Emergence of Non-Abelian Magnetic Monopoles in a Quantum Impurity Problem

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(Received 26 May 2017; published 6 December 2017)

Recently, it was shown that molecules rotating in superfluid helium can be described in terms of the angulon quasiparticles [Phys. Rev. Lett. **118**, 095301 (2017)]. Here, we demonstrate that in the experimentally realized regime the angulon can be seen as a point charge on a two-sphere interacting with a gauge field of a non-Abelian magnetic monopole. Unlike in several other settings, the gauge fields of the angulon problem emerge in the real coordinate space, as opposed to the momentum space or some effective parameter space. Furthermore, we find a topological transition associated with making the monopole Abelian, which takes place in the vicinity of the previously reported angulon instabilities. These results pave the way for studying topological phenomena in experiments on molecules trapped in superfluid helium nanodroplets, as well as on other realizations of orbital impurity problems.

DOI: 10.1103/PhysRevLett.119.235301

In Maxwell's unification of electricity and magnetism, there was one piece missing that would make the electric and magnetic forces perfectly symmetric with respect to each other—the magnetic monopoles. As demonstrated by Dirac [1], the existence of a single magnetic monopole would explain quantization of electric charge everywhere in the Universe. Since Dirac's work, the existence of magnetic monopoles—as real elementary particles or effective quasiparticles—has preoccupied physicists working in several different fields. 't Hooft [2] and Polyakov [3] demonstrated the existence of non-Abelian magnetic monopoles in the context of the unification of the fundamental interactions [4]. Despite the lack of experimental evidence for *elementary* monopoles in nature [5], collective phenomena exhibiting the behavior of magnetic monopoles have been predicted to emerge in condensed matter systems [6–10] and were subsequently observed in experiments [11–14].

As shown by Berry [15], magnetic monopoles can also emerge in an external parameter space of a simple quantum mechanical problem [16,17]. Moreover, the parameter space can be further generalized to coordinates of an interacting particle [18–20]. For example, Moody *et al.* [18] showed that the effective Hamiltonian describing nuclear rotation of a diatomic molecule can be rewritten as that of a charged particle interacting with a gauge field of a magnetic monopole. In follow-up studies the emerging gauge fields were studied in various contexts, from different types of atomic problems [21–25] to spinor Bose-Fermi mixtures [26] to the fractional quantum Hall effect [27] to Bose-Einstein condensates [8–10]. Finally, emergence of monopolelike gauge fields represents an important tool for computing topological invariants of quantum systems using Chern numbers [28], and thereby classifying the topology of the problem [8,29–32]. Such a topological classification of quantum states is particularly relevant in the context of current research on topological states of matter [33–38].

In this Letter, we demonstrate that non-Abelian magnetic monopole fields emerge in the recently introduced angulon impurity problem [39–41]. The angulon represents a quantum impurity exchanging orbital angular momentum with a many-particle bath, and it serves as a reliable model for the rotation of molecules in superfluids [42–45]. We hereby show that superfluid helium nanodroplets, which have been used as a tool of molecular spectroscopy for more than two decades [46–50], behave as effective non-Abelian magnetic monopoles with respect to molecular impurities trapped inside them. Furthermore, our analysis reveals a topological transition taking place around the previously reported "angulon instabilities" [39,51,52], where orbital angular momentum is resonantly transferred between the impurity and the bath. Such a transition corresponds to an Abelianization of the magnetic monopole.

Our approach is based on the idea of Moody *et al.* [18], which we herein extend to the case of quantum impurity problems. Let us start from the most general Hamiltonian describing a quantum impurity interacting with a many-particle environment:

$$\hat{H}(\boldsymbol{r}) = -\mu\boldsymbol{\nabla}^2 + \hat{H}_{\mathrm{mb}}(\boldsymbol{r}). \tag{1}$$

Here, the Laplacian in the generalized coordinates of the impurity, $\boldsymbol{r}$, represents the kinetic energy of the impurity. For a linearly moving impurity (the polaron problem [53]), there is $-\mu\boldsymbol{\nabla}^2 \equiv 1/(2m)\hat{\boldsymbol{P}}^2$, where m is the mass of an electron moving at momentum $\boldsymbol{P}$ (units of $\hbar \equiv 1$ are used hereafter). In the angulon problem, for a rotating impurity we have $-\mu\boldsymbol{\nabla}^2 \equiv B\hat{\boldsymbol{L}}^2$, where $\hat{\boldsymbol{L}}$ is the orbital angular momentum operator and $B = 1/(2I)$ the rotational constant with the moment of inertia I. The second term of Eq. (1) corresponds to the many-body part of the Hamiltonian, which includes the kinetic energy of the bath and the impurity-bath interactions that depend on $\boldsymbol{r}$.

