Instability of Insulators near Quantum Phase Transitions

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(Received 24 June 2016; revised manuscript received 2 August 2017; published 13 December 2017)

Thin films of amorphous indium oxide undergo a magnetic field driven superconducting to insulator quantum phase transition. In the insulating phase, the current-voltage characteristics show large current discontinuities due to overheating of electrons. We show that the onset voltage for the discontinuities vanishes as we approach the quantum critical point. As a result, the insulating phase becomes unstable with respect to any applied voltage making it, at least experimentally, immeasurable. We emphasize that unlike previous reports of the absence of linear response near quantum phase transitions, in our system, the departure from equilibrium is discontinuous. Because the conditions for these discontinuities are satisfied in most insulators at low temperatures, and due to the decay of all characteristic energy scales near quantum phase transitions, we believe that this instability is general and should occur in various systems while approaching their quantum critical point. Accounting for this instability is crucial for determining the critical behavior of systems near the transition.

DOI: 10.1103/PhysRevLett.119.247001

The superconducting insulator transition (SIT) [1,2] observed in highly disordered superconductors is a quantum phase transition (QPT) [3] driven by varying the magnetic field (B) [4–6], disorder [7], film thickness [8], or charge density [9].

In the B-driven SIT, beyond the critical B (B_c), Cooper pairs persist and become spatially localized [10–16], leading to a strongly insulating state [5,12,14,17–19]. In this insulating state, at low T ($T < 200$ mK), the current-voltage characteristics (I-V's) exhibit large I discontinuities (ΔI) [13] (Fig. 1). The I-V's can be separated into two regions, a high resistance (HR) state at low V and a low resistance (LR) state at high V. We denote the threshold V where the HR → LR (LR → HR) occurs as V_{escape} (V_{trap}).

Recently Altshuler *et al.* showed that the ΔI's could be explained by a thermal bistability where the electrons can thermally decouple from the phonon bath resulting in a well-defined electron T (T_{el}) [20]. The central assumption of this model is that deviations from a linear I-V result from an increase in T_{el}. The steady state T_{el} is determined by the heat balance between the experimentally applied Joule heating ($I \cdot V$) and cooling via the phonons. This nonequilibrium state is analyzed by solving the heat-balance equation

$$\frac{V^2}{R(T_{\mathrm{el}})} = \Gamma\Omega(T_{\mathrm{el}}^{\beta} - T_{\mathrm{ph}}^{\beta}), \qquad (1)$$

where Ω is the volume of the sample, Γ is the electron-phonon coupling strength, β is an exponent that determines the power-law decay of the electron-phonon coupling as $T \to 0$ and $R(T_{\mathrm{el}}) = R_0 \exp[(\Delta/T_{\mathrm{el}})^{\gamma}]$, typical of insulators (Δ is the insulator's activation energy and γ is typically ≤ 1).

The central result of this model is that, below a critical phonon T ($T_{\mathrm{ph}}^{\mathrm{cr}}$), Eq. (1) has two stable solutions for T_{el}. The ΔI's are a result of a change in R that occurs when the electrons abruptly switch between the low T_{el} solution, where the HR state exists, and the high T_{el} solution. This electron-heating approach gained support from several experiments [21–23].

At first sight, the far-from-equilibrium ΔI's and the underlying electron-phonon decoupling appear to be not

