# Communication Games Reveal Preparation Contextuality

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A communication game consists of distributed parties attempting to jointly complete a task with restricted communication. Such games are useful tools for studying limitations of physical theories. A theory exhibits preparation contextuality whenever its predictions cannot be explained by a preparation noncontextual model. Here, we show that communication games performed in operational theories reveal the preparation contextuality of that theory. For statistics obtained in a particular family of communication games, we show a direct correspondence with correlations in spacelike separated events obeying the no-signaling principle. Using this, we prove that all mixed quantum states of any finite dimension are preparation contextual. We report on an experimental realization of a communication game involving three-level quantum systems from which we observe a strong violation of the constraints of preparation noncontextuality.

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*Introduction.*—Communication games are tools by which one can study fundamental limiting features of physical theories in terms of their ability to process information [1–3]. In these games, a number of parties intend to jointly solve a task despite the amount and type of communication being constrained by some rules. Thus, the task can be solved only with some probability, which depends on the theory by which they are assumed to operate. Therefore, communication games are frequent tools for identifying and quantifying quantum advantages over classical theories [4–10].

Interestingly, there are known examples of communication games in which the better-than-classical performance constitutes a certificate of the system lacking a preparation noncontextual ontological model [11–13]. An ontological model is a way of explaining the physics of an operational theory, by assuming that there are independent and objective (ontic) states subject to experiment. However, specifying a preparation does not necessarily specify the ontic state. A preparation may be represented by a distribution $\mu$ over the ontic states. Let two preparations $P_1$ and $P_2$ associated to distributions $\mu_1$ and $\mu_2$ be indistinguishable, i.e., satisfy $p(b|P_1, M) = p(b|P_2, M)$ for any measurement $M$ with outcome $b$. The assumption of preparation noncontextuality asserts that no additional features (called *contexts*) influence the physics of the preparations and, therefore, asserts that both preparations have equivalent representation in the ontological model: $\mu_1 = \mu_2$ [14]. If a theory does not satisfy this assumption, it is said to be preparation contextual. Preparation contextuality has been shown relevant for many foundational topics [3,15–18].

Here, we show that the performance of an operational theory in communication games constitutes a certificate of that theory exhibiting preparation contextuality. Specifically, we introduce communication constraints which keep the receiver oblivious about subsets of the information held by the sender. Preparation noncontextuality imposes a bound on the performance of any communication game executed under such an obliviousness constraint. This bound is violated by preparation contextual theories. Subsequently, we show how to understand no-signaling correlations from spacelike separated measurements (perhaps violating a Bell inequality) through a subclass of communication games. In particular, we find that quantum preparation contextuality manifested in communication games imposes a quantitative bound on quantum nonlocality (i.e., Bell inequality violations). Furthermore, we apply this result to resolve an open problem in this field: Which quantum states are preparation contextual? We show that all mixed quantum states in any finite dimension are preparation contextual. Finally, we present an experimental implementation of a quantum strategy in a specific communication game, inspired by the Collins-Gisin-Linden-Massar-Popescu (CGLMP) Bell inequality, in which three-level quantum systems are communicated. We certify a large violation of a preparation noncontextuality inequality.

*Communication games.*—In a two-player communication game, a party Alice (Bob) holds a set of data denoted $x \in I_A$ ($y \in I_B$) with associated probability distribution $p_A(x)$ [$p_B(y)$]. Alice encodes $x$ by preparing a state which is sent to Bob, who attempts to decode it with a measurement labeled $y$. This returns an outcome $b$. Subsequently, a payoff $\mathcal{C}_{x,y}^b \in \mathbb{R}$ is awarded. The average payoff earned by the partnership is

$$A \equiv \sum_{x\in I_A} \sum_{y\in I_B} \mathcal{C}_{x,y}^b p_A(x) p_B(y) p(b|x, y). \quad (1)$$

 

Equation (1) quantifies the *performance* in the game. However, the content of Alice's communication to Bob is restricted by some communication constraints. These ensure that the game is nontrivial; i.e., Alice cannot simply send $x$ to Bob. A suitable choice of these constraints enables the connection to tests of preparation contextuality.

