Optimal Quantum Spatial Search on Random Temporal Networks

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To investigate the performance of quantum information tasks on networks whose topology changes in time, we study the spatial search algorithm by continuous time quantum walk to find a marked node on a random temporal network. We consider a network of n nodes constituted by a time-ordered sequence of Erdös-Rényi random graphs $G(n, p)$, where p is the probability that any two given nodes are connected: After every time interval τ, a new graph $G(n, p)$ replaces the previous one. We prove analytically that, for any given p, there is always a range of values of τ for which the running time of the algorithm is optimal, i.e., $\mathcal{O}(\sqrt{n})$, even when search on the individual static graphs constituting the temporal network is suboptimal. On the other hand, there are regimes of τ where the algorithm is suboptimal even when each of the underlying static graphs are sufficiently connected to perform optimal search on them. From this first study of quantum spatial search on a time-dependent network, it emerges that the nontrivial interplay between temporality and connectivity is key to the algorithmic performance. Moreover, our work can be extended to establish high-fidelity qubit transfer between any two nodes of the network. Overall, our findings show that one can exploit temporality to achieve optimal quantum information tasks on dynamical random networks.

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Temporal networks are ubiquitous: Natural, technological, and social networks typically have time-varying topologies. Recently, such networks have been extensively studied at the classical level [1–5]. However, quantum dynamics on temporal networks has largely been unexplored. Intuitively, one could expect that the uncontrolled dynamical loss and emergence of links would hinder the performance of quantum information tasks realized on networks, namely, for communication, computation, and sensing. But could this temporal character actually yield any advantages for such tasks? In this work, we consider the spatial search algorithm by continuous time quantum walk (CTQW) [6] to find a marked node on a temporal network and establish analytically that there are regimes where its performance is optimal.

This algorithm was first introduced in Ref. [6] and has been extensively studied on particular static graphs [7–10]. Furthermore, the analog version of Grover's algorithm [11] can be perceived as a spatial search by quantum walk on the complete graph [6]. Recently, the algorithm was proven to be optimal for Erdös-Rényi random graphs, i.e., graphs of n nodes with each link existing between any two nodes with probability p [12,13], as long as $p \geq p_{\text{static}} = \log^{3/2}(n)/n$ [14]. Moreover, as a random graph can also be obtained by randomly removing links from a complete graph, these results can be seen as an analysis of the robustness of quantum search on the complete graph to a random loss of links. Note that quantum dynamics on static Erdös-Rényi random graphs and other complex networks has been studied in Refs. [15,16]. Also, some properties of the evolution of quantum walks on dynamical percolation graphs such as the mixing time, return probabilities, and spreading were studied in Refs. [17–19].

In this Letter, we study how the quantum spatial search algorithm performs on random temporal networks. These networks are obtained as a sequence of Erdös-Rényi random graphs $G(n, p)$: After every time interval τ, a new graph $G(n, p)$ replaces the previous one. This problem can also be viewed as a spatial search on a complete graph with dynamical structural defects, i.e., where links can randomly vanish and reappear over time, as in a dynamical percolation problem.

We define the *temporality* of a network as the frequency with which a given network changes its topology as compared to the relevant energy scale of the Hamiltonian representing the network, and thus $1/\tau$ is a measure of temporality. Naturally, the introduction of this new feature leads to a much richer behavior in the algorithmic dynamics, as compared to the static scenario. In fact, now the optimality of the algorithm depends crucially on the interplay between τ and p.

In our work, we find a new threshold of p, namely, $p_{\text{temp}} = \log(n)/\sqrt{n}$, such that for $p \geq p_{\text{temp}}$ the algorithm is optimal irrespective of the temporality of the network. On the other hand, we show that a sufficiently high temporality ensures that the algorithm retains its optimality for arbitrarily low values of p. This holds even when the underlying random graphs are comprised of mostly isolated nodes and small trees which are graphs where, in the static case, quantum search would not provide any speedup.