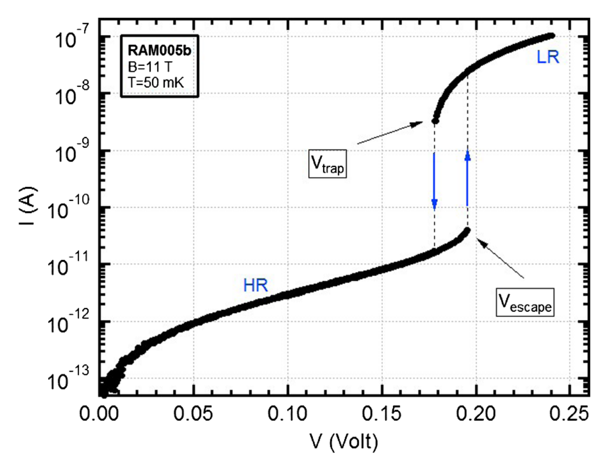


FIG. 1. Discontinuities in the I-V's. I (log scale) vs V measured at $T = 50$ mK and $B = 11$ T (in the insulating phase). The measured data points are marked by full circles, the dashed line connecting the data points is a guide for the eye. The LR → HR (HR → LR) transition is marked by a blue arrow pointing down (up). At $V = 0$ the sample is in the HR state. By increasing V the I-V's exhibit a discontinuity at $V_{\mathrm{escape}} = 0.195$ V, where I jumps by 3 orders of magnitude. Decreasing V results in a hysteresis where I drops back to the HR state at $V_{\mathrm{trap}} = 0.178$ V.

relevant to the study of the equilibrium phases, and of the SIT itself. The main conclusion from the results presented in this Letter is that near the quantum critical point (QCP) of the SIT this is not the case. By systematically following the B evolution of ΔI we show that V_{escape} vanishes as $B \to B_c$. Consequently, close enough to B_c, the finite V required for transport measurements will inevitably exceed V_{escape}, driving the system out of equilibrium with a discontinuous transition to the high T_{el} state. The significance of the discontinuous departure from equilibrium is that not only the equilibrium state cannot be measured, but near the QCP, where like all other energy scales $V_{\mathrm{escape}} \to 0$, it is not experimentally possible to extrapolate the equilibrium properties from the measurable R.

Our data were obtained by measuring thin films of amorphous indium oxide. The films were deposited by e-gun evaporation of In_2O_3 onto a SiO_2 substrate in an O_2 rich environment. Both samples have a Hall-bar geometry, their lengths and widths are $2 \times 0.5\ \mathrm{mm}^2$ (sample RAM005b) and $10 \times 5\ \mu\mathrm{m}^2$ (BT1c) and their thickness is 30 nm (see Sec. S2 of the Supplemental Material [24] for a discussion about the contact). The data presented were measured in a two-probe dc configuration, which agrees with our four-terminal measurements in their overlapping regime of applicability.

The main results of our work are summarized in Fig. 2, were we display V_{escape} vs $\delta B \equiv B - B_c/B_c$ at our base $T = 11$ mK. On this log-log plot V_{escape} follows a power law that spans almost two decades in δB and four decades in V_{escape}, indicating that V_{escape} vanishes upon approaching B_c,

$$V_{\mathrm{escape}}(B) \propto (B - B_c)^{\alpha}, \tag{2}$$

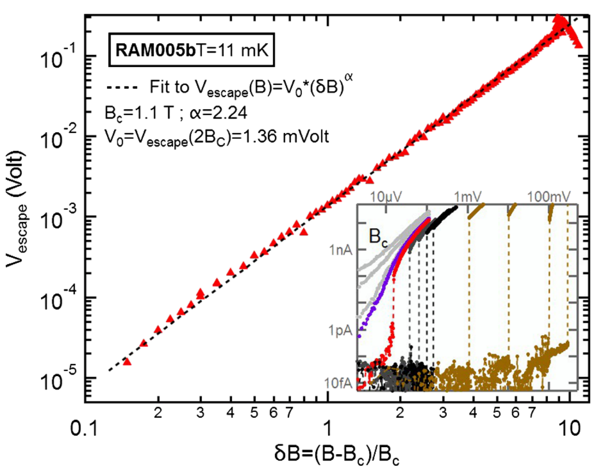


FIG. 2. Magnetic-field evolution of V_{escape}. V_{escape} vs $\delta B \equiv B - B_c/B_c$ (log-log scale) at $T = 11$ mK. The data are presented as red triangles while a dashed black line marks a power-law fit. Inset: I vs V (log-log scale) at 11 mK measured at $B = 10$, 8, 3.5, 2 T (brown I-V's), $B = 1.45$, 1.4, 1.35, 1.3 T (black), $B = 1.25$ T (red), $B = 1.225$ T (purple), and $B = 1.2$, 1.15, 1.1 T (gray).

