Beyond the Boost: Measuring the Intrinsic Dipole of the Cosmic Microwave Background Using the Spectral Distortions of the Monopole and Quadrupole

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We present a general framework for the accurate spectral modeling of the low multipoles of the cosmic microwave background (CMB) as observed in a boosted frame. In particular, we demonstrate how spectral measurements of the low multipoles can be used to separate the motion-induced dipole of the CMB from a possible intrinsic dipole component. In a moving frame, the leakage of an intrinsic dipole moment into the CMB monopole and quadrupole induces spectral distortions with distinct frequency functions that, respectively, peak at 337 and 276 GHz. The leakage into the quadrupole moment also induces a geometrical distortion to the spatial morphology of this mode. The combination of these effects can be used to lift the degeneracy between the motion-induced dipole and any intrinsic dipole that the CMB might possess. Assuming the current peculiar velocity measurements, the leakage of an intrinsic dipole with an amplitude of $\Delta T = 30\ \mu$K into the monopole and quadrupole moments will be detectable by a PIXIE-like experiment at ~40 nK (2.5σ) and ~130 nK (11σ) level at their respective peak frequencies.

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Introduction.—The measurements of the COBE/FIRAS instrument show that the intensity of the cosmic microwave background (CMB) has an almost perfect blackbody spectrum [1]. Even though in a frame moving with respect to the CMB the observed intensity is effectively a blackbody in every direction, the intensity harmonic multipoles in this frame generally contain frequency spectral distortions. These distortions are a result of the leakage of the nearby multipoles into each other due to the aberration and Doppler effects [2–8]. The most prominent motion-induced leakage component is that of the monopole into the dipole (i.e., kinematic dipole). The kinematic dipole has a frequency dependence identical to a differential blackbody spectrum which makes it degenerate with any intrinsic (or nonkinematic) dipole that the CMB might possess. Current modeling of the CMB dipole includes only the leakage of the monopole but ignores any intrinsic dipole component as well as other kinematic corrections to this mode (e.g., the leakage of the quadrupole). Here we present an accurate description of the frequency spectrum of the low multipoles of CMB and show how the kinematic (motion-induced) corrections to these modes can be used by the next generation of CMB surveys to lift the dipole degeneracy.

A kinematic dipole is not the only observational consequence of our motion with respect to the CMB. The motion-induced leakage of the intensity multipoles into each other causes a *boost coupling* between the nearby multipoles. Measuring this boost coupling in a wide range of harmonic modes can actually lead to an independent measure of the peculiar velocity of an observer with respect to the CMB [9–12]. In the CMB rest frame, all motion-induced effects (including the kinematic dipole and the boost coupling) vanish; however, there is no compelling reason for us to believe that the intrinsic dipole moment of the CMB in this frame is precisely zero.

It has been shown that, in a flat ΛCDM universe with adiabatic initial perturbations, the intrinsic dipole of the CMB is strongly suppressed [13,14]. For this reason, the intrinsic dipole of the CMB is usually either ignored or set to zero, and the observed dipole of the CMB is interpreted entirely as a kinematic effect. This results in a peculiar velocity of $\beta \equiv v/c = 0.001\,23$ in the direction $\hat{\boldsymbol{\beta}} = (264°, 48°)$ in galactic coordinates [15]. If the observed dipole moment has only a kinematic origin, it can be used to define a natural rest frame for CMB (namely, the frame in which the whole dipole vanishes). However, the unintended subtraction of an existent nonkinematic dipole in this process will result in obtaining an incorrect CMB rest frame. This can in turn lead to unexpected anomalies, such as the observed power and parity asymmetries in the CMB [16,17] and the mismatch between the CMB rest frame and the matter rest frame [18–21]. Studying the angular variance of the Hubble parameter over different redshifts (in the CMB dipole-inferred frame) also indicates the presence of a nonkinematic dipole component in the CMB [22,23]. Furthermore, since isocurvature initial perturbations and multifield inflationary scenarios typically invoke a non-negligible intrinsic dipole moment, a detection of this component could have important implications for prerecombination physics [24–28].