Interestingly, there also exists an intermediate regime $p_{\text{static}} \leq p < p_{\text{temp}}$ where the spatial search algorithm is

optimal on the underlying random graphs, whereas for a certain interval of τ this is no longer the case. We find that, when the temporality of the network coincides with the energy scale of the Hamiltonian representing the network, the algorithmic running time is peaked. By gradually lowering or increasing the temporality, the running time of the algorithm decreases and, after a certain threshold of temporality, becomes optimal—a behavior also observed in Ref. [20] for the analog version of Grover's algorithm albeit in a different context. Our results show that quantum information processing tasks can be performed optimally on dynamically disordered structures.

Quantum spatial search on random temporal networks.—A temporal network is a dynamically evolving network of n vertices that alters its topology after a given time interval. As a result, links appear and disappear after every time interval. If initially the network is represented by a graph G_1, then after a time interval τ the topology of the network changes, and we obtain a new graph G_2, and so on. Thus, within a time t, a temporal network may be represented by a sequence of static graphs $G_{\text{temp}} = \{G_1, G_2, \ldots, G_m\}$, where $t = m\tau$ and $m \in \mathbb{N}$.

Naturally, a random temporal network is represented by a network that is a sequence of random graphs. Let us consider Erdös-Rényi random graphs $G(n, p)$. A random temporal network $G_{\text{temp}}(n, p, \tau)$ is a temporal sequence of Erdös-Rényi random graphs such that, after a time $t = m\tau$, the network will be defined as $G_{\text{temp}}(n, p, \tau) = \{G_1(n, p), G_2(n, p), \ldots, G_m(n, p)\}$, where $G_j(n, p)$ represents the random graph at the jth time interval. We shall focus on the optimality of the spatial search algorithm by CTQW on these networks and thus first introduce the algorithm briefly.

Let G represent a graph of n vertices $V = \{1, \ldots, n\}$. We consider the Hilbert space spanned by the localized quantum states at the vertices of the graph $\mathcal{H} = \text{span}\{|1\rangle, |2\rangle, \ldots, |n\rangle\}$. The search Hamiltonian corresponding to G is given by

$$H_{\text{search}} = -E|w\rangle\langle w| - \gamma A_G, \tag{1}$$

where $|w\rangle$ corresponds to the solution node of the search problem marked by the local site energy E, γ is a real number, and A_G is the adjacency matrix of the graph G [21]. We set the energy scale E to be 1, such that the quantum simulation of $|w\rangle\langle w|$ for time t would correspond to $\mathcal{O}(t)$ queries to the standard Grover oracle [6]. The initial state of the algorithm is usually chosen to be the equal superposition of all vertices, i.e., the state $|s\rangle = \sum_{i=1}^{n} |i\rangle/\sqrt{n}$. The quantum search algorithm is said to be optimal on graph G if there exists a value of γ such that, after a time $T = \mathcal{O}(\sqrt{n})$, the probability of obtaining the solution upon a measurement in the basis of the vertices is $|\langle w|e^{-iH_{\text{search}}T}|s\rangle|^2 = \mathcal{O}(1)$ [6].

In order to analyze this algorithm on $G_{\text{temp}}(n, p, \tau)$, we use two separate approaches to prove our results for different ranges of p. For $p \geq p_{\text{static}} = \log^{3/2}(n)/n$, we use the fact that the maximum eigenvalue of the adjacency matrix of each of the random graphs appearing during the time evolution is separated from the bulk of the spectrum, and the eigenstate corresponding to it is almost surely the initial condition of the algorithm $|s\rangle$, as was shown to be the case in Lemma 2 of Ref. [14]. To obtain the regime where the optimality of the algorithm is maintained as a function of τ and p, we use time-dependent perturbation theory. However, this property about the spectrum of adjacency matrices of random graphs does not hold when p is below the aforementioned threshold. So, for such regimes, we construct a linear superoperator that describes the average dynamics of the algorithm on random temporal networks. We present each of these approaches separately.