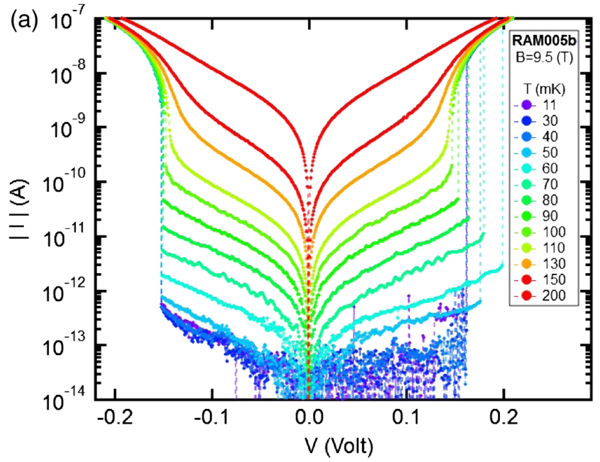
where $\alpha = 2.24$ is extracted using a power-law fit (α appears to be nonuniversal: it is sample and T dependent).

Equation (2) reflects the inherent experimental difficulty one is faced with when measuring equilibrium properties near B_c. While conducting transport measurements, it is essential to apply a finite V across the sample. This applied V must exceed the noise present during the experiment (either instrumental or inherent such as Johnson-Nyquist noise) and must also be large enough to induce a measurable I response from the sample (typically $V > \mu$V). The vanishing of V_{escape} suggests a losing cause: whatever small V is, there will always be a B range, close to the SIT, where it will exceed V_{escape} and drive the system out of equilibrium.

In the inset of Fig. 2 we display I-V's from which the data of the main figure were extracted. Close to B_c (black color) V_{escape} decreases down to $B = 1.25$ T (red), which is the lowest B where a discontinuity was observed (at $V_{\mathrm{escape}} = 15\ \mu$V). According to the power-law fit of Eq. (2), $V_{\mathrm{escape}}(B = 1.225\ \mathrm{T}) \sim 9\ \mu$V, but, at $B = 1.225$ T (purple) the data already appear continuous. A possible reason for this is that for this B range, the measurement $T = 11$ mK might become larger than $T_{\mathrm{ph}}^{\mathrm{cr}}$. As we discuss in Sec. S1 of the Supplemental Material [24] this is unlikely. A more probable explanation is that the integrated voltage-noise surpasses V_{escape}, the I-V's will appear continuous and measure only the LR state.

It is known that systems exhibit a nonlinear response near QPTs [3,25–27] where, at $T = 0$, any finite V will drive them out of equilibrium. The pivotal difference reported in this Letter is that, not only our system has no linear response, but it departs from equilibrium in a discontinuous fashion. In the discussion section we present several consequences of this discontinuous response.

Because, strictly speaking, the QPT takes place at $T = 0$ it is worthwhile to examine the T evolution of V_{escape}. If V_{escape} increases sufficiently as $T \to 0$, at low enough T's, the HR state might span a large V interval and become measurable. In Fig. 3(a) we display the I-V's measured at $B = 9.5$ T at different T's. Below $T = 100$ mK, the I-V's become discontinuous. As predicted [20], V_{trap} (the ΔI at $V < 0$) is nearly independent of the phonon T (T_{ph}). V_{escape}, on the other hand, initially increases as T_{ph} is reduced down to $T_{\mathrm{ph}} = 60$ mK (green to light blue) as expected. As T_{ph} is further lowered (dark blue), the trend changes and V_{escape} begins to decrease. In Fig. 3(b) we display V_{escape} and V_{trap} (up and down pointing triangles respectively) vs T. It appears that $\lim_{T_{\mathrm{ph}} \to 0} V_{\mathrm{escape}}(T_{\mathrm{ph}}) \sim V_{\mathrm{trap}}$. In the inset of Fig. 3(b) we display V_{escape} and V_{trap} vs T at various B's. For all measured B's, V_{escape} follows a similar pattern. These results suggest that lowering T will not increase V_{escape} and make the HR state measurable. In the discussion we show that this T dependence of V_{escape} can be explained by considering some inhomogeneity in the system.