Furthermore, $\hat{H}_{\rm mb}(\boldsymbol{r})$ can include any external potential, such as that due to an electromagnetic field [54–56].

The eigenvalue equation for Hamiltonian (1) can be written as

$$\hat{H}(\boldsymbol{r})|\Psi^{\alpha}(\boldsymbol{r})\rangle = E^{\alpha}|\Psi^{\alpha}(\boldsymbol{r})\rangle, \tag{2}$$

where $|\Psi^{\alpha}(\boldsymbol{r})\rangle \equiv \langle \boldsymbol{r}|\Psi^{\alpha}\rangle$ is the eigenstate in the impurity coordinate and α is the quantum number labeling the eigenstate. Next, we perform the Born-Oppenheimer (BO) expansion of the eigenstates [18–20]:

$$|\Psi^{\alpha}(\boldsymbol{r})\rangle = \sum_{n} \Phi_{n}^{\alpha}(\boldsymbol{r})|\varphi_{n}(\boldsymbol{r})\rangle, \tag{3}$$

where $|\varphi_{n}(\boldsymbol{r})\rangle$ are the basis vectors formed from the eigenstates of $\hat{H}_{\rm mb}(\boldsymbol{r})$ with any possible quantum numbers n in the corresponding Fock space, and $\Phi_{n}^{\alpha}(\boldsymbol{r}) \equiv \langle \varphi_{n}(\boldsymbol{r})|\Psi^{\alpha}(\boldsymbol{r})\rangle$ is the n-component impurity wave function. After we plug the BO expansion (3) into Eq. (2) and project onto the basis vector $\langle \varphi_{m}(\boldsymbol{r})|$, we obtain

$$\sum_{n} H_{mn}(\boldsymbol{r})\Phi_{n}^{\alpha}(\boldsymbol{r}) = E^{\alpha}\Phi_{m}^{\alpha}(\boldsymbol{r}), \tag{4}$$

with the effective impurity Hamiltonian

$$H_{mn}(\boldsymbol{r}) = -\mu \sum_{l} \boldsymbol{D}_{ml} \cdot \boldsymbol{D}_{ln} + A_{mn}^{0}(\boldsymbol{r}). \tag{5}$$

Here, $A_{mn}^{0}(\boldsymbol{r}) = \langle \varphi_{m}(\boldsymbol{r})|\hat{H}_{\rm mb}(\boldsymbol{r})|\varphi_{n}(\boldsymbol{r})\rangle$ is the non-Abelian scalar potential, and $\boldsymbol{D}_{mn} \equiv \boldsymbol{\nabla}\delta_{mn} - i\boldsymbol{A}_{mn}(\boldsymbol{r})$ is the covariant derivative. The particular object we are interested in is the non-Abelian gauge field

$$\boldsymbol{A}_{mn}(\boldsymbol{r}) = \langle \varphi_{m}(\boldsymbol{r})|i\boldsymbol{\nabla}|\varphi_{n}(\boldsymbol{r})\rangle \tag{6}$$

which contains all of the information about the geometry and topology of the problem.