Quantum spatial search on random temporal networks having $p \geq p_{\text{static}}$.—As long as $p \geq \log^{3/2}(n)/n$, the eigenstate corresponding to the maximum eigenvalue of the adjacency matrix of an Erdös-Rényi random graph is almost surely the state $|s\rangle$ with eigenvalue np [14]. Thus, the adjacency matrix of each of the random graphs appearing in $G_{\text{temp}}(n, p, \tau)$ satisfies this property. Let A_j denote the adjacency matrix of the random graph appearing at the jth time instance (i.e., after a time $t = j\tau$). Then, each off-diagonal entry of A_j is 1 with probability p and 0 with probability $1 - p$. Let $B_j = A_j - np|s\rangle\langle s| + pI$, where B_j is a random matrix with each off-diagonal entry having mean 0 and variance p, with the diagonal entries being zero, and I is the identity matrix. We define the search Hamiltonian for $G_{\text{temp}}(n, p, \tau)$ as in Eq. (1) by choosing $\gamma = 1/(np)$. By expressing each of the adjacency matrices appearing in $G_{\text{temp}}(n, p, \tau)$ as mentioned previously, we obtain the following search Hamiltonian:

$$H_{\text{search}}(t) = \underbrace{-|w\rangle\langle w| - |s\rangle\langle s|}_{H_0} \underbrace{- \sum_{j=1}^{m} \gamma B_j f_j(t, \tau)}_{V(t)}, \tag{2}$$

where $f_j(t, \tau) = \Theta\big(t - (j-1)\tau\big) - \Theta(t - j\tau)$, where $\Theta(x)$ is the Heaviside function, and $m = T/\tau$ is the number of instances of random graphs appearing throughout the evolution time of $T = \mathcal{O}(\sqrt{n})$. Here H_0 induces a rotation in the two-dimensional subspace spanned by $|w\rangle$ and $|s\rangle$, whereas $V(t)$ will induce a coupling between this subspace and the $n - 2$ degenerate eigenspace of H_0. Also, H_0 is the search Hamiltonian corresponding to the quantum walk on a complete graph, where the search algorithm runs optimally [6,11]. In this case, we treat $V(t)$ as a perturbation to H_0 and use the time-dependent perturbation theory. Let $|\psi(t)\rangle$ be the wave function of the quantum walk obtained by evolving under $H_{\text{search}}(t)$. The error probability induced by the perturbation is thus $\epsilon = 1 - |\langle w|\psi(T)\rangle|^2$, where $T = \mathcal{O}(\sqrt{n})$.

We are interested in calculating when the average error probability $\langle\epsilon\rangle$ is bounded for a given τ and p. Whenever

$\langle\epsilon\rangle \sim o(1)$, the algorithm outputs the solution state $|w\rangle$ with probability $1-o(1)$ in $\mathcal{O}(\sqrt{n})$ time. Without the loss of generality, we intend to bound $\langle\epsilon\rangle = \mathcal{O}(1/\log(n))$ [22]. We prove that the average error probability is given by (for the derivation, see Sec. I of Supplemental Material [23]):

$$\langle\epsilon\rangle = \begin{cases} \mathcal{O}\left(\frac{\tau}{p\sqrt{n}}\right) & \text{if } \tau < \mathcal{O}(1), \\ \mathcal{O}\left(\frac{1}{p\tau\sqrt{n}}\right) & \text{if } \tau \geq \mathcal{O}(1). \end{cases} \quad (3)$$

First, we are interested in finding the regime of p for which the algorithm is robust to temporality. From Eq. (3), we find that, as long as $p \geq p_{\text{temp}} = \log(n)/\sqrt{n}$, the average error is bounded irrespective of any $0 < \tau \leq \mathcal{O}(\sqrt{n})$. For lower values of p, temporality becomes crucial to the optimality of the algorithm, and in fact for the range of p between p_{static} and p_{temp} there exist two separate regimes of temporality that determine the optimality of the algorithm: a fast temporality regime and a slow temporality regime such that, if the topology of the network alters faster than $\tau_{\text{fast}} = \mathcal{O}(p\sqrt{n}/\log(n))$ or slower than $\tau_{\text{slow}} = \mathcal{O}(\log(n)/(p\sqrt{n}))$, the algorithm remains optimal. The behavior of the algorithm in the intermediate regime of $\tau_{\text{fast}} < \tau < \tau_{\text{slow}}$ is also interesting, albeit suboptimal. As the temporality of the network increases from τ_{fast}, the algorithmic running time increases with it, peaking at $\tau = \mathcal{O}(1)$, after which it gradually decreases until $\tau = \tau_{\text{slow}}$. To confirm this, we plot in Fig. 1 the average running time of $G_{\text{temp}}(200, 0.06, \tau)$ (blue dots) and $G_{\text{temp}}(200, 0.1, \tau)$ (red squares) as a function of τ. As predicted, the average running time peaks when the temporality $1/\tau \approx 1$ and approaches the optimal running time (solid line) away from the peak.