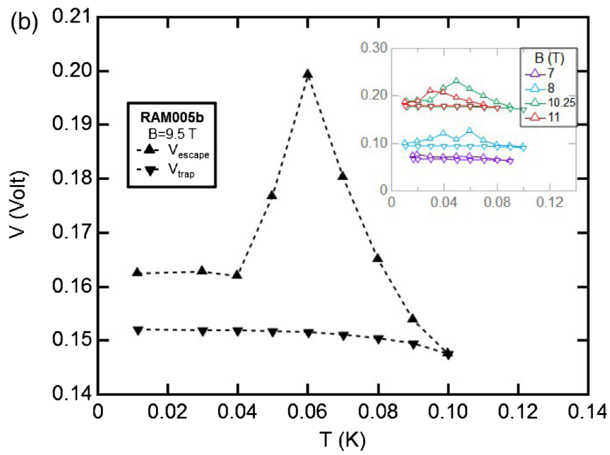


FIG. 3. Saturation of $\boldsymbol{V_{\mathrm{escape}}}$ as $\boldsymbol{T_{\mathrm{ph}} \to 0}$. (a) $|I|$ (log scale) vs V measured at $B = 9.5$ T. The color coding describes different T_{ph} isotherms ranging from 11 (purple) to 200 mK (red). At $T_{\mathrm{ph}} = 100$ mK, the I-V's become discontinuous with $V_{\mathrm{escape}} \sim 150$ mV. While cooling, V_{escape} initially increases up to $V_{\mathrm{escape}} = 200$ mV at $T_{\mathrm{ph}} = 60$ mK. At lower T_{ph}, V_{escape} drops and saturates at a finite V. (b) V_{escape} and V_{trap} vs T_{ph} at $B = 9.5$ T. V_{escape} and V_{trap} are marked by up and down pointing triangles, respectively. At low T's, V_{escape} saturates at a value which is comparable to V_{trap}. Inset: V_{escape} and V_{trap} vs T_{ph} at different B's.

An important question is how does the magnitude of the ΔI's evolve while approaching the QCP. If the ΔI's vanishes sufficiently fast, the transition from the LR to the HR states becomes practically continuous in the sense that one could extrapolate equilibrium, HR, properties from the LR state. The B dependence of the ΔI's at $T = 11$ mK is displayed in Fig. 4, where we focus on the trapping side where the LR $\to$ HR transition occurs. The blue triangles correspond to the left (blue) axis and represent I on both sides of V_{trap}, where upwards pointing triangles correspond to the last measured I in the LR state before the jump (I_{LR}) and downwards pointing triangles stand for I_{HR}, the first measured I in the HR state (for most B's I_{HR} was in the noise level). The red triangles mark V_{trap} and correspond to