Thus, we have rewritten the Hamiltonian (1) in the gauge invariant form (5) corresponding to the gauge group $U(\infty)$. Note that no approximations were introduced during this step. The origin of the emerging gauge symmetry, or, more properly, the gauge redundancy [57], follows from the fact that one could unitarily transform the basis vectors such that the BO expansion (3) is invariant [20]. Now, we assume that there exists a certain physical configuration where the restriction of the basis vectors is legitimate. A standard technique would be to use the product-state ansatz (the BO approximation), where the eigenstate (3) is approximated by $|\Psi^{\alpha}(\boldsymbol{r})\rangle \approx \Phi_{n}^{\alpha}(\boldsymbol{r})|\varphi_{n}(\boldsymbol{r})\rangle$, which results in a $U(1)$ gauge field. This approximation is, in principle, applicable to any impurity problem. A non-Abelian gauge field, on the other hand, can only be obtained by considering more than one many-body state. The latter can be realized, for instance, within the adiabatic approximation. There it is assumed that the many-body state $|\varphi_{n}(\boldsymbol{r})\rangle$ remains in a certain energy level, and an N-fold degenerate level yields a $U(N)$ gauge field [16]; see also Refs. [58,59] for emerging non-Abelian fields in nondegenerate systems. In many-body systems, however, the latter can be challenging to achieve unless the many-body state of interest is separated from the rest of the spectrum by an energy gap. Here, to truncate the number of basis vectors without employing adiabaticity, we introduce a new method based on the variational principle. As long as the quasiparticle has a discrete energy spectrum (as the angulon does), the variational state can be written as the BO expansion with a small number of basis vectors. We note that the discussion below is applicable to other quantum impurity problems with discrete spectrum, such as a polaron interacting with a magnetic field [54], or a particle in a double-well potential coupled to a bosonic bath [60].

Let us consider the angulon quasiparticle [39–41,52,55,61,62]. For the angulon, the truncation of basis states through the variational principle was shown to provide good agreement with experiments [42,44]. The angulon Hamiltonian, originally derived in Ref. [39], is

$$\hat{H}^{A} = B\boldsymbol{L}^{2} + \sum_{k\lambda\mu} \omega(k)\hat{b}_{k\lambda\mu}^{\dagger}\hat{b}_{k\lambda\mu} + \sum_{k\lambda\mu} U_{\lambda}(k)[Y_{\lambda\mu}^{*}(\hat{\Omega})\hat{b}_{k\lambda\mu}^{\dagger} + Y_{\lambda\mu}(\hat{\Omega})\hat{b}_{k\lambda\mu}]. \tag{7}$$

Here, $\sum_{k} \equiv \int dk$, $\omega(k)$ is the dispersion relation for the bosonic bath, and $\hat{b}_{k\lambda\mu}^{\dagger}$ and $\hat{b}_{k\lambda\mu}$ are the bosonic creation and annihilation operators in the angular momentum representation, with k, λ, and μ labeling the bosonic linear momentum, the angular momentum, and its projection on the laboratory-frame z axis, respectively [41]. The last term of Eq. (7) describes the interaction of the impurity with the bosonic bath, where $Y_{\lambda\mu}(\hat{\Omega})$ are the spherical harmonic operators [63] that depend on the impurity orientation in the laboratory frame, $\hat{\Omega} \equiv (\hat{\theta}, \hat{\phi})$, and $U_{\lambda}(k)$ is the angular-momentum-dependent coupling strength.

In Ref. [40], it was shown that in the strong-coupling regime, $U_{\lambda}(k) \gg B$, the angulon can be described by the variational state

$$|\Psi^{LM}\rangle = \hat{S}_{1}\hat{S}_{2}\left(g_{0}|0\rangle|LM0\rangle + \sum_{k\lambda n} \alpha_{k\lambda n}\hat{b}_{k\lambda n}^{\dagger}|0\rangle|LMn\rangle\right), \tag{8}$$

where g_{0} and $\alpha_{k\lambda n}$ are the variational parameters, and L, M, and n label the total orbital angular momentum, its projection on the quantization axis in the laboratory, and the body-fixed frames, respectively [64]. The first transformation, $\hat{S}_{1} = \exp(-i\hat{\phi} \otimes \hat{\Lambda}_{z})\exp(-i\hat{\theta} \otimes \hat{\Lambda}_{y})\exp(-i\hat{\gamma} \otimes \hat{\Lambda}_{z})$, brings the bath degrees of freedom into the frame corotating with the quantum rotor, where $\hat{\boldsymbol{\Lambda}} = \sum_{k\lambda\mu\nu} \boldsymbol{\sigma}_{\mu\nu}^{\lambda}\hat{b}_{k\lambda\mu}^{\dagger}\hat{b}_{k\lambda\nu}$ is the total angular momentum operator of the bath and

$\boldsymbol{\sigma}^\lambda$ the λ's representation of the rotation group [63]. The second transformation, $\hat{S}_2 = \exp[-\sum_{k\lambda} f_\lambda(k)(\hat{b}^\dagger_{k\lambda 0} - \hat{b}_{k\lambda 0})]$, is the coherent state transformation with $f_\lambda(k) = U_\lambda(k)\times\sqrt{(2\lambda+1)/(4\pi)}/[\omega(k) + B\lambda(\lambda+1)]$.