Recently, the Planck team has obtained an independent value for the peculiar velocity of the Solar System using the boost coupling of the CMB multipoles. Their result $\beta = 0.00128 \pm 0.00026(\mathrm{stat}) \pm 0.00038(\mathrm{syst})$ [9] is consistent with the kinematic interpretation of the dipole and shows that most of the dipole that we observe is induced by our peculiar motion. However, the error bars still allow for a nonkinematic dipole component that remains to be measured.

In this Letter, we show how the kinematic and non-kinematic dipoles can be separated by measuring the motion-induced spectral distortions in the observed low multipoles of the CMB in our local frame. Future microwave surveys, such as *PIXIE* [29] with a sensitivity of 5 Jy/sr, will be able to measure these effects with high precision.

Lorentz boosting the CMB.—We define the rest frame of the CMB as the frame in which there is no Doppler and aberration effect and the boost coupling between the harmonic modes of the CMB vanishes. [Indeed, in this frame all the other kinematic effects including the kinematic dipole (the leakage of the monopole into the dipole) and the ones that we are about to discuss will vanish as well [30].] We still allow the CMB to have a nonkinematic dipole in this frame. Then we argue that the full frequency spectrum of the low-intensity multipoles in the boosted frame can be exploited to separate the intrinsic dipole from the kinematic part induced by a boost. We assume that the CMB frequency spectrum in its rest frame can be described as a pure blackbody by neglecting any prerecombination and secondary μ and y distortions (see Fig. 12 in Ref. [29] and also [31]). In this frame, we expand the intensity and the thermodynamic temperature in spherical harmonic multipoles, respectively, as

$$I_{\nu_{\rm CMB}}(\hat{\boldsymbol{\gamma}}_{\rm CMB}) = \sum_{\ell=0}^{\infty}\sum_{m}^{\ell} a_{\ell m}^{I_{\rm CMB}}(\nu_{\rm CMB})Y_{\ell m}(\hat{\boldsymbol{\gamma}}_{\rm CMB}) \tag{1}$$

and

$$T(\hat{\boldsymbol{\gamma}}_{\rm CMB}) = \sum_{\ell=0}^{\infty}\sum_{m}^{\ell} a_{\ell m}^{T_{\rm CMB}}Y_{\ell m}(\hat{\boldsymbol{\gamma}}_{\rm CMB}), \tag{2}$$

where the sum notation $\sum_m^\ell$ is shorthand for $\sum_{m=-\ell}^{\ell}$. The frequency dependence of the intensity harmonic coefficients for a blackbody—with an average temperature T_0—can be expanded to first order in thermodynamic temperature harmonics as

$$a_{00}^{I_{\rm CMB}}(\nu) = \tilde{B}_\nu(T_0)a_{00}^{T_{\rm CMB}}, \tag{3a}$$

$$a_{\ell m}^{I_{\rm CMB}}(\nu) = \tilde{F}_\nu(T_0)a_{\ell m}^{T_{\rm CMB}}(\ell > 0), \tag{3b}$$

where $\tilde{B}_\nu(T_0) \equiv T_0^{-1}B_\nu(T_0)$, $B_\nu(T) \equiv (2h\nu^3/c^2)(1/e^{h\nu/kT}-1)$ is the blackbody spectrum, and $\tilde{F}_\nu(T_0) \equiv \tilde{B}_\nu(T_0)f(x)$ is the differential blackbody spectrum with $f(x) \equiv (xe^x/e^x-1)$ and $x = h\nu/kT_0$.

In order to find the observed multipoles in the boosted frame, we use the Lorentz invariance of I_ν/ν^3 to write the observed incoming intensity along the line-of-sight unit vector $\hat{\boldsymbol{\gamma}}$ at frequency ν as

$$I_\nu(\hat{\boldsymbol{\gamma}}) = \left(\frac{\nu}{\nu_{\rm CMB}}\right)^3 I_{\nu_{\rm CMB}}(\hat{\boldsymbol{\gamma}}_{\rm CMB}), \tag{4}$$