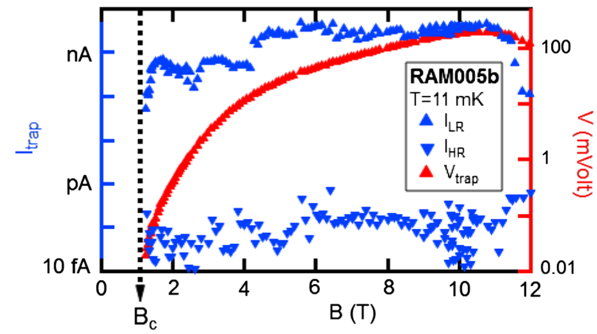


FIG. 4. $\boldsymbol{B}$ dependence of the $\boldsymbol{\Delta I}$'s. $I_{\mathrm{trap}} \equiv I(V_{\mathrm{trap}})$ and V_{trap} vs B. The blue triangles correspond to the left (blue) axis and mark I on both sides of V_{trap} where the upwards (downwards) pointing triangles correspond to the LR (HR) side of the jump. The red triangles correspond to the right (red) axis and mark V_{trap}. The vertical dashed black line marks $B_c = 1.1$ T.

the right (red) axis. While V_{trap} vanishes rapidly over a vast B range, the magnitude of the ΔI's does not seem to vary significantly. This observation has a great impact on the reliability of I-bias transport measurements as we will argue in the discussion.

Discussion.—The data presented raise several points. We demonstrated above the instability of the insulating state near the SIT. It is interesting to consider the relevance of our findings to other systems exhibiting a QPT involving insulators such as the metal insulator and the quantum Hall transitions. According to Ref. [20], $V_{\mathrm{trap}} \propto \Delta^{\beta/2}$, guaranteeing its vanishing at the SIT. Because all relevant energy scales, including Δ of the insulators, must vanish at all QCP, we expect similar inherent difficulties to arise in all QPTs involving insulators (provided that $\beta > 0$). In Sec. S6 of the Supplemental Material [24] we display preliminary results we obtained by measuring the I-V's of a silicon MOSFET sample in the insulating phase near the metal-insulator transition. These results strongly support our claim as, similarly to the SIT data, they also show discontinuous I-V's with V_{escape} vanishing while approaching the QPT.

So far we have stated the difficulties in V-biased measurements and showed that the close-to-equilibrium HR state can only be probed far from B_c. It turns out that there are similarly severe implications regarding I-biased, four-terminal measurements. These are vividly illustrated in Fig. 5, where we plot data obtained from sample BT1c at $T = 15$ mK. The red triangles are R values extracted from the $V = 0$ limit of two-probe dc I-V's and represent our best estimation of the true Ohmic R with the caveat that, close to B_c, we probably probe the LR state. In contrast, the blue line is a result of a standard, low frequency (< 10 Hz), ac four-probe measurement with $I_{\mathrm{rms}} = 1$ nA [28]. A pronounced discrepancy reaching several orders of magnitude exists between the two types of measurement for $B > B_c$. The roots of this discrepancy can be traced to Fig. 4. The 1 nA used in the four-terminal measurement in Fig. 5 falls in the

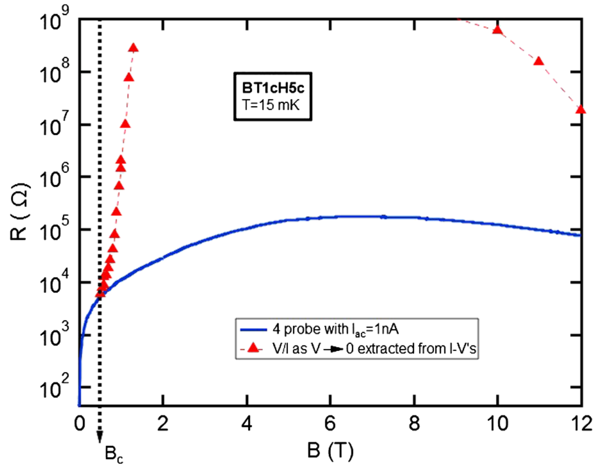


FIG. 5. Unavoidable "transport catastrophe." R (log scale) vs B of sample BT1c at $T = 15$ mK. The red triangles were extracted from the I-V's in the limit $V \to 0$. The blue line was measured using a standard four-probe ac measurement with $I_{\mathrm{rms}} = 1$ nA.

unstable regime of the I-V's and far exceeds the maximum I's allowed to observe the HR state. For sample BT1c, $I = 1$ nA is in the LR state for all $B > B_c$ (see Fig. S2a of the Supplemental Material [24] for the I-V's).