*Communication games as tests of preparation contextuality.*—An operational theory is said to be preparation noncontextual [14] if operationally equivalent preparations imply equivalent distributions over the ontic states:

$$\forall y \quad \forall\, b:\; p(b|x,y) = p(b|x',y) \Rightarrow p(\lambda|x) = p(\lambda|x'), \tag{2}$$

where $\lambda$ is a hidden variable, $x$ and $x'$ are two preparations, and $y$ denotes a measurement.

We will now define a class of communication constraints which enables a connection to the premise of Eq. (2). The assumption of preparation noncontextuality then leads to a preparation noncontextuality inequality in which the performance in the communication game is the operator.

Construct $L$ subsets of the space $I_A$; $S_k \subset I_A$ for $k = 1, \ldots, L$. Now, choose communication constraints as follows: Impose an *obliviousness* constraint

$$\forall\; y, b, k, k':\; \frac{1}{q_k}\sum_{x\in S_k} p(x|b,y) = \frac{1}{q_{k'}}\sum_{x\in S_{k'}} p(x|b,y). \tag{3}$$

Here $q_k = p(x \in S_k) = \sum_{x\in S_k} p_A(x)$ serves as a normalization. In other words, Eq. (3) states that, no matter the performed measurement and observed outcome, Bob gains no information, as compared to what he knew before the communication, about to which set $S_k$ the data $x$ of Alice belong. Let us now apply Bayes' rule to the above summands: $p(x|b,y) = p(b|x,y)p(x|y)/p(b|y)$. Since $x$ and $y$ are independent, Eq. (3) becomes

$$\forall y \quad \forall\, b:\; \sum_{x\in S_k} p(b|x,y)\frac{p_A(x)}{q_k} = \sum_{x\in S_{k'}} p(b|x,y)\frac{p_A(x)}{q_{k'}}. \tag{4}$$

Note that each side is a convex combination, since $\{p_A(x)/q_k\}_{x\in S_k}$ is a probability distribution over the set $S_k$. Now, note that the probability that the outcome $b$ was obtained from a measurement on a preparation associated to $S_k$ is the convex mixing of its constitutes: $p(b|x \in S_k, y) = \sum_{x\in S_k} p(b|x,y)p_A(x)/q_k$. Similarly, the distribution of the hidden variable is $p(\lambda|x \in S_k) = \sum_{x\in S_k} p(\lambda|x)p_A(x)/q_k$. Putting it all together, we have $\forall y \quad \forall\, b:\; p(b|x \in S_k, y) = p(b|x \in S_{k'}, y)$, which takes the form of the premise of the preparation noncontextuality statement in Eq. (2). Thus, preparation noncontextuality imposes that $p(\lambda|x \in S_k) = p(\lambda|x \in S_{k'})$. Using Bayes' rule, we find that $p(x \in S_k|\lambda)/q_k = p(x \in S_{k'}|\lambda)/q_{k'}$. This means that, despite knowledge of the hidden variable, Eq. (3) remains satisfied.

Given any $\lambda$, Alice encodes $x$ classically knowing that the obliviousness constraint is satisfied. Therefore, the preparation noncontextual bound $p^{\text{pnc}}$ of Eq. (1) is obtained from maximizing Eq. (1) over all classical encodings respecting the obliviousness constraint. Hence, $A \leq p^{\text{pnc}}$ is a preparation noncontextuality inequality. ■

Clearly, for a given communication game, there are a plethora of ways in which one can choose the obliviousness constraint and construct the associated preparation noncontextuality inequality. In what follows, we will examine some interesting cases of the presented framework.