where

$$\nu_{\rm CMB} = \left(\frac{1-\beta\mu}{\sqrt{1-\beta^2}}\right)\nu \tag{5}$$

and

$$\hat{\boldsymbol{\gamma}}_{\rm CMB} = \left(\frac{(1-\sqrt{1-\beta^2})\mu - \beta}{1-\beta\mu}\right)\hat{\boldsymbol{\beta}} + \left(\frac{\sqrt{1-\beta^2}}{1-\beta\mu}\right)\hat{\boldsymbol{\gamma}} \tag{6}$$

are the frequency and line-of-sight unit vector, respectively, in the CMB rest frame and $\mu = \hat{\boldsymbol{\gamma}}\cdot\hat{\boldsymbol{\beta}}$. Equations (5) and (6), respectively, represent the Doppler and aberration effects. Expanding both sides of Eq. (4) in harmonic space allows us to find the observed multipoles in the moving frame as

$$a_{\ell' m'}^{I}(\nu) = \sum_{\ell=0}^{\infty}\sum_{m}^{\ell}\int\left(\frac{\nu}{\nu_{\rm CMB}}\right)^3 a_{\ell m}^{I_{\rm CMB}}(\nu_{\rm CMB})Y_{\ell m} \times(\hat{\boldsymbol{\gamma}}_{\rm CMB})Y^*_{\ell' m'}(\hat{\boldsymbol{\gamma}})\mathrm{d}^2\hat{\boldsymbol{\gamma}}. \tag{7}$$

Substituting Eqs. (5) and (6) into (7) will, respectively, result in the *Doppler and aberration leakage* of the nearby multipoles into each other. To nth order in β, the observed multipoles $a_{\ell' m'}^{I}(\nu)$ will have a contribution from $a_{\ell'\pm n, m'}^{I_{\rm CMB}}(\nu)$ of the rest frame. This integral has been computed analytically in Ref. [2]. We do not repeat the calculations here and use only the results hereafter. We also acquire the same notation for the frequency functions.

The boosted dipole.—First, we calculate the observed dipole in the moving frame to illustrate the dipole degeneracy problem. By setting $\ell' = 1$ in Eq. (7), we find [Eq. (B.37) in Ref. [2]]

$$\begin{aligned} a_{1m'}^{I}(\nu) = {} & \overbrace{\tilde{F}_\nu(T_0)a_{1m'}^{T_{\rm CMB}}}^{\text{Intrinsic dipole}} + \overbrace{\frac{2\sqrt{\pi}}{3}Y^*_{1m'}(\hat{\boldsymbol{\beta}})\tilde{F}_\nu(T_0)a_{00}^{T_{\rm CMB}}}^{\text{Kinematic dipole}} \\ & + \beta\sum_{m,n}^{2,1}{}_{0}^{1}\mathcal{G}_{1m'}^{2m}(\hat{\boldsymbol{\beta}})\tilde{F}_\nu^{(11)}(T_0)a_{2m}^{T_{\rm CMB}} \\ & + \beta\sum_{m,n}^{2,1}{}_{1}^{0}\mathcal{G}_{1m'}^{2m}(\hat{\boldsymbol{\beta}})\tilde{F}_\nu(T_0)a_{2m}^{T_{\rm CMB}} \\ & + O(\beta^2), \end{aligned} \tag{8}$$

where $\tilde{F}_\nu^{(11)}(T) = \tilde{F}_\nu(T)[g(x)-1]$ with $g(x) \equiv x\coth(x/2)$, while ${}_0^1\mathcal{G}_{1m'}^{2m}(\hat{\boldsymbol{\beta}})$ and ${}_1^0\mathcal{G}_{1m'}^{2m}(\hat{\boldsymbol{\beta}})$ are numerical factors of the order of ~ 1.

The first term in Eq. (8) is the intrinsic dipole of the CMB with the differential blackbody spectrum $\tilde{F}_\nu(T_0)$. The second term is what is normally identified as the kinematic dipole, which is a result of the Doppler leakage of the monopole into the observed dipole moment. Notice that the frequency dependence of this term is identical to the intrinsic dipole which makes the two components degenerate. The third and the fourth terms are, respectively, the Doppler and aberration leakages of the quadrupole into the dipole. These terms have never been considered in the analysis of the CMB dipole.