The overwhelming discrepancy in R's between the two types of measurement displayed in Fig. 5 emphasizes that, for insulators at low T's, transport data without studying the I-V's are unreliable. In Sec. S3 of the Supplemental Material [24] we show that the four-probe R can be extracted from the LR state, in S4 [24] we show that this discrepancy also occurs in larger samples, in S5 we discuss the two- and four-probe measurement configurations [24].

A notable feature in Fig. 3(b) is the nonmonotonic T dependence of V_{escape}, which initially increases on lowering T, reaches a maximum at $T = 60$ mK, then decreases and eventually saturates at low T. Altshuler *et al.* noted that the ΔI's occur within a bistability V interval (ΔV), $V_{\mathrm{trap}}^{\mathrm{min}} < V < V_{\mathrm{escape}}^{\mathrm{max}}$, where $V_{\mathrm{trap}}^{\mathrm{min}}$ and $V_{\mathrm{escape}}^{\mathrm{max}}$ are the lower and upper bounds of V_{trap} and V_{escape} satisfying the parametric dependences:

$$V_{\mathrm{trap}}^{\mathrm{min}} \propto \Delta^{\beta/2}$$
$$V_{\mathrm{escape}}^{\mathrm{max}} \propto \Delta^{-\gamma/2} T_{\mathrm{ph}}^{\beta+\gamma/2} e^{\frac{1}{2}(\Delta/T_{\mathrm{ph}})^{\gamma}}. \qquad (3)$$

Recalling that $\lim_{B\to B_c} \Delta = 0$ these equations imply that, as $T_{\mathrm{ph}} \to 0$ and $B \to B_c$, $V_{\mathrm{trap}}^{\mathrm{min}} \to 0$ and $V_{\mathrm{escape}}^{\mathrm{max}} \to \infty$. Our data show that, for $T < 60$ mK, both V_{trap} and V_{escape} occur prematurely, i.e., at the low limit of the ΔV. This can be understood by considering a mapping that we established between our nonequilibrium I discontinuities and first-order phase transitions in a van der Waals liquid [29], where the actual transition occurs not at the limit of stability but according to the Maxwell area rule. Only in near-ideal samples where no nucleation centers are found, it is possible to observe a supercooled liquid close to the limit of stability [30]. If V_{escape} is governed by a Maxwell-law equivalent, it is expected to increase upon cooling [31]. This is the case initially but, for $T < 60$ mK [Fig. 3(b)], an opposite trend develops.

These premature jumps can be explained by inhomogeneity in the samples [32] (indications for nonstructural inhomogeneity were reported in similar systems [15,19, 33–35]). At low T's, there is a competition between inefficient cooling via phonons and the small Joule heating (due to the sample's large R). While cooling is probably less sensitive to imperfections, heating may be affected near the impurities giving rise to "hot spots" that act similarly to nucleation centers in the van der Waals liquid.

We would like to note an earlier work [36] that remarks on the possibility that several observations near the SIT are due to electron heating, and shows that one cannot consider electric field and T scaling independently. They did not consider discontinuous responses.

In summary, our findings question the stability of an equilibrium, insulating state bordering the SIT. Our main result is that V_{escape}, above which only the LR state persists, vanishes as we approach the QCP ($T \to 0$ and $B \to B_c$). This seemingly innocuous behavior has far-reaching consequences on transport: Because transport measurements require a finite V in order to probe the sample, only the LR state can be accessed. In addition, any V noise will already heat the electrons and drive the sample to the LR state. A central question that remains is whether, theoretically, there is a V range where the HR state is stable.

We are grateful to S. Kravchenko for providing a Si-MOSFET sample, and to V. Kravtsov and I. Aleiner for fruitful discussions. This work was supported by the Israeli Science Foundation Grant No. 556/17.

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