Recognizing Axionic Dark Matter by Compton and de Broglie Scale Modulation of Pulsar Timing

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Light axionic dark matter, motivated by string theory, is increasingly favored for the "no weakly interacting massive particle era". Galaxy formation is suppressed below a Jeans scale of $\simeq 10^8\ M_\odot$ by setting the axion mass to $m_B \sim 10^{-22}$ eV, and the large dark cores of dwarf galaxies are explained as solitons on the de Broglie scale. This is persuasive, but detection of the inherent scalar field oscillation at the Compton frequency $\omega_B = (2.5\ \text{months})^{-1}(m_B/10^{-22}\ \text{eV})$ would be definitive. By evolving the coupled Schrödinger-Poisson equation for a Bose-Einstein condensate, we predict the dark matter is fully modulated by de Broglie interference, with a dense soliton core of size $\simeq 150$ pc, at the Galactic center. The oscillating field pressure induces general relativistic time dilation in proportion to the local dark matter density and pulsars within this dense core have detectably large timing residuals of $\simeq 400\ \text{nsec}/(m_B/10^{-22}\ \text{eV})$. This is encouraging as many new pulsars should be discovered near the Galactic center with planned radio surveys. More generally, over the whole Galaxy, differences in dark matter density between pairs of pulsars imprints a pairwise Galactocentric signature that can be distinguished from an isotropic gravitational wave background.

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Introduction.—Axions are a compelling choice for extending the standard model in particle physics, generating oscillating dark matter (DM) with symmetry broken by the misalignment mechanism [1–3]. Such fields are generic to string theory, arising from dynamical compactification to four space-time dimensions, so that multiple stabilized complex scalar fields naturally appear (e.g., Ref. [4]). Such an axion is effectively massless until the Universe cools below some critical temperature, and rolls down a small nonpertubatively generated potential, oscillating about the minimum, corresponding to a coherent zero-momentum axion. It is argued that very light axions are very natural in this context [5] and furthermore, in a "string theory landscape," the cosmological constant may also be naturally small and accompanied by correspondingly very light scalar bosons [6].

Axion oscillation generates field pressure at frequency $2m_B$, with static energy density to leading order that is coherent below the de Broglie scale. At a certain scale, self-gravity is balanced by field pressure, yielding a static centrally located density peak, or soliton. The soliton scale depends on the gravitational potential corresponding to only 150 pc for the Milky Way [7]. Throughout the Galaxy, gravity is balanced by the axion pressure forming density "granules" of similar scale to the soliton from interference of many large-amplitude de Broglie waves; these are unstable, and regenerate themselves with a lifetime of about 1 Myr for our galaxy [8].

Khmelnitsky and Rubakov made the very interesting observation that the field pressure oscillations on the Compton scale leads to an oscillating gravitational potential that can affect pulsar timing measurements [9]. They assume a constant local dark matter density so that pulsar timing and our Earth clock will be affected in the same way, with a net relative timing modulation from the phase difference that will be very challenging to detect. However, we show here that a much stronger signal is expected towards the Galactic center, in and around the solitonic core that we predict for light axionic dark matter. This additional de Broglie structure is absent in [9], which predates the first simulations of galaxies in this context [7]. We show this greatly enhances the expected pulsar timing modulation by 2–3 orders of magnitude within the central kiloparsec of our Galaxy, for our favored axion mass (0.8×10^{-22} eV [7]), well within the current bounds of detectability, providing a direct, practical test for axions.

Here we calculate this additional de Broglie effect self-consistently by including our simulated halo structure. This is an important feature that is absent in Ref. [9], which if observed would directly support light bosonic dark matter. The parameters assumed are Hubble constant $H_0 = 70\ \text{km/sec/Mpc}$ with critical density $\rho_c = 6\ \text{GeV m}^{-3}$, and present dark matter density $\rho_0 = 1.5\ \text{GeV m}^{-3}$.