*Communication games based on Bell inequalities.*—Consider a general bipartite Bell experiment in which Alice and Bob share a two-particle state with each of them choosing measurements $X \in \{1, \ldots, m_A\}$, for some positive integer $m_A$, and $Y \in \{1, \ldots, m_B\}$, for some positive integer $m_B$, sampled from a distribution $p_A(X)$ and $p_B(Y)$, respectively. Each measurement returns an outcome $a, b \in \{1, \ldots, d\}$. From the resulting probability distribution $p(a,b|X,Y)$, one constructs a general Bell inequality

$$I_b \equiv \sum_{abXY} \mathcal{C}^{a,b}_{X,Y} p_A(X)p_B(Y)p(a,b|X,Y) \leq C, \tag{5}$$

where $C$ is the local realist bound and $\mathcal{C}^{a,b}_{X,Y}$ are real coefficients.

In the following, we construct a family of communication games and obliviousness constraints inspired by such Bell experiments. Alice is given inputs $(x, x_0) \in \{1, \ldots, m_A\} \times \{1, \ldots, d\}$ admitting the distribution $p(x_0, x) = p_g(x_0|x)p_A(x)$, with $p_A(x = i) = p_A(X = i)$ whereas $p_g(x_0|x)$ is yet to be specified. Bob has an input $y \in \{1, \ldots, m_B\}$ with distribution $p_B(y = i) = p_B(Y = i)$. The inputs $(x_0, x, y)$ in the communication game, respectively, correspond to $(a, X, Y)$ in the Bell experiment. Having received Alice's communication, Bob earns a payoff $\mathcal{C}^{x_0,b}_{x,y}$ if he outputs $b$ given a measurement of $y$ and that Alice held $(x_0, x)$. The performance is written

$$I_g[\{p_g(x_0|x)\}_x] \equiv \sum_{x_0xyb} \mathcal{C}^{x_0,b}_{x,y} p_A(x)p_B(y)p_g(x_0|x)p(b|x_0,x,y). \tag{6}$$

Notice that, for every choice of $\{p_g(x_0|x)\}_x$, we have a different communication game.

Alice's communication must satisfy the following obliviousness constraint. Partition Alice's $m_A d$ possible inputs into $m_A$ sets each containing $d$ elements; we define $S_k = \{x_0 x | x = k\}$ for $k = 1, \ldots, m_A$. The obliviousness constraint requires that Bob gains no information about to which $S_k$ the data $(x_0, x)$ belong. Inserting this into Eq. (4) with $q_k = p_A(x = k)$ and using Bayes' rule, we obtain

$$\forall\, b, y, k, k' : \sum_{x_0=1}^{d} p(x_0, b|x = k, y) = \sum_{x_0=1}^{d} p(x_0, b|x = k', y). \quad (7)$$

This constraint is an analogy of the directed no-signaling principle imposed by special relativity on correlations in spacelike separated measurement events: The probability of Bob's outcome marginalized over Alice's input $x_0$ is independent of Alice's other input $x$. One needs only to relabel $x_0$ by $a$ and $(x, y)$ by $(X, Y)$ to recover the corresponding statement in Bell experiments.

On the one hand, imagine we run a Bell experiment and achieve some value of $I_b$. Using Bayes' theorem and the obliviousness constraint (7), it is straightforwardly shown that if we choose the communication game in which $p_g$ coincides with the observed marginals of Alice, $p(a|X)$, one finds $I_g = I_b$. We explicitly consider the case of the quantum theory. In a Bell experiment, when Alice performs her measurement $X$, she renders Bob's local state in one of $d$ possible states labeled $\varrho_l^X$ for $l = 1, \ldots, d$. The probability of Bob's local state being $\varrho_l^X$ is the probability of Alice obtaining outcome $l$, i.e., $p(a = l|X)$. No signaling implies that the average state of Bob is independent of the measurement choice $X$ of Alice. We associate for every $X$ the set $\{\varrho_l^X\}_{l=1}^d$ to the states in $S_X$ prepared by Alice in our communication game. As shown, these will necessarily satisfy the obliviousness constraint (7) while by construction returning the same performance in the communication game (6) as in the Bell experiment, namely, $I_g = I_b$.

