures in current experiments.

To be complete, we give a low-temperature equivalent of the damping rate of the γ quasiparticle $\mathbf{k}$ due to interaction with thermal phonons. With $k-k_0 = O(T^{1/2})$ as in Eq. (8), we find $\hbar\Gamma_{\mathbf{k}}^{\gamma\phi} \sim (\pi I/42)(k_BT)^7/(\hbar c\rho^{1/3})^6$, where the factor $2s+1$ is gone (no summation over σ is needed) but I is the same angular integral as in Eq. (12). Here scattering dominates [15]. Using $\tau_\gamma \simeq 1/\Gamma_{\mathbf{k}}^{\gamma\phi}$, we checked that the Figs. 1 and 2 below are in the collisionless regime $\omega_{\mathbf{q}}\tau_\gamma \gg 1$. Similarly, we checked that $\omega_{\mathbf{q}}\tau_\phi \gg 1$ on the figures.

Application to helium.—Precise measurements of the equation of state (relating ρ to pressure) and of the roton dispersion relation for various pressures were performed in liquid ^{4}He at low temperature ($k_BT \ll mc^2, \Delta$). They give access to the parameters k_0, Δ, their derivatives, and m_*. The measured sound velocities agree with the thermodynamic relation $mc^2 = \rho(d\mu/d\rho)$, where μ is the zero-temperature chemical potential of the liquid. We plot in Fig. 1 the phonon damping rates $\Gamma_{\mathbf{q}}$ as functions of temperature, at fixed angular frequency $\omega_{\mathbf{q}}$. At the chosen high pressure, the phonon dispersion relation is concave at low q; therefore, the Beliaev-Landau [16–21] three-phonon process $\phi\leftrightarrow\phi\phi$ is energetically forbidden and the Landau-Khalatnikov [1,6,21] process $\phi\phi\leftrightarrow\phi\phi$ is dominant. Our high yet experimentally accessible [22,23] value of $\omega_{\mathbf{q}}$ leads to sound attenuation lengths $2c/\Gamma_{\mathbf{q}}$ short enough to be measured in centimetric cells. As visible on Fig. 1, sound

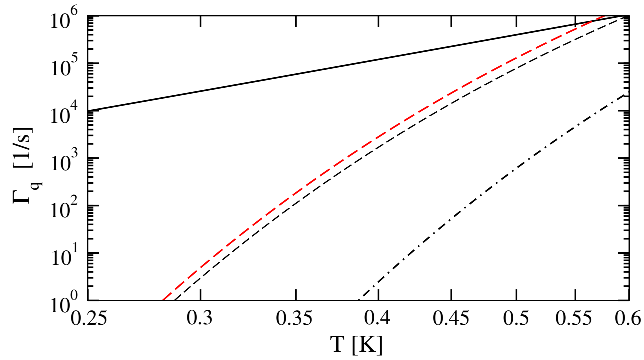


FIG. 1. Phonon damping rates at angular frequency $\omega_{\mathbf{q}} = 2\pi \times 165$ GHz ($q = 0.3$ Å^{-1}) in liquid ^{4}He at pressure $P = 20$ bar as functions of temperature. Solid line: purely phononic damping $\Gamma_{\phi\phi}$ due to Landau-Khalatnikov four-phonon processes [1,6,21]; it depends on the curvature parameter γ defined as $\omega_{\mathbf{q}} = cq[1 + (\gamma/8)(\hbar q/mc)^2 + O(q^4)]$. Interpolating measurements of $P \mapsto \gamma(P)$ in Refs. [25,26] gives $\gamma = -6.9$. Dashed black line [dash-dotted black line]: damping due to scattering [absorption or emission] by rotons, see Eq. (12) [Eq. (15)]. Red dashed line: original formula of Ref. [1] for the damping rate due to phonon-roton scattering. The roton parameters are extracted from their dispersion relation $k \mapsto \epsilon_{\mathbf{k}}$ measured at various pressures [27]: $\Delta/k_B = 7.44$ K, $k_0 = 2.05$ Å^{-1}, $m_*/m = 0.11$, $\rho k_0'/k_0 = 0.39$, $\rho\Delta'/\Delta = -1.64$, $\rho^2\Delta''/\Delta = -8.03$, $\rho m_*'/m_* = -4.7$. In Eq. (15), parabolic approximation Eq. (9) is used (hence, $\epsilon_{k_*}/\Delta \simeq 1.43$). The speed of sound $c = 346.6$ m/s, and the Grüneisen parameter $d\ln c/d\ln\rho = 2.274$ entering in $\Gamma_{\phi\phi}$, are taken from equation of state Eq. (A1) of Ref. [28]. The low values $\hbar q/mc = 0.13$ and $k_BT/mc^2 < 10^{-2}$ justify our use of quantum hydrodynamics.

damping is actually dominated by four-phonon Landau-Khalatnikov processes up to temperatures $T \simeq 0.6$ K. In this regime one would directly observe this phonon-phonon damping mechanism, which would be a premiere. The sound attenuation measurements of Ref. [24] in helium at 23 bars and $\omega_{\mathbf{q}} = 2\pi \times 1.1$ GHz are indeed limited to $T > 0.8$ K where damping is still dominated by rotons.