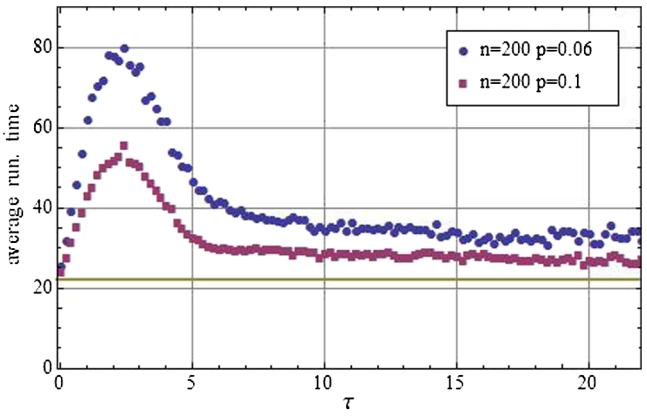

FIG. 1. Average running time of the quantum spatial search algorithm as a function of τ for $G_{\text{temp}}(200, 0.06, \tau)$ (in blue dots) and $G_{\text{temp}}(200, 0.1, \tau)$ (in red squares). Each point is averaged over 100 realizations. As predicted, the average running time peaks at $\tau \sim 1$, when the temporality coincides with the energy scale of the search Hamiltonian. Away from this peak, the average running time decreases gradually towards the optimal running time (indicated by the solid line).

A similar behavior has also been observed in Ref. [20] for the analog version of Grover's algorithm, for the following noise model: The authors consider a perturbation to the search Hamiltonian in the form of a random matrix, with each entry being a time-dependent random variable with a predefined autocorrelation function and with a certain cutoff frequency. The authors find that, when the cutoff frequency of noise scales much faster or slower than the energy scale of the Hamiltonian, the algorithm retains its optimality. On the other hand, when they scale similarly [i.e., when the cutoff frequency of noise is $\mathcal{O}(1)$], the average error is bounded by a constant only when the ratio of the norm of the perturbation Hamiltonian and that of the unperturbed search Hamiltonian scales as $\mathcal{O}(n^{-1/4})$. Analogously, we find that, for networks with constant temporality, the average error is constant when $p \sim 1/\sqrt{n}$, in which case the aforementioned ratio is also $||V(t)||/||H_0|| = ||B_j|| = \mathcal{O}(n^{-1/4})$, where we have used the fact that $||B_j|| = \mathcal{O}(\sqrt{np})$ [24,25]. This shows that the global features of the response of this algorithm with respect to the typical noise time scales for these two models are quite similar.

Note that we also recover the scenario of the spatial search algorithm on a static random network by choosing $\tau = \mathcal{O}(\sqrt{n})$. In this case, the average error is always bounded for $p \geq p_{\text{static}}$, thereby recovering the results of Ref. [14].

Quantum spatial search on random temporal networks having $p < p_{\text{static}}$.—Here we prove that, for random temporal networks with a sufficiently high temporality, the spatial search algorithm is optimal for arbitrarily low p. For this regime of p, the results obtained previously no longer hold, as $|s\rangle$ is not an eigenstate of the adjacency matrix (and np is no longer the maximum eigenvalue) of an Erdös-Rényi random graph. For $p < \log(n)/n$, the underlying random graphs are no longer connected [26]. Moreover, for $p \ll 1/n$, the static random graphs appearing during the time evolution of the algorithm are extremely sparse and are mostly comprised of isolated nodes and trees. In particular, we shall focus on finding a regime of optimality of the search algorithm for $p \leq 1/n$, while we refer the reader to Sec. II of Supplemental Material for results when $1/n < p < p_{\text{static}}$ [23].