Compton scale pressure oscillation and pulsar timing.— For a single real scalar field ϕ with harmonic potential oscillation, $\phi(\mathbf{x},t) = A(\mathbf{x})\cos[m_B t + \alpha(\mathbf{x})]$, where m_B is the associated boson mass. We let $\mathrm{Re}[\psi]\cos(m_B t) + \mathrm{Im}[\psi]\sin(m_B t) \equiv m_B^{1/2}\phi$ to relate ϕ to the complex wave function of Schrödinger's equation, so $m_B^{1/2}A$ and α are the amplitude and phase of ψ, respectively. From the energy-momentum tensor one obtains an oscillating pressure $p(\mathbf{x},t) = -\frac{1}{2}m_B^2 A^2 \cos(\omega t + 2\alpha)$ with frequency $\omega = 2\pi\nu = 2m_B$. These oscillations are usually neglected as the average pressure over the period is zero. However, at the Compton scale of interest here, the oscillating scalar field generates an oscillating gravitational potential in proportion to the local mass density [9]. All clocks are modulated by such an oscillating field, including Earth clocks and pulsars.

It is customary to write the time-dependent part of the time residuals as the relative frequency shift of the pulse,

$$\delta t(t) = -\int_0^t \frac{\nu(t') - \nu_0}{\nu_0} dt', \tag{1}$$

where $\nu(t')$ is the pulse frequency at the detector, emitted at a distance D at the time $t' = t - D/c$, and ν_0 is the pulsar emission frequency. To rewrite Eq. (1) in terms of the oscillating gravitational potential we consider the linearized Einstein equations in the Newtonian gauge with two potentials, $h_{00} = 2\Phi$ and $h_{ij} = -2\Psi\delta_{ij}$. Then, the frequency shift is [10]

$$\frac{\nu(t) - \nu_0}{\nu_0} \approx \Psi(\mathbf{x_p}, 0) - \Psi(\mathbf{x}, t), \tag{2}$$

where $\mathbf{x_p}$ is the source position. Thus, to compute the frequency shift we need only consider the oscillating contribution to Ψ. Therefore, from the spatial components of the linearized Einstein equations the amplitude is

$$\Psi_c(\mathbf{x}) = \pi \frac{G\rho_{\mathrm{DM}}(\mathbf{x})}{m_B^2}. \tag{3}$$

We stress the amplitude of the oscillation will vary over the Galaxy density with large de Broglie scale modulations about the mean, which is close to the Navarro-Frenk-White (NFW) profile [11] (see Fig. 1).

Plugging Eqs. (2) and (3) into Eq. (1), we obtain the time-dependent part of the time residuals for the ith pulsar with respect to the average signal ($\Delta t_i = \delta t_i - \langle \delta t_i \rangle$),

$$\Delta t_i(t) = \frac{1}{\omega}\left[\Psi(\mathbf{x_i})\sin\left(\omega\left(t - \frac{D_i}{c}\right) + 2\alpha_i\right) - \Psi(\mathbf{x_e})\sin(\omega t + 2\alpha_e)\right], \tag{4}$$

where $\alpha_i \equiv \alpha(\mathbf{x_i})$ and D_i are the phase and the pulsar distance and $\alpha_e \equiv \alpha(\mathbf{x_e})$ is the phase of the Earth clock; hence, cancellation of timing residuals is possible, and when pairs of clocks or pulsars are maximally out of phase their relative timing residual is enhanced, see Fig. 2.

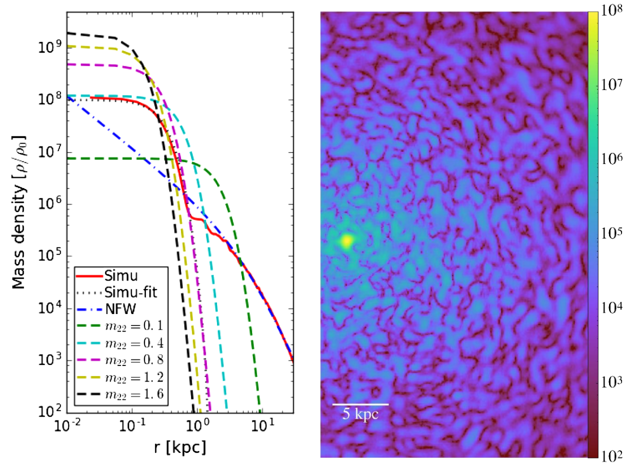

FIG. 1. The left panel shows predicted soliton profiles for a range of m_B (dashed curves), including the profile of a massive simulated halo of 10^{11} $M_\odot$ (solid red curve), corresponding to the simulation on the right (using the YT package [12]), where the granular de Broglie scale structure is visible on all scales, including the dense central soliton. Also indicated is a NFW profile (blue dashed curve) that fits well the azimuthally averaged galaxy profile (solid red curve).