Application to fermions.—In cold-atom Fermi gases, interactions occur in s wave between opposite-spin atoms. Of negligible range, they are characterized by the scattering length a tunable by Feshbach resonance [29–34].

Precise measurements of the fermionic excitation parameters k_0 and Δ were performed at unitarity $a^{-1} = 0$ [35]. Because of the unitary-gas scale invariance [36–38], k_0 is proportional to the Fermi wave number $k_F = (3\pi^2\rho)^{1/3}$, $k_0 \simeq 0.92 k_F$ [35], and Δ is proportional to the Fermi energy $\epsilon_F = \hbar^2 k_F^2/2m$, $\Delta \simeq 0.44\epsilon_F$ [35]. This also determines their derivatives with respect to ρ. Similarly, the equation of state measured at $T = 0$ is simply $\mu = \xi\epsilon_F$, where $\xi \simeq 0.376$ [34], and the critical temperature is $T_c \simeq 0.167\epsilon_F/k_B$ [34]. For the effective mass of the fermionic excitations and their dispersion relation at nonvanishing $k - k_0$, we must rely on results of a dimensional $\epsilon = 4 - d$ expansion, $m_*/m \simeq 0.56$ and $\epsilon_k \simeq \Delta + [\hbar^2(k^2 - k_0^2)^2/8m_* k_0^2]$ [39]. We also trust

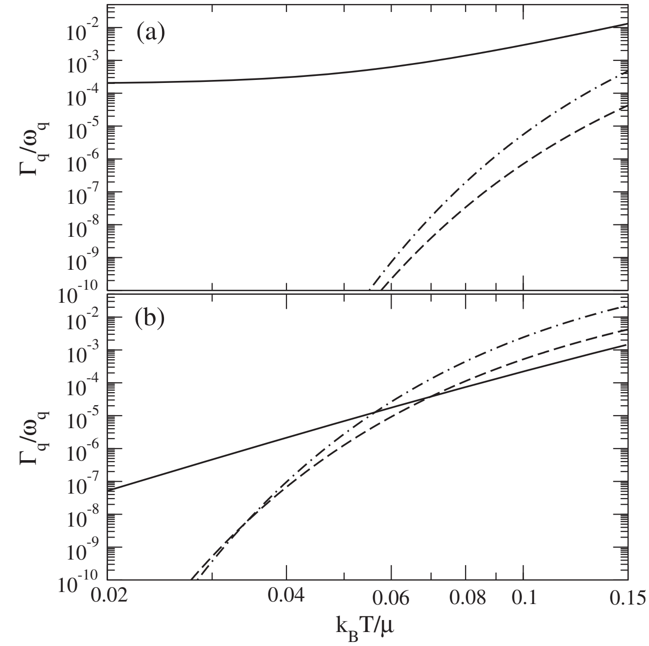


FIG. 2. Phonon damping rates at wave number $q = mc/2\hbar$ in unpolarized homogeneous cold-atom Fermi gases in thermodynamic limit as functions of temperature. (a) At unitarity $a^{-1} = 0$, where most parameters of the phonons and fermionic quasiparticles are measured (see text). (b) On the BCS side $1/k_Fa = -0.389$, these parameters are estimated in BCS theory ($\mu/\epsilon_F \simeq 0.809$, $\Delta/\mu \simeq 0.566$, $m_*/m = \Delta/2\mu$, $\rho\mu'/\mu \simeq 0.602$, $\rho\Delta'/\Delta \simeq 0.815$, $\rho^2\Delta''/\Delta \simeq -0.209$, $d\ln c/d\ln\rho \simeq 0.303$). In both cases the curvature parameter γ defined in the caption of Fig. 1 is estimated in the RPA [42]. Solid line: phonon-phonon (a) Beliaev-Landau damping $\phi \leftrightarrow \phi\phi$ (for $\gamma > 0$) as in Eqs. (121) and (122) of Ref. [21] (independent of $|\gamma|$) and (b) Landau-Khalatnikov damping $\phi\phi \leftrightarrow \phi\phi$ (for $\gamma \simeq -0.30 < 0$) [6,21]. Dashed line [dash-dotted line]: scattering [absorption or emission] phonon-fermionic quasiparticle processes, as in Eq. (12) [Eq. (15)]. In Eq. (15), we took for ϵ_k (a) the form proposed in Ref. [39] (hence, $\epsilon_{k_*}/\Delta \simeq 1.12$) and (b) the BCS form (hence, $\epsilon_{k_*}/\Delta \simeq 1.14$). μ is the $T = 0$ gas chemical potential, and the plotted quantities are in fact inverse quality factors. Here $k_BT/mc^2 > 0.03$ in contrast to Fig. 1 where $k_BT/mc^2 < 0.01$: cold atoms are effectively farther from the $T \to 0$ limit than liquid helium, hence the inversion of the $\Gamma_{\mathbf{q}}^{\rm scat}$-$\Gamma_{\mathbf{q}}^{a\,{\rm or}\,e}$ hierarchy.