On the other hand, imagine we have not specified $p_g(x_0|x)$. Let $\lambda$ index all functions $f_\lambda(x) : \{1, \ldots, m_A\} \to \{1, \ldots, d\}$. By choosing a suitable probability distribution $\mu(\lambda)$, we can write $p_g(x_0|x) = \sum_\lambda \mu(\lambda) D_A(x_0|x\lambda)$, where $D_A(x_0|x\lambda) = \delta_{f_\lambda(x), x_0}$. Alice then communicates $\lambda$, which contains no information about $x$, to Bob, who decodes the message using some strategy $D_B$. We find

$$I_g = \sum_{x_0 x y b} \mathcal{C}_{x,y}^{x_0,b} p_A(x) p_B(y) \sum_\lambda \mu(\lambda) D_A(x_0|x\lambda) D_B(b|y\lambda). \quad (8)$$

This is precisely the notion of local realist models for the Bell experiment (5). Hence, if we choose $p_g(x_0|x)$ such that there is a local hidden variable strategy that both (i) has $p_g(x_0|x) = p(a|X)$ as a marginal of Alice and (ii) saturates the local realist bound $C$ of (5), the preparation noncontextuality inequality $I_g \leq C$ will be tight. Of particular interest is to choose $p_g(x_0|x)$ such that it coincides with Alice's marginals in a maximal violation of a Bell inequality given some operational no-signaling theory. Then, we assert that $I_g$ can witness a violation of preparation noncontextuality corresponding to the maximal Bell inequality violation.

Note that only very particular obliviousness constraints and communication games retain the analogy to the no-signaling principle through our construction. In Ref. [19], we present a family of games that is not of the type presented in this section. The corresponding preparation noncontextuality inequalities are many-outcome generalizations of the those based on parity-oblivious multiplexing [11].

*Quantum preparation contextuality limits maximal quantum nonlocality.*—If Alice and Bob share entangled states, all mixed states can be prepared on Bob's side by considering the average of his local state computed over the outcomes of Alice obtained from some measurement. Thus, due to our previous discussion, it follows that the maximal quantum violation of a bipartite Bell inequality is a limitation imposed by the preparation contextuality allowed in the quantum theory. This generalizes the result of Ref. [3], showing this statement for the Clauser-Horne-Shimony-Holt inequality [20]. We exemplify this generalization by shining light on the numerical quantum violations of the preparation noncontextuality inequalities considered in Ref. [13]. These inequalities were based on communication games which happen to admit an obliviousness constraint of the form considered in the above section. The corresponding Bell inequalities were in fact studied in Ref. [21] in a different context. Comparing the numerics for quantum preparation contextuality [13] and the quantum nonlocality [21], one indeed finds that these agree very accurately.

*All mixed states are preparation contextual.*—The maximally mixed quantum state of dimension $d = 2, 3, 4, 5$ is known to be preparation contextual [13,14]. So is every mixed qubit state [22]. Our mapping between communication games and Bell inequalities allows us to straightforwardly show that all mixed quantum states of any dimension $d$ are preparation contextual. For this purpose, consider the CGLMP Bell inequality [23], which is a bipartite facet Bell inequality with $d$ outcomes for both observers. For any $d$, this inequality can be violated by all pure bipartite entangled states of dimension $d$ [24]. Hence, all possible mixed quantum states of dimension $d$ can appear as the average state of Bob after either of Alice's measurements. That average state is just the state of Bob's part of the entangled system. Since quantum strategies in the Bell scenario can be mapped to quantum strategies in a communication game (of the form previously discussed) testing preparation contextuality, it follows that all mixed quantum states of dimension $d$ are preparation contextual.