In order to build some intuition, instead of working with the $a_{1,m}^{T_{\rm CMB}}$ coefficients, we parametrize the three degrees of freedom for the intrinsic dipole in terms of an amplitude and two angles via the definition

$$a_{1,m}^{T_{\rm CMB}} \equiv \frac{4\pi}{3} dY_{1,m}^{*}(\theta_d, \phi_d). \quad (9)$$

We define the dipole vector $\vec{d} = d\hat{\boldsymbol{d}}$, where d and $\hat{\boldsymbol{d}} \equiv (\theta_d, \phi_d)$ are the amplitude and direction, respectively, of the maximum of the dipole on the sky.

With this new definition, we set out to study the observable effects of an intrinsic dipole of the order of $\sim 10^{-5}$ on the local dipole, monopole, and quadrupole of the CMB. In order to gauge the expected magnitude of the effect, we will consider two different dipoles with the amplitudes $d = 30\ \mu$K and $d = 60\ \mu$K [motivated by Ref. [13], Eqs. (31)–(33)]. We will refer to these dipoles, respectively, as $d30$ and $d60$.

The observed dipole intensity in the direction (θ, ϕ) is defined as $\delta I_\nu^{(1)}(\theta, \phi) \equiv \sum_{m'}^{1} a_{1m'}^{I}(\nu) Y_{1m'}(\theta, \phi)$. Figure 1 shows the contribution of each term in Eq. (8) to $\delta I_\nu^{(1)}(\theta_\beta, \phi_\beta)$ at different frequencies. Unless the intrinsic dipole is much larger than the one we chose, the dominant term in this equation is the leakage of the monopole into the dipole (kinematic dipole) with the thermodynamic temperature $\delta T^{(1)} \equiv \delta I_\nu^{(1)} / \tilde{F}_\nu(T_0) = 3.35$ mK. The next order contribution is due to the intrinsic dipole with the same frequency function as that of the kinematic dipole. The leakage of the quadrupole into the dipole is a motion-induced effect which does not depend on the intrinsic dipole at all. Since this term has a different frequency dependence, technically it could be used as an independent measure of β. However, the peak amplitude of this component—assuming the observed value of the quadrupole as input—is lower than the sensitivity of *PIXIE*, and therefore it is not likely to be useful for lifting the dipole degeneracy. Nevertheless, this extra leakage component should be taken into account for a precise analysis of the observed dipole in the future CMB surveys.

Now we show how the dipole degeneracy can be removed by looking at the motion-induced spectral distortions in the dipole's neighbors: the monopole ($\ell' = 0$) and the quadrupole ($\ell' = 2$).

The boosted monopole.—Using Eq. (7), it is easy to find the monopole of the CMB in a boosted frame [Eq. (B.36) in Ref. [2]]

$$\begin{aligned} a_{00}^{I}(\nu) = {} & \tilde{B}_\nu(T_0) a_{00}^{T_{\rm CMB}} + \beta^2 \tilde{B}_\nu^{(20)}(T_0) a_{00}^{T_{\rm CMB}} \\ & + \beta \sum_{m}^{1} \frac{2\sqrt{\pi}}{3} Y_{1m}(\hat{\boldsymbol{\beta}}) \tilde{F}_\nu^{(11)}(T_0) a_{1m}^{T_{\rm CMB}} \\ & - \beta \sum_{m}^{1} \frac{4\sqrt{\pi}}{3} Y_{1m}(\hat{\boldsymbol{\beta}}) \tilde{F}_\nu(T_0) a_{1m}^{T_{\rm CMB}} + O(\beta^2), \end{aligned} \quad (10)$$

with $\tilde{B}_\nu^{(20)}(T_0) = \frac{1}{6} \tilde{F}_\nu(T)[g(x) - 3]$. Here the first term is the well-known $T = 2.725$ blackbody spectrum, the second term is the second-order Doppler correction to the monopole, and the third and fourth terms are, respectively, the Doppler and aberration leakages of the dipole into the monopole.