In this regime, we follow a different approach: We consider the evolution of the quantum state averaged over all possible realizations of a random graph using the density matrix formalism. The number of possible realizations of $G(n, p)$ is $|G| = 2^N$, where $N = \binom{n}{2}$. The average dynamics of the algorithm on a random temporal network after one time step τ is described by the following superoperator:

$$\Phi(\rho) = \sum_{r=1}^{|G|} p_r e^{-iH_r\tau}\rho e^{iH_r\tau} = \langle e^{-iH_r\tau}\rho e^{iH_r\tau}\rangle, \quad (4)$$

where p_r is the probability of the rth realization and $H_r = |w\rangle\langle w| + \gamma A_{G_r}$ with A_{G_r} being the adjacency matrix corresponding to the rth realization of $G(n, p)$. Let $\langle X\rangle$ represent

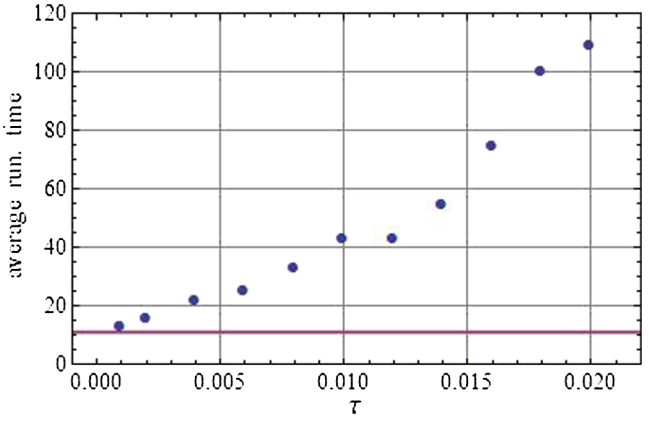

FIG. 2. Average running time of the quantum spatial search algorithm on $G_{\text{temp}}(50, 0.0008, \tau)$ (dots) as a function of τ. Each point is averaged over 50 realizations. Note that $p = 2/n^2$, and even then, for small enough τ, the algorithm runs in optimal time (solid line). As τ is increased, the algorithmic running time increases.

the expected value of X. The evolution of the algorithm after $m = \mathcal{O}(\sqrt{n}/\tau)$ time steps is given by $\Phi^m(\rho)$. The first-order expansion of the superoperator yields

$$\Phi(\rho) = \langle \rho - i\tau[H_r, \rho] \rangle + \delta \tag{5}$$

$$= \rho - i\tau[|w\rangle\langle w| + |s\rangle\langle s|, \rho] + \delta \tag{6}$$

$$= \Phi_0(\rho) + \delta, \tag{7}$$

where the second step follows because the expected value of each entry of A_{G_r} is p and so $\langle A_{G_r} \rangle = np|s\rangle\langle s|$. Thus, $\langle H_r \rangle = |w\rangle\langle w| + |s\rangle\langle s|$, which is the same as H_0 defined in Eq. (2) and is optimal for quantum spatial search. Here δ is the error induced by truncating the superoperator Φ after the first order and is given by $\delta \leq \sum_{k=2}^{\infty} (\tau^k/k!)\langle ||H_r^k|| \rangle$. Note that the superoperator Φ_0^m describes approximately the standard evolution of the algorithm under the Hamiltonian $\langle H_r \rangle$, and thus we intend to bound the error obtained by using the superoperator Φ instead of the superoperator Φ_0 to describe the dynamics for each of the m time steps. This is given by

$$\epsilon = ||\Phi^m(\rho) - \Phi_0^m(\rho)|| \leq m\delta. \tag{8}$$

Thus, to bound ϵ we need to bound $\langle ||A_{G_r}|| \rangle$. Since $p \leq 1/n$, the underlying random networks are extremely sparse, containing isolated nodes and few links. Thus, $||A_{G_r}||$ is bounded by the sum of the individual links of the random graphs. As p decreases further (i.e., $p \ll 1/n$), the aforementioned bound is better, as the underlying networks have fewer and fewer links. For a given range of p, we find the bound for τ where $\epsilon \leq \mathcal{O}(1/\log(n))$. In fact, we obtain that (for the derivations, refer to Sec. II of Supplemental Material [23])

$$\tau \leq \begin{cases} \frac{1}{n^{5/2}\log(n)} & \text{if } 1/n^2 \leq p \leq 1/n, \\ \frac{p}{\sqrt{n}\log(n)} & \text{if } p < 1/n^2. \end{cases} \tag{9}$$