*A specific communication game.*—Let us focus on the CGLMP Bell inequality with $d = 3$ and construct the preparation noncontextuality inequality based on the associated communication game. Following our previous discussion, we let Alice hold $x = x_0 x \in \{0, 1, 2\} \times \{0, 1\}$ with $p(x_0, x) = 1/6$ and Bob hold $y \in \{0, 1\}$ with $p(y) = 1/2$. In order to satisfy the obliviousness constraint, Alice's communication $\rho_{x_0 x}$ must in the quantum theory obey $\sum_{x_0=0}^{2} p(x_0|x = 0)\rho_{x_0 0} = \sum_{x_0=0}^{2} p(x_0|x = 1)\rho_{x_0 1}$. Since the preparation noncontextual bound coincides with the local bound of the CGLMP inequality (which achieves its maximal quantum violation with uniform marginals on Alice), our preparation noncontextuality inequality reads

$$A_3 \equiv \frac{1}{12}\sum_{x_0 xyk}(-1)^k p(b = T_k|x_0, x, y) \leq 1/2, \quad (9)$$

where $T_k = x_0 - (-1)^{x+y+k}k - xy \bmod 3$ for $k = 0, 1$. The maximal quantum violation of the CGLMP Bell inequality is $A_3 = (3 + \sqrt{33})/12 \approx 0.7287$ [25], which immediately translates into an equal quantum violation of the inequality (9). In Ref. [19], we give the details of the corresponding quantum strategy in the communication game.

*Experiment.*—We experimentally confirm the above prediction of quantum preparation contextuality. The experimental implementation of the communication game uses three-path encoding for preparing qutrits. Single photons are initially prepared in the $|H\rangle$ polarization state by the use of polarization fiber controllers in a single-mode fiber (SMF). The qutrit state is prepared using the two spatial modes of three polarization beam splitters (PBSs) (see Fig. 1). The states required for the game, $|\psi_{\mathrm{in}}\rangle = \cos(2\chi_1)|0\rangle + \sin(2\chi_1)\sin(2\chi_2)|1\rangle + \sin(2\chi_1)\cos(2\chi_2)|2\rangle$, are prepared by suitably orienting the half-wave plates (HWPs) $\chi_1$ and $\chi_2$. Details are given in Ref. [19].

We use a heralded single-photon source generating twin photons at 780 nm by spontaneous parametric down-conversion. In this process, a nonlinear crystal type II ($\beta$-barium borate) is pumped using a high-power femtosecond laser such that a pump photon probabilistically converts into two lower-energy photons, called the signal and idler. The twin photons pass through a 3 nm filter and are coupled into single-mode fibers to have well-defined spatial and spectral properties. A detection of the idler then heralds the signal photon.

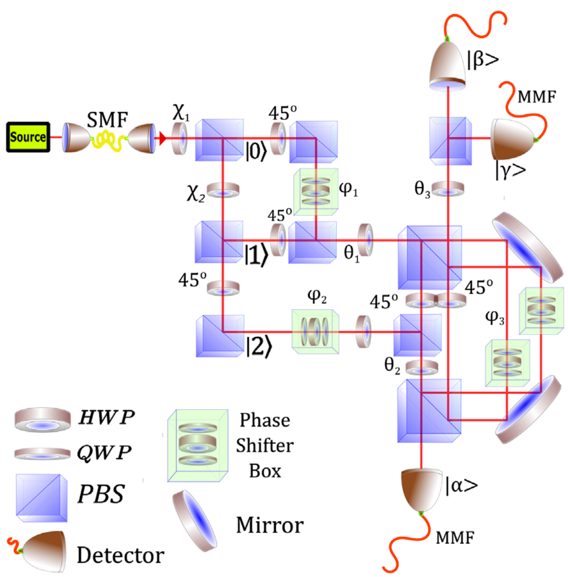


FIG. 1. Experimental setup. Suitable settings of $\chi_1$ and $\chi_2$ allow us to produce the desired qutrit states for the task. Measurement basis selection is implemented by appropriate settings of HWPs $\theta_1$, $\theta_2$, and $\theta_3$ and by setting the total experimental phase $(\phi_i; i \in 1, 2, 3)$ between path modes by employing a phase shifter box (QWP-HWP-QWP) inside the setup. Detection events in detectors $|\alpha\rangle$, $|\beta\rangle$, and $|\gamma\rangle$ are used to obtain the respective probabilities.