The observed monopole intensity $I_\nu^{(0)}(\theta, \phi) = a_{00}^{I}(\nu) Y_{00}(\theta, \phi) = a_{00}^{I}(\nu)/2\sqrt{\pi}$ is plotted in Fig. 2 for different amplitudes and orientations of the intrinsic dipole. Using Eq. (9), we can rewrite Eq. (10) as

$$\delta I_\nu^{(0)} = \tilde{B}_\nu(T_0) T_0 + \beta \tilde{B}_\nu^{(20)}(T_0)[\beta T_0 + 2d(\hat{\boldsymbol{d}} \cdot \hat{\boldsymbol{\beta}})]. \quad (11)$$

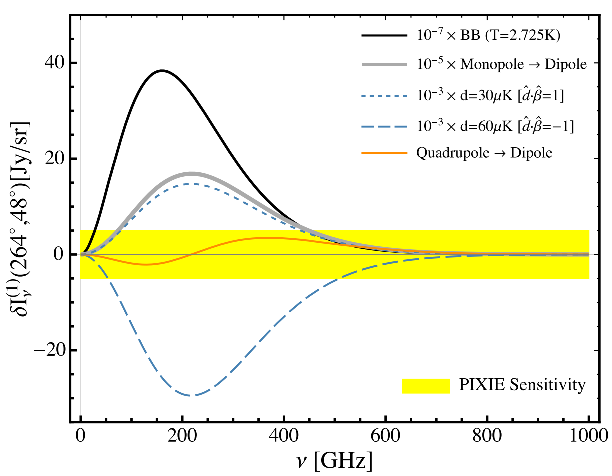

FIG. 1. The CMB dipole constituents observed in a moving frame with $\beta = 0.00128$ and $\hat{\boldsymbol{\beta}} = (264°, 48°)$. The intrinsic dipoles $d30$ and $d60$ have identical frequency functions as the kinematic dipole. The average $T = 2.725$ blackbody spectrum (solid black curve) is depicted in all plots for reference.

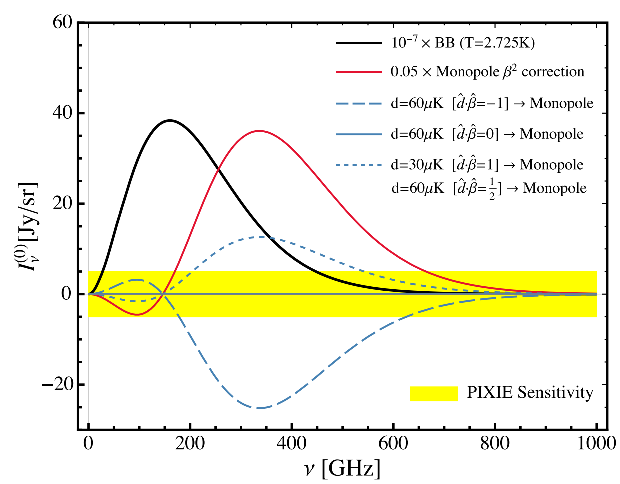

FIG. 2. The motion-induced spectral distortions of the observed CMB monopole.

Since the frequency dependence of the intrinsic monopole T_0 is different from the motion-induced terms, it can be fit and measured separately. Since the motion-induced spectral distortions depend the combination of the kinematic dipole (βT_0) and the projection of the intrinsic dipole along the direction of motion $[d(\hat{\boldsymbol{d}}\cdot\hat{\boldsymbol{\beta}})]$, it might seem like these two components still remain degenerate. However, combining this with the observed dipole in $\hat{\boldsymbol{\beta}}$ direction (with the quadrupole leakage term dropped, assuming it is negligible)

$$\delta I_\nu^{(1)}(\hat{\boldsymbol{\beta}}) = \tilde{F}_\nu(T_0)[\beta T_0 + d(\hat{\boldsymbol{d}}\cdot\hat{\boldsymbol{\beta}})], \qquad (12)$$

reveals that the monopole spectral distortion adds an independent equation that allows one to separate βT_0 and $d(\hat{\boldsymbol{d}}\cdot\hat{\boldsymbol{\beta}})$.