Following Eq. (3), the basis vectors of the state $|\Psi^{LM}(\Omega)\rangle$ is

$$|\varphi_n(\Omega)\rangle = \hat{S}_1(\Omega)\hat{S}_2 \frac{1}{c_n}\left(\sum_{k\lambda}\alpha_{k\lambda n}\hat{b}^\dagger_{k\lambda n}|0\rangle + \delta_{n0}g_0|0\rangle\right), \quad (9)$$

with $|c_n|^2 = \sum_{k\lambda}|\alpha_{k\lambda n}|^2 + \delta_{n0}|g_0|^2$. For impurities of experimental interest (such as molecules in superfluids), only a few coupling constants $U_\lambda(k)$ are of substantial magnitude [65]. We assume that only the isotropic term, $U_0(k)$, as well as the leading anisotropic term, $U_1(k)$, is present. In this case, the gauge group of interest is $U(3)$. Using Eq. (6), we compute $\boldsymbol{A} = A_\phi\hat{\phi} + A_\theta\hat{\theta}$ with the physical (or Cartesian) components of $\boldsymbol{A}$ given by [66]

$$A_\phi = \begin{pmatrix} -\cot\theta & \frac{-\kappa}{\sqrt{2}} & 0 \\ \frac{-\kappa^*}{\sqrt{2}} & 0 & \frac{-\kappa^*}{\sqrt{2}} \\ 0 & \frac{-\kappa}{\sqrt{2}} & \cot\theta \end{pmatrix}, \quad A_\theta = \begin{pmatrix} 0 & \frac{i\kappa}{\sqrt{2}} & 0 \\ \frac{-i\kappa^*}{\sqrt{2}} & 0 & \frac{i\kappa^*}{\sqrt{2}} \\ 0 & \frac{-i\kappa}{\sqrt{2}} & 0 \end{pmatrix}. \quad (10)$$

Here, $\hat{\theta}$ and $\hat{\phi}$ are unit vectors, and $\kappa = \sum_k \alpha^*_{k11}[\alpha_{k10} - f_1(k)g_0]/(c_1c_0)$. We further calculate the field strength,

$$\begin{aligned} F_{\phi\theta} &= \frac{1}{\sin\theta}[\partial_\phi A_\theta - \partial_\theta(A_\phi\sin\theta)] - i[A_\phi, A_\theta] \\ &= (1-|\kappa|^2)\Sigma_z, \end{aligned} \quad (11)$$

where $\boldsymbol{\Sigma} = \boldsymbol{\sigma}^1$. $F_{\phi\theta}$ in Eq. (11) is the strength of a $U(3)$ monopole with charge $g = 1 - |\kappa|^2$. This allows us to interpret the angulon (or a molecule immersed in a droplet of superfluid ^{4}He) as a three-component impurity interacting with the field of a non-Abelian magnetic monopole.

In order to provide quantitative results, we set the model parameters to the values used in Ref. [40]. $U_\lambda(k) = u_\lambda\sqrt{8k^2\varepsilon(k)n_0/[\omega(k)(2\lambda+1)]}\int dr r^2 v_\lambda(r) j_\lambda(kr)$, where u_λ and $v_\lambda(r)$ define the strength and shape of the molecule-boson interaction potential and $j_\lambda(kr)$ is the spherical Bessel function. We model the two-body potentials using Gaussian form factors $v_\lambda(r) = (2\pi)^{-3/2}e^{-r^2/(2r_\lambda^2)}$ and adapt a Bogoliubov-type dispersion relation $\omega(k) = \sqrt{\varepsilon(k)[\varepsilon(k) + 2g_{\rm bb}n_0]}$, where $\varepsilon(k) = k^2/(2m)$ is the boson kinetic energy. The boson-boson contact interaction is set to $g_{\rm bb} = 418(m^2u_0)^{-1/2}$. Furthermore, we take the potential anisotropy to be $u_1 = 5u_0$ and the range to be $r_0 = r_1 = 15(mu_0)^{-1/2}$, with $u_\lambda \equiv 0$ for $\lambda > 1$. In what follows, we study the behavior of the system as a function of the dimensionless rotational constant, $\xi = \ln[B/u_0]$, and the dimensionless density, $\tilde{n}_0 = \ln[n_0(mu_0)^{-3/2}]$.