Anderson's RPA prediction [40,41] that the $q = 0$ third derivative of the phononic dispersion relation is positive [42]. The damping rates of phonons with wave number $q = mc/2\hbar$ are plotted in Fig. 2(a). The contribution of the three-phonon Landau-Beliaev processes $\phi \leftrightarrow \phi\phi$, here energetically allowed, is dominant; it is computed in the quantum-hydrodynamic approximation where it is independent of the aforementioned third derivative.

The phononic excitation branch becomes concave in the BCS limit $k_Fa \to 0^-$ [43]. As visible on Fig. 2(b), the phonon-phonon damping (now governed by the Landau-Khalatnikov processes mentioned earlier) is much weaker, and dominates the ϕ-γ damping only at very low temperatures. At commonly reached temperatures $T > 0.05\epsilon_F/k_B$ [44], the damping is in fact dominated by absorption-emission ϕ-γ processes which, unlike in liquid helium, prevail over scattering ones because of the smaller value of ϵ_{k_*}/Δ. Although the associated quality factors $\omega_{\mathbf{q}}/\Gamma_{\mathbf{q}}$ may seem impressive, the mode lifetimes $\Gamma_{\mathbf{q}}^{-1}$ do not exceed 1 s in a gas of ^{6}Li with a typical Fermi temperature $T_F = 1\ \mu$K, which is shorter than what was observed in a Bose-Einstein condensate [45]. Our predictions, less quantitative than on Fig. 2(a), are based on the BCS approximation for the equation of state and the fermionic excitation dispersion relation $\epsilon_k \simeq \epsilon_k^{\rm BCS} = [(\hbar^2 k^2/2m - \mu)^2 + \Delta_{\rm BCS}^2]^{1/2}$ and on the RPA for the $q = 0$ third derivative of ω_q (whose precise value matters here). A cutting remark on Ref. [46]: even in the BCS approximation to which it is restricted, we disagree with its expression of $\Gamma_{\mathbf{q}}^{a\ \mathrm{or}\ e}$.

Conclusion.—By complementing the local density approximation in Ref. [1] with a systematic low-temperature expansion, we derived the definitive leading order expression of the phonon-roton coupling in liquid helium and we generalized it to the phonon-pair-breaking excitation coupling in Fermi gases. The ever-improving experimental technics in these systems give access to the microscopic parameters determining the coupling and allow for a verification in the near future. Our result also clarifies the regime of temperature and interaction strength in which the purely phononic $\phi\phi \leftrightarrow \phi\phi$ Landau-Khalatnikov sound damping in a superfluid, unobserved to this day, is dominant.

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[13] We use energy conservation and relations $1 + (-1)^{2s}\bar{n} = e^{\epsilon/k_B T}\bar{n}$ to transform the difference of the gain and loss quantum statistical factors.
[14] $I/(4\pi)^2 = (\hbar^2 k_0^2/2m_* mc^2)^2\{\frac{1}{25} - (4\alpha/15) + \frac{28}{45}\alpha^2 + (2\beta^2/9) + A[\frac{2}{9} - (4\alpha/3)] + A^2 + 4\beta B[\frac{1}{15} - (\alpha/9)] + B^2[\frac{2}{15} - (4\alpha/9) + (2\alpha^2/3) + (4\beta^2/3)] + (4\beta/9)B^3 + (B^4/9)\}$, with $\alpha = (\rho k_0'/k_0)$, $\beta = (m_* c/\hbar k_0)$, $A = [m_* \rho^2 \Delta''/(\hbar k_0)^2] + \alpha^2$, $B = (\rho\Delta'/\hbar c k_0)$.
[15] At low T, k is close to k_0, emission $\mathbf{k} \leftrightarrow \mathbf{q} + \mathbf{k}'$ is forbidden by energy conservation, and absorption $\mathbf{k} + \mathbf{q} \leftrightarrow \mathbf{k}'$, conserving energy for $q \geq q_* \simeq 2m_* c/\hbar$, is $O(e^{-\hbar\omega_{q_*}/k_B T})$.
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