The timing differences between two well-separated pulsars will be enhanced by the modulation of the axion density field on the de Broglie scale throughout the Galaxy, as shown in Figs. 1 and 2, as the density structure is

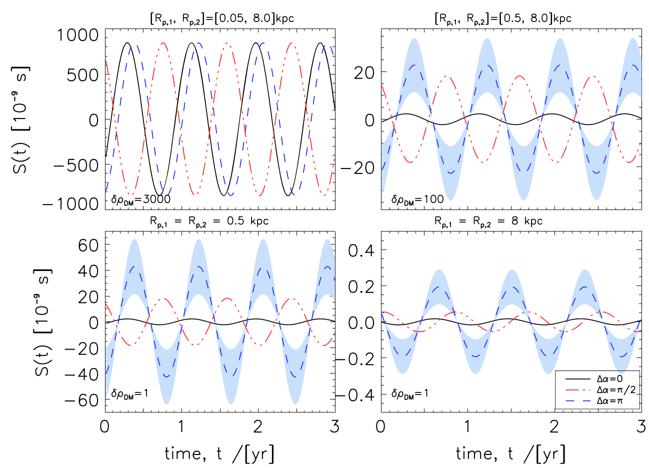

FIG. 2. Predicted timing signal between pulsar pairs, for a light scalar field of $m_B \sim 0.8 \times 10^{-22}$ eV. The top panels show the relative timing signal between local pulsars at 8 kpc, and pulsars close to the Galactic center at a radius of 50 and 500 pc. In the bottom left and right panels both members of the pair are located at the same Galactocentric radius of 0.5 and 8 kpc, with relative phases chosen indicated in the inset box, bottom right, illustrating that the signal can cancel in such cases. The shaded regions indicate how the density modulation on the de Broglie scale can enhance or diminish the pairwise relative timing signal.

predicted by our simulations to be fully modulated on the de Broglie scale.

de Broglie scale galactic structure.—If sufficiently light, bosonic dark matter, such as axions, can satisfy the ground state condition, where the de Broglie wavelength exceeds the mean particle separation set by the density of dark matter. This is simply described by a coupled Schrödinger-Poisson equation, analogous to the Gross-Pitaevskii equations for a Bose-Einstein condensate. Expressed in comoving coordinates we have

$$\left(i\frac{\partial}{\partial\tau}+\frac{\nabla^2}{2}-aV\right)\psi=0, \tag{5}$$

$$\nabla^2 V=4\pi(|\psi|^2-1), \tag{6}$$

where ψ is the wave function, V is the gravitation potential, and a is the cosmological scale factor. The system is normalized to the time scale $d\tau=\chi^{1/2}a^{-2}dt$ and to the scale length $\xi=\chi^{1/4}(m_B/\hbar)^{1/2}\mathbf{x}$, where $\chi=\frac{3}{2}H_0^2\Omega_0$ and Ω_0 is the current density parameter [13].

The simplest "fuzzy dark matter" case of no self-interaction was first advocated by Hu *et al.* [14], for which the boson mass is the only free parameter, with further work in relation to dwarf galaxies [15,16]. New cosmological simulations in this context, dubbed $\psi_{\rm DM}$, by [7] have uncovered a rich nonlinear structure by solving the above equation, evolved from an initial standard power spectrum truncated at the inherent Jeans scale. A solitonic core forms within each virialized halo, naturally explaining dark-matter-dominated cores of dwarf spheroidal galaxies [7]. The central soliton is surrounded by an extended halo with "granular" texture on the de Broglie scale in Fig. 1, which on average follows the NFW form outside the soliton [7,8], as seen in the figure. This agreement can be understood because the pressure from the uncertainty principle is limited to the de Broglie radius, beyond which is it negligible and behaves as collisionless cold dark matter.