Since the leakage of the dipole into the monopole depends only on the projection of $\vec{\boldsymbol{d}}$ along $\hat{\boldsymbol{\beta}}$ (see Fig. 2), by only looking at the monopole alone one cannot find all three components of $\vec{\boldsymbol{d}}$; there remains an azimuthal degeneracy between the two vectors. Now we show that the leakage of the intrinsic dipole into the quadrupole can be exploited to find both the amplitude and direction of $\vec{\boldsymbol{d}}$.

The boosted quadrupole.—In a boosted frame, the intrinsic dipole also leaks into the observed quadrupole [Eq. (B.38) in Ref. [2]]

$$\begin{aligned} a^I_{2m'}(\nu) = {}& \tilde{F}_\nu(T_0)a^{T_{\rm CMB}}_{2m'} + \beta^2\frac{2\sqrt{\pi}}{5}Y^*_{2m'}(\hat{\boldsymbol{\beta}})\tilde{B}^{(22)}_\nu(T_0)a^{T_{\rm CMB}}_{00} \\ & + \beta\sum\nolimits^{1,1}_{m,n}{}^{1}_{0}\mathcal{G}^{1m}_{2m'}(\hat{\boldsymbol{\beta}})\tilde{F}^{(11)}_\nu(T_0)a^{T_{\rm CMB}}_{1m} \\ & + \beta\sum\nolimits^{1,1}_{m,n}{}^{0}_{1}\mathcal{G}^{1m}_{2m'}(\hat{\boldsymbol{\beta}})\tilde{F}_\nu(T_0)a^{T_{\rm CMB}}_{1m} \\ & + \beta\sum\nolimits^{3,1}_{m,n}{}^{1}_{0}\mathcal{G}^{3m}_{2m'}(\hat{\boldsymbol{\beta}})\tilde{F}^{(11)}_\nu(T_0)a^{T_{\rm CMB}}_{3m} \\ & + \beta\sum\nolimits^{3,2}_{m,n}{}^{0}_{1}\mathcal{G}^{3m}_{2m'}(\hat{\boldsymbol{\beta}})\tilde{F}_\nu(T_0)a^{T_{\rm CMB}}_{3m} + O(\beta^2), \end{aligned} \qquad (13)$$

where $\tilde{B}^{(22)}_\nu \equiv \frac{1}{3}\tilde{F}_\nu(T)g(x)$. The largest term here is the intrinsic quadrupole, followed by the leakage of the monopole into the quadrupole (the second term). The third and fourth terms represent the Doppler and aberration leakage of the intrinsic dipole into the quadrupole, respectively. (The frequency function of the dipole leakage is different from Ref. [32], which does not account for the aberration effect.) Figure 3 shows the contribution of different terms in Eq. (13) to the observed quadrupole intensity $\delta I^{(2)}_\nu(\theta,\phi) \equiv \sum^2_{m'} a^I_{2m'}(\nu)Y_{2m'}(\theta,\phi)$.

Note that the leakage of the intrinsic dipole $d30$ aligned with $\hat{\boldsymbol{\beta}}$ induces the same signal in the line-of-sight direction $\hat{\boldsymbol{\beta}}$ as the $d60$ dipole with $\hat{\boldsymbol{d}}\cdot\hat{\boldsymbol{\beta}} = 1/2$. However, in contrast to the case of the monopole, the spatial morphology of the dipole leakage into the quadrupole is not uniform over the whole sky and depends on $\hat{\boldsymbol{d}}$. Figure 4 shows this difference for two cases of dipoles with the same parallel component along $\hat{\boldsymbol{\beta}}$ but different $\hat{\boldsymbol{d}}$'s. Therefore, the whole sky map of the leakage component can be used to lift the degeneracy between the amplitude and the orientation of the dipoles. The dipole leakage into the quadrupole adds five independent equations to Eqs. (8) and (10) which, combined together, are more than enough for simultaneous determination of $\vec{\boldsymbol{d}}$ and $\vec{\boldsymbol{\beta}}$.