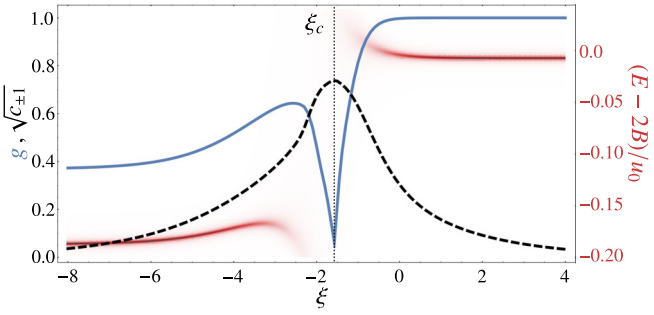

FIG. 1. Density plot of the angulon spectral function (the red curves), the charge of the $U(3)$ magnetic monopole, g (the blue solid curve), and the amplitude of the ±1 component of the impurity wave function, $c_{\pm1}$ (the black dashed curve) as a function of the dimensionless rotational constant, $\xi = \ln[B/u_0]$, for the $L = 1$ state. The dimensionless bath density is set to $\tilde{n}_0 = \ln[0.014]$. The vertical dashed line indicates the critical value, ξ_c, corresponding to the topological transition. See the text.

In Refs. [39–41] it was shown that the variational calculation with a state of the form (8) allows one to access to the entire spectrum of the system through the spectral function [67]. Figure 1 shows the spectral function for $L = 1$ (and $M = 0$ hereafter) state at the density $\tilde{n}_0 = \ln[0.014]$ (in red), which corresponds to the case presented in Ref. [40]. One can see that around $\xi_c \approx -1.6$ there is a discontinuity in the spectrum, which corresponds to the so-called angulon instability [39,40]. Such an instability corresponds to a resonant transfer of angular momentum between the impurity and the bath. Recently, the angulon instabilities were identified in experimentally observed spectra of CH_3 and NH_3 molecules trapped in superfluid helium nanodroplets [51].

In the same figure, we present the charge of the magnetic monopole g. First of all, we observe that g approaches zero around the instability point ξ_c, which corresponds to the limit of $\kappa \to 1$. In this limit the magnetic field (11) vanishes; hence, the vector potential (10) can be gauged away, $\boldsymbol{A} \to \boldsymbol{A}' = 0$. This result can be understood as follows. When $\kappa \to 1$, the basis vectors form a representation of the rotation group, $\langle\varphi_n|\hat{S}_1(\Omega)\hat{\boldsymbol{\Lambda}}\hat{S}_1^{-1}(\Omega)|\varphi_m\rangle = \boldsymbol{\Sigma}_{nm}$. As rotational invariance represents a global symmetry, there cannot exist a gauge field. As a consequence, the impurity and the bath interact only through the electric potential, A^0.

Away from the instability point ξ_c, the monopole charge assumes a finite value. For $\xi < \xi_c$ and for $\xi > \xi_c$, however, the behavior of the gauge field is quite different. In the former regime, the impurity interacts with an effective gauge field which, as we show below, is truly non-Abelian, and the monopole charge takes values in the range of ~0.4–0.6. For $\xi > \xi_c$, on the other hand, the monopole charge is identically 1, which corresponds to $\kappa = 0$. In this situation, the monopole gauge field becomes "Abelianized", i.e.,

$$\boldsymbol{A} = \cot\theta\Sigma_z\hat{\phi}. \quad (12)$$

In other words, for $\xi > \xi_c$ the gauge field can be decomposed into three $U(1)$ gauge fields, $\boldsymbol{A} = \boldsymbol{A}_- \oplus \boldsymbol{A}_0 \oplus \boldsymbol{A}_+$. While $\boldsymbol{A}_0 = 0$, the $U(1)$ gauge field $\boldsymbol{A}_\pm$ is the so-called Dirac monopole field with charge $g_\pm = \pm 1$ [1,68].