The identification of the centrally stable soliton with the large cores of dwarf spheroidal (dSph) galaxies has allowed the boson mass to be estimated with little model dependence [7,8,17]. The best constraint comes from the well-studied Fornax dSph, with an estimated halo mass of $4\times10^9\ M_\odot$, yielding the soliton peak density of 2 GeV cm^{-3}, core width 1 kpc [7], and $m_B=0.8\times10^{-22}$ eV, with somewhat larger m_B derived using dSph galaxies from the SDSS survey [18]. This allows us to predict the soliton scale expected for the Milky Way by using the scaling between halo mass and soliton mass scaling law, $\rho_{\rm peak}\propto M_{\rm halo}^{4/3}$ and $r_c\propto M_{\rm halo}^{-1/3}$ derived from simulations [7,8], which for our Galaxy with a mass $M_{\rm halo}^{\rm MW}=2\times10^{12}\ M_\odot$ [19] implies a Milky Way soliton peak density $\rho_{\rm peak}^{\rm MW}=8\times10^3$ GeV cm^{-3} and soliton core width $r_c^{\rm MW}=120$ pc (Fig. 1). The Milky Way halo is generally taken to have a dark matter density 0.3 GeV cm^{-3} in the solar neighborhood, with recent careful studies by Portail *et al.* [19] and Sivertsson *et al.* [20], revising upward this figure to 0.5–0.6 GeV cm^{-3}. Our predicted soliton for our Galaxy ranges over 4–8×10^3, and from Eq. (5) above, we have $\Delta t=\pi G\rho_{\rm DM}/2m_B^3=(0.7-1.4)\times10^{-27}m_B^{-3}$ sec^{-2}. Since $m_B=1.5\times10^{-7}$ sec^{-1}, we arrive at $\Delta t=200$–400 nsec; this Δt is within the reach of current pulsar time arrays. The central soliton then provides an enhancement of approximately 2 orders of magnitude of the DM density within $r\lesssim100$ pc, dominating the Earth clock and pulsars elsewhere in the galaxy as shown in Fig. 2 (top left). For pairs of pulsars within this soliton region, separated by more than the Compton wavelength of (>pc scale), the combined amplitude cancels or adds constructively to a factor of 2, as shown in Fig. 2. The above assumes $m_B=10^{-22}$ eV, but allowing this to vary we have $\rho\propto m_B^2$ and $r_c\propto m_B^{-1}$, and thus $\Delta t\propto m_B^{-1}$, increasing as the soliton density becomes higher, but the core radius is reduced.

Pairwise timing amplitudes.—It is important to appreciate that all clocks are modulated within an oscillating scalar field, including Earth clocks, and so in practice we can work only with relative timing residuals, either between pairs of pulsars or between any pulsar and a time standard based on precise Earth clocks. The ticks of an individual clock are cyclically slowed and increased at the Compton frequency, with a magnitude that is proportional to the mass density of the scalar field local to each pulsar, where the "amplitude" of this effect is the time difference induced by Eq. (3). In practice, pulsar timing measurements are typically averaged over a sizable number of pulses on a relatively short time scale of hours and this rate is then compared on longer time scales, a practice that is well suited to the larger than monthly Compton frequency modulation that we seek, set by the axion mass. Note that this time scale is unaffected by the slow changes in structure on the de Broglie scale of $\simeq1$ Myr that set the amplitude of the timing residual for a given axion mass via Eq. (3).

The timing amplitude of any such modulation is determined by the de Broglie scale density modulation that is largest for a pulsar within the central soliton and also when the pulsars are spatially located such that they are out of phase relative to the Compton frequency. In general this phase difference means that any given pair of pulsars, for which the local axion density is equal, range in relative amplitude from zero to double the amplitude of each separately, as shown by the blue shaded area in Fig. 2. Furthermore, for well-separated pulsars (on a scale greater than the de Broglie scale) the timing amplitude range can be enhanced by another factor of 2 because of the de Broglie scale interference that fully modulates the local density about the mean level, shown in Fig. 2.