In general, our results imply that, although p is well below the percolation threshold, and in fact the temporal network consists of graphs that do not have giant components and are mostly composed of isolated nodes and trees of $\mathcal{O}(1)$ nodes, a sufficiently high temporality can still lead to optimal search. This cannot be achieved by performing a quantum walk on any of these structures appearing as a static network. This has been confirmed in Fig. 2, wherein we plot (in blue points) the average running time of the quantum spatial search algorithm on random temporal networks $G_{\text{temp}}(50, 0.0008, \tau)$ with a value of p that is way below the percolation threshold $(p = 2/n^2)$. As expected, for sufficiently low values of τ, the running time of the algorithm is close to the optimal running time of $T = \pi\sqrt{n}/2$ (solid line) and increases as τ is increased. We summarize the regimes of τ and p where the algorithm is optimal in Fig. 3. See Supplemental Material for derivations [23].

Discussion.—We have proven analytically that, for any given p, there is always a range of values of τ for which the running time of the spatial search algorithm by CTQW on a random temporal network $G_{\text{temp}}(n, p, \tau)$ is optimal, i.e., $\mathcal{O}(\sqrt{n})$. Indeed, we find that the nontrivial interplay between p and the temporality of the network is key to the algorithm's performance (see Fig. 3).

We obtain a threshold $p_{\text{temp}} = \log(n)/\sqrt{n}$, above which the algorithm is optimal irrespectively of τ, i.e., of how fast or slowly the links appear and disappear in the dynamical network.

We also find that, for sufficiently low values of τ, the algorithm is optimal for any value of p. This means that a high temporality allows an optimal performance even when p is well below the static percolation threshold, i.e., when the underlying static graphs are comprised mostly of isolated nodes and trees of constant depth.

Interestingly, for $p_{\text{static}} < p < p_{\text{temp}}$, the algorithm is optimal on each static random graph but not always on the

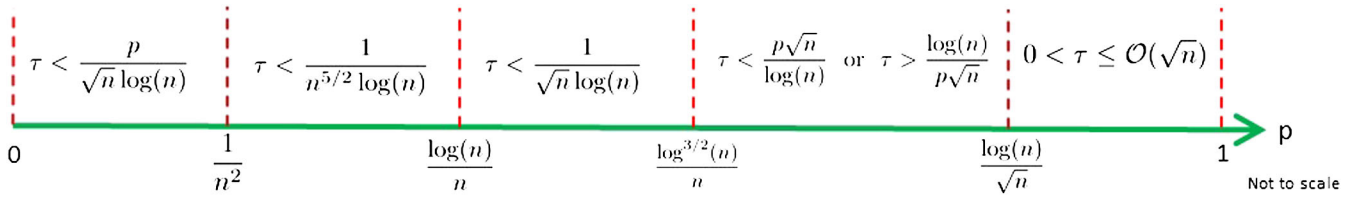

FIG. 3. Summary of analytical results: thresholds of τ above or below which the quantum spatial search algorithm on a random temporal network of n nodes is optimal for a given range of p.

temporal network composed by the sequence of such graphs. In the suboptimal regime, the algorithmic running time is peaked when the temporality of the network coincides with the energy scale of the search Hamiltonian. We can move away from this regime by decreasing or increasing the temporality: The running time of the algorithm will then decrease accordingly, reaching the optimal performance at τ_{slow} or τ_{fast}, respectively.

Note that our results on spatial search can also be extended to perform high-fidelity state transfer of a qubit between any two nodes of a random temporal network [14,27,28].

Finally, our findings can also be interpreted as an analysis of the robustness of the quantum spatial search algorithm and the state transfer protocol on a complete graph with dynamical structural defects. Furthermore, they pave the way to study quantum dynamics on non-Markovian temporal networks [29], as well as to exploit temporality as a control mechanism to improve or protect the effectiveness and efficiency of quantum information tasks on dynamical networks.

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[22] In our analysis when $p \geq \log^{3/2}(n)/n$, we neglect terms of $\mathcal{O}(1/(np))$, and we fix $\langle\epsilon\rangle = 1/\log(n)$ to be the upper bound of error throughout and obtain bounds for τ and p.
[23] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.119.220503 for the detailed derivations of the results of the Letter.
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