In the Abelian regime, one can define the total angular momentum operator, which commutes with the Hamiltonian (5), as $\boldsymbol{\mathcal{J}} = \boldsymbol{r} \times (\boldsymbol{p} - \boldsymbol{A}) - \Sigma_z \boldsymbol{r}/r$, where the last term corresponds to the three Dirac monopoles [69]. Moreover, as shown in Ref. [70], $\boldsymbol{\mathcal{J}}$ is the total angular momentum operator in the non-Abelian regime as well. This suggests that the general solutions of the Hamiltonian are given by superposition of the corresponding angular momentum eigenstates, which explains the formation of the angulon. For each Dirac monopole the corresponding eigenstate is given by the spin-weighted spherical harmonics, ${}_nY_{LM}(\Omega)$ [69,71,72]. Then, the quasiparticle wave function yielding the spectral function of Fig. 1 can be written as $\Phi^{LM}(\Omega) = \sum_n {}_nY_{LM}(\Omega) c_n \chi_n$, with χ_n being the eigenvectors of Σ_z. Here, the amplitude $c_{\pm 1}$ corresponds to the impurity components that interact with the monopole. However, we note that for the regimes $\xi \ll 1$ and $\xi \gg 1$ the amplitude vanishes, as shown by the dashed curve in Fig. 1, which is a consequence of the vanishing impurity-bath interaction.

In fact, ${}_nY_{LM}(\Omega)$ is already given by the impurity part of the variational state (8), $|LMn\rangle$, which is the solution for the linear molecule in the corotating frame. First, this shows the self-consistency of our method because, unlike in the other studies [18,23], here the magnetic monopole solution directly emerges from the variational state (8). More importantly, however, since a linear molecule in the corotating frame can be regarded as a symmetric top [73], there is a correspondence between a nonlinear symmetric-top molecule and a particle in the field of a monopole. Namely, the quantum number of the projection of angular momentum on the body-fixed quantization axis corresponds to the charge of a Dirac monopole. This provides further insights into the physics of magnetic monopoles.

The Abelianization of the gauge field after the instability point is reminiscent of the 't Hooft–Polyakov monopole, which reduces to the Dirac monopole after the spontaneous symmetry breaking [2,3]. In fact, it is possible to introduce a gauge in which the gauge field is written as $\boldsymbol{A}' = (1-\kappa)\hat{\boldsymbol{r}} \times \boldsymbol{\Sigma}$ [22,70], which has the same structure as the 't Hooft–Polyakov monopole. Furthermore, in this gauge the Hamiltonian is $H' = B[(\boldsymbol{\mathcal{J}}' - \kappa\boldsymbol{\Sigma})^2 - (1-\kappa)^2(\hat{\boldsymbol{r}}\cdot\boldsymbol{\Sigma})^2] + A^{0\prime}$. Here, $\boldsymbol{\mathcal{J}}' = \boldsymbol{L} + \boldsymbol{\Sigma}$, $\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$, and $\boldsymbol{\Sigma}$ correspond to the total, orbital, and spin angular momentum, respectively. The Hamiltonian can be further simplified to $H' = B\boldsymbol{L}^2 + 2B(1-\kappa)\boldsymbol{L}\cdot\boldsymbol{\Sigma} + V(\kappa)$, where the second term is nothing but the spin-orbit coupling. This indicates that the initially spinless linear molecule behaves like a spin-1 particle when it interacts with the bath. This phenomenon, the emergence of the spin degrees of freedom of an initially spinless particle, is known as the isospin-spin conversion [58,74,75]. We are confident that the angulon offers a great opportunity for realization of this phenomenon.

In order to see that the vector potential is truly non-Abelian for $\xi < \xi_c$, one can argue as follows. Assume there exists a gauge transformation that brings the vector potential to the form $\boldsymbol{A} = \boldsymbol{A}_- \oplus \boldsymbol{A}_0 \oplus \boldsymbol{A}_+$. Using Eq. (11), $1/(2\pi)\int_{S^2} F_{\phi\theta} d\Omega = 2\mathrm{diag}(-g, 0, g)$, where $d\Omega = \sin\theta d\theta d\phi$. It is well known that those numbers are the first Chern numbers (topological invariants) of the line bundles associated with $\boldsymbol{A}_-$, $\boldsymbol{A}_0$, and $\boldsymbol{A}_+$ [28]. By definition, they can take only integer values. This reflects the well-known fact that $U(1)$ magnetic monopoles are necessarily quantized. Since g lies in the range of ~0.4–0.6, this contradicts the assumption that $\boldsymbol{A}$ is Abelian and we conclude that it is truly non-Abelian. For non-Abelian monopoles the above reasoning fails since, in this case, the only topological invariant is the Chern number of the whole bundle [76], which is given by $1/(2\pi)\int_{S^2} \mathrm{tr}(F_{\phi\theta}) d\Omega$ and equals zero in our case.