Ideally, the relative pulsar timing may not have to rely on being referred to any Earth clock for simultaneous observations of different pulsars through the same telescope, or for telescopes that are highly synchronous, with the advantage that any vagaries in precision of Earth clocks cancel. Thus, here we calculate the relative timing amplitudes for pairs of pulsars $S(t)$,

$$S(t) = \Delta t_1(t) - \Delta t_2(t) = \frac{1}{\omega}(\Psi(\mathbf{x_1})\sin(\omega t + \alpha_1') - \Psi(\mathbf{x_2})\sin(\omega t + \alpha_2')), \quad (7)$$

where we have defined $\alpha_i' = 2\alpha_i - \omega D_i/c$. To illustrate this difference we calculate the relative timing signal in Fig. 2 between local pulsars, and pulsars close to the Galactic center at a radius of 50 and 500 pc, fixing the boson mass to $m_B \sim 0.8 \times 10^{-22}$ eV. In this case, the relative timing amplitude is dominated by the pulsar closer to the Galactic center and it can reach an amplitude of the order of 600 ns. While taking the pulsars at approximately the same distance from the Galactic center, the relative timing amplitude signal strongly depends on the phase of the pulse, canceling when the signals as seen from Earth are in phase.

For convenience, we define ratio of the DM density at the locations of the two pulsars as

$$\delta\rho_{\mathrm{DM}} = \left(\frac{\Psi(\mathbf{x_1})}{\Psi(\mathbf{x_2})}\right) = \frac{\rho_{\mathrm{DM}}(\mathbf{x_1})}{\rho_{\mathrm{DM}}(\mathbf{x_2})}. \quad (8)$$

To estimate the detectability of forthcoming pulsar timing arrays to the axion oscillation we compute the average square signal over all the phases

$$\sqrt{\langle S^2(t)\rangle} = \frac{\sqrt{2}}{2\omega}\sqrt{\Psi(\mathbf{x_1})^2 + \Psi(\mathbf{x_2})^2}, \quad (9)$$

and relate it to the gravitational wave (GW) strain, and since the $\Psi(\mathbf{x})$ amplitude of the oscillation only depends on the axion density we can always rewrite $\Psi(\mathbf{x_2})$ in term of $\Psi(\mathbf{x_1})$,

$$h_c = \frac{\sqrt{6}}{2}\Psi(\mathbf{x_2})\sqrt{1 + \delta\rho_{\mathrm{DM}}^2}. \quad (10)$$

We compute the characteristic amplitude shown in Fig. 3, highlighting the relatively strong signal expected for pulsars within ~0.5 kpc of the Galactic center, We also overplot the confidence regions for the current results from Pulsar Timing Array (PTA), Parkes Pulsar Timing Array (PPTA), and Square Kilometre Array (SKA) experiments to allow a direct comparison with expected sensitivities of these new surveys [21], where even one such central pulsar can provide a sufficient timing residual to test for the presence of light axionic dark matter.

Discussion and conclusions.—We have considered Compton and de Broglie scale modulations within the axionic interpretation of dark matter and examined their combined effect on pulsar timing. For our Galaxy we estimate the de Broglie scale is approximately 100 times larger than the Compton scale, corresponding to ≃150 pc for the Milky Way, with the favored axion mass of $m_B \simeq 10^{-22}$ eV. Within virialized halos our simulations reveal that the density distribution is fully modulated on the de Broglie scale, as shown in Fig. 1, with a dense soliton at the center of radius ≃150 pc where the pulsar timing effect is strong. Compton oscillation will be coherent within de Broglie sized patches, becoming unrelated on larger scales. We have made self-consistent predictions for pulsar timing residuals, including this spatial dependence revealed in our ψ_{DM} simulations [7].