Discussion.—Future generations of microwave experiments are going to make accurate measurements of the

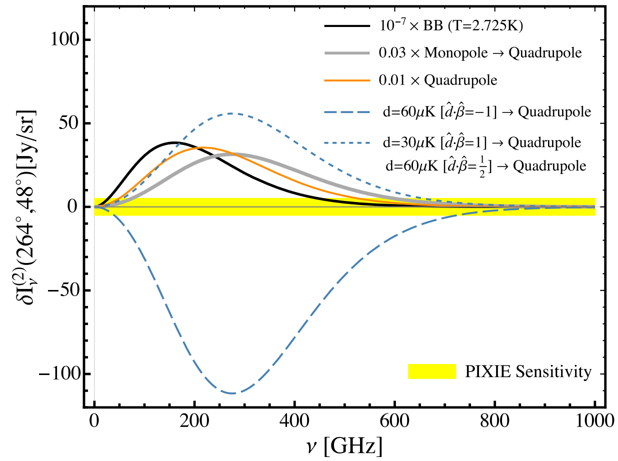

FIG. 3. The motion-induced spectral distortions of the observed CMB quadrupole. Since the leakage of the octupole has a different frequency function compared to the other components, we have assumed that this term can be identified and subtracted and therefore is not shown here. In this specific direction in the sky, the leakage of the $d30$ is not distinguishable from a $d60$ with the same projection along $\hat{\boldsymbol{\beta}}$ (short dashed blue curve). However, the amplitude of these two leakage components are different at other lines of sight (see Fig. 4).

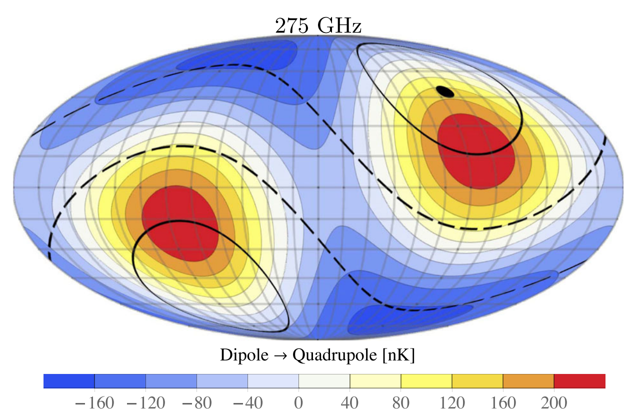

FIG. 4. Mollweide projection of the leakage of an intrinsic dipole $d60$ with $\hat{\boldsymbol{d}}\cdot\hat{\boldsymbol{\beta}} = 1/2$ into the observed quadrupole. The solid (dashed) black lines are the −40 (80) nK contour lines for the leakage of a smaller intrinsic dipole $d30$ with a different orientation $\hat{\boldsymbol{d}} = \hat{\boldsymbol{\beta}}$. Even though the two dipole leakage components have the same amplitude along the $\hat{\boldsymbol{\beta}}$ direction (black dot), their spatial morphology is different over the whole sky.

frequency spectrum of the CMB. We presented a framework for the accurate spectral modeling of the low multipoles of the CMB in a moving frame that should be considered in the future CMB surveys. In particular, we showed how measuring the spectral distortions in the CMB multipoles can be used to distinguish between the motion-induced and intrinsic dipole components of the CMB. The main idea is that our peculiar motion with respect to the CMB rest frame causes the low multipoles of the CMB to leak into each other. These leakage components induce distinct frequency distortions that can be used to determine both the amplitude and orientation of a possible intrinsic dipole in the CMB and separate it from the kinematic dipole.

Considering instrument sensitivity only, a *PIXIE*-like experiment will be able to detect the leakage of a 30μK dipole into the monopole and quadrupole at the peak frequencies (337 and 276 GHz) with $\sim 2.5\sigma$ and 11σ. The main limiting factor in the detection will, however, be foreground subtraction. The wide frequency coverage of *PIXIE* (400 channels), as well as the addition of external auxiliary data, will mitigate foreground effects and allow signals with magnitudes of the ones presented here to be detectable even in the presence of foregrounds [33]. A proper foreground subtraction analysis aimed at the detection of these specific signatures has not yet been carried out and would be needed to infer final statements on actual detectability.

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