The above discussion reveals that the transition from a non-Abelian vector potential with no topological restriction on g for $\xi < \xi_c$ to an Abelian vector potential, with the topological restriction that g has to be an integer for $\xi > \xi_c$, is also a topological transition of the underlying vector bundle. This topological transition is clearly visible in Fig. 2, where we plot the monopole charge g as a function of both ξ and $\tilde{n}_0$. In the non-Abelian domain (NA), the monopole charge takes a range of values between zero and 1. On the other hand, for a large positive ξ value, the monopole charge is equal to 1 with high precision, which corresponds to the Abelian region (A). The topological transition from the non-Abelian to the Abelian monopole takes place across the angulon instability (I). The fact that the spectral function of Fig. 1 remains constant for all values of ξ in the Abelian region strongly indicates that $\partial(E/B)/\partial B \propto d\kappa/dB$. As a result, the robustness of the Abelian domain against the change of parameters can be

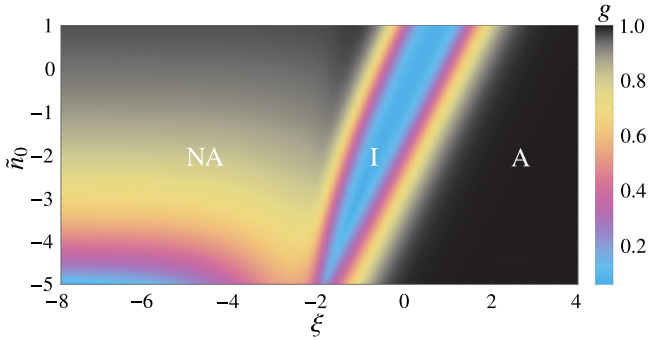

FIG. 2. The monopole charge of the angulon $L = 1$ state, g, as a function of the dimensionless rotational constant, $\xi = \ln[B/u_0]$, and the dimensionless density, $\tilde{n}_0 = \ln[n_0(mu_0)^{-3/2}]$. Topological transition from the non-Abelian (NA) to the Abelian (A) monopole takes place in the angulon instability region (I). See the text.

tested experimentally by measuring the spectral function for different molecules. Furthermore, as the monopole charge can be identified in terms of the angular momentum of the bath, $\kappa = i\sqrt{2}\langle\varphi_1|\hat{S}_1(\Omega)\hat{\Lambda}_y\hat{S}_1^{-1}(\Omega)|\varphi_0\rangle$, the topological transition, in principle, is accessible through time-of-flight measurements [77] or momentum-resolved Bragg scattering [78].

In conclusion, we have demonstrated that a rotating impurity coupled to a many-particle bath (the "angulon quasiparticle") can be interpreted as a quantum particle on the two-sphere interacting with a gauge field of a non-Abelian magnetic monopole. Intuitively, in the corotating frame, the cloud of bosons rotates around the molecule fast, and this rotation induces a gauge field of a magnetic monopole, similar to the case of electrons orbiting the nuclei [18]. In the particular setting considered here, a superfluid helium droplet manifests itself as a $U(3)$ non-Abelian magnetic monopole in the real space of the molecular impurity. We demonstrate that the $U(3)$ gauge field vanishes exactly at the angulon instabilities. Furthermore, on one side of the instability the gauge field is truly non-Abelian, whereas on the other side the gauge field Abelianizes and the components of the impurity effectively interact with separate Dirac monopoles. The Abelianization of the gauge field around the instability corresponds to a topological transition of the underlying vector bundle. Since the angulon instabilities have been recently identified in an experiment [51], our results pave the way for the study of topological transitions and related physics using molecules in helium nanodroplets.

We are grateful to G. Bighin, S. Castrignano, S. Meuren, B. Midya, D. Monaco, R. Schmidt, M. Serbyn, and H. Weimer for the valuable discussions. E. Y. acknowledges the financial support received from the People Programme (Marie Curie Actions) of the European Union's Seventh Framework Programme (FP7/2007–2013) under the Research Executive Agency (REA) Grant Agreement No. 291734. A. D. acknowledges support from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation program (Grant Agreement No. 694227). M. L. acknowledges support from the Austrian Science Fund (FWF) under Project No. P29902-N27.

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