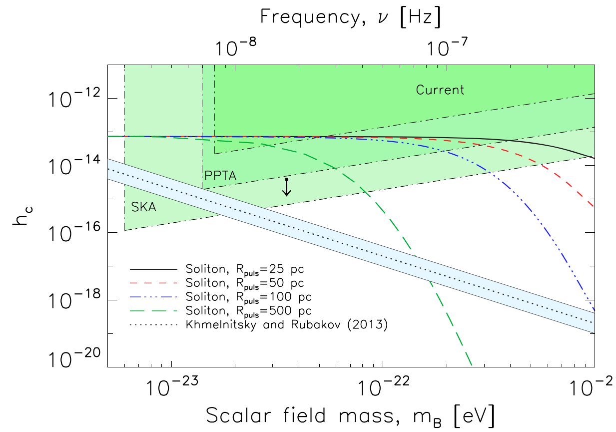

FIG. 3. Characteristic strain measured between pairs of pulsars, with Galactocentric radii chosen as in Fig. 2 and compared with the expected sensitivities from the current and forthcoming PTA experiments (adapted from [9,21]), with the corresponding oscillation frequency shown above. We highlight the relatively high signal strength we calculate for pulsars within the Galactic soliton region as a function of axion mass in a series of curves, demonstrating that this signal is already detectable for central Galactic pulsars. For comparison, the diagonal dotted line is the prediction obtained by [9] for local pulsars, assuming a smooth density distribution, with the blue shaded region representing the wider range we predict that includes our de Broglie scale DM density modulation. The local upper limit obtained by [22] is also shown (black square with arrow).

The de Broglie interference is most conspicuous by the formation of central soliton on the de Broglie scale, representing a stable, time-independent ground state, where the pulsar timing residuals are expected to be 2–3 orders of magnitude higher than those imprinted on local pulsars, due to the relatively high central density of the soliton which much exceeds in density a corresponding NFW profile. Such central millisecond pulsars are expected in large numbers within the bulge and near the Galactic center [23,24], and can account for the GeV γ-ray excess [25,26]; they are being searched for with some success [27,28]. Detection will be compromised within the inner 100 pc where the high plasma density is high and the interstellar medium contains small-scale irregularities, causing dispersion of pulse arrival time and pulse smearing, although “corridors” of lower scattering may be evident [29,30]. The smearing is a low-pass filter making millisecond pulsars undetectable at low frequency, but it decreases rapidly at high frequency (<10 GHz) as ν^{-4}. Blind searches are not yet very practical at these frequencies with single dishes and the pulse amplitude is typically relatively weak, but soon the SKA will be capable of efficiently detecting the putative population of central millisecond pulsars; comprehensive searches are already underway [29,31].

In terms of the central potential of the Galaxy, the stellar bar dominates, but the signature of this soliton feature is expected dynamically on a scale of $\lesssim 150/m_B$ pc, which lies between the scale length of the stellar bulge (1 kpc) and the smaller pc-scale region of influence of the central Milky Way black hole. Careful dynamical modeling by Portail *et al.* [19] has recently uncovered a central shortfall of $2\times10^9\ M_\odot$ of "missing matter," which may help account for the 100 km/s motion of stars within the central $\simeq 120$ pc of the galaxy [32–34] and which we aim to examine in the context of the near-spherical soliton potential that we predict here.

By using pulsars spread over a wide range of Galactocentric radii, we may detect the radial dependence of pulsar timing amplitude on the dark matter density, beyond current limits claimed in [22] and pulsar binary resonance [35]. This dependence helps distinguish the monotonic Compton scale modulation we seek from an isotropic, stochastic GW background that has broad band variance. In making such pairwise measurements we have stressed that only relative timing variations can be detected, as all clocks are modulated on the same Compton frequency scale, including Earth clocks. Furthermore, we can anticipate a relatively large variance of such timing residuals because of the large density modulation from de Broglie scale interference. The timing residual will absent for pulsars at density minima but can be enhanced by 100% for pairs that lie near density maxima and that are located out of phase with respect to the Compton oscillation. This means that we cannot rely on only one pair of pulsars when assessing the presence of this Compton frequency, but must examine an ensemble to average over the combined Compton and de Broglie modulation for a statistical detection.

For pulsars that may be detected in local dwarf spheroidal galaxies, where the entire visible content lies within a solitonic core [7], the time scale is set by m_B but the timing amplitude will be lower, set by the soliton density which scales as $M_{\rm halo}^{4/3}$ [8], about a factor of 10^{-3} smaller than for the Galactic center, close to the current PTA capability limit.

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