The corresponding experimental setup consists of three subsequent interferometers comprising of single-photon interferometers between all three paths followed by a stable and compact Sagnac interferometer, such that, while performing a measurement in a given measurement basis, the state is projected into basis vectors of the chosen basis. The protocol requires measurements in the computational basis and a second basis defined in Ref. [19]. Moreover, state tomography is performed using measurements in four mutually unbiased bases (MUBs), so that the total set of measurements is informationally complete [26]. For this purpose, the choice of a given measurement basis is enabled by suitable orientations of the HWPs $\theta_1$, $\theta_2$, and $\theta_3$ (see Table I in Ref. [19]) and by the introduction of a phase $(\phi_i; i \in 1, 2, 3)$ between the special modes by employing a set of three wave plates QWP-HWP-QWP (phase shifter box) geometries at different tilding positions [27].

A measurement projects the state onto the basis vectors. These are represented by the spatial modes of the two PBSs in the Sagnac interferometer (denoted by $|\alpha\rangle$, $|\beta\rangle$, and $|\gamma\rangle$). In our experiment, the photons arriving at $|\alpha\rangle$, $|\beta\rangle$, and $|\gamma\rangle$ are collected by multimode fibers that are in turn coupled to single-photon silicon avalanche photodiodes from Excelitas Technologies with an effective detection efficiency $\eta_d = 0.55$. A home-built field-programmable gate array-based timing system records the coincidence events between the arriving and trigger (idler) photons with a detection time window of 1.7 ns. The number of detection events at each detector is used to compute the respective probabilities. In each measurement round, approximately 60000 photons were detected per second. The measurement time was 10 s.

From the measured probabilities, we computed $A_3^{\mathrm{pri}} \approx 0.7172 \pm 0.0365$, which is in good agreement with the theoretical prediction. We reconstructed the states using variational quantum tomography [26,28] and the experimental results from four MUBs. We found the following fidelities for the six states: $|\psi_{11}\rangle \sim 0.9826$, $|\psi_{12}\rangle \sim 0.9804$, $|\psi_{13}\rangle \sim 0.9893$, $|\psi_{21}\rangle \sim 0.9838$, $|\psi_{22}\rangle \sim 0.9876$, and $|\psi_{23}\rangle \sim 0.9840$. These small imperfections cause the obliviousness constraint not to be perfectly satisfied. Next, we shall see how to overcome this issue.

*Data analysis.*—Reference [29] constructed a method in which one maps measured outcome probabilities (primary data), which does not perfectly satisfy a strict equivalence constraint, into another set of probabilities (secondary data) that satisfies that equivalence constraint. Then, one uses the secondary data to calculate the parameter of interest in the experiment. We will use this method to strictly enforce the obliviousness constraint and then compute $A_3$.

The primary data in our experiment consist of six $2 \times 3$ matrices [one for each preparation $(x_0, x)$] with elements $\mathbf{P}_{i,j}^{x_0 x} \equiv P^{\mathrm{lab}}(j|x_0, x, i)$ corresponding to performing measurement $i$ in the laboratory and obtaining outcome $j$. We will assume that the underlying physical theory governing the system is linear, allowing us to search for secondary data in the form of six other matrices $\{\mathbf{P}'^{x_0x}\}_{x_0,x}$ that are in the convex hull of $\{\mathbf{P}^{x_0x}\}_{x_0,x}$. That is, we let $\forall\, x_0,x\colon \mathbf{P}'^{x_0,x} = \sum_{x_0'=0}^{2}\sum_{x'=0}^{1} w_{x_0',x'}^{x_0x}\mathbf{P}^{x_0',x'}$, where $\forall\ x_0,x\colon\ w_{x_0',x'}^{x_0x}$ is a probability distribution. We seek secondary data which (i) satisfy the obliviousness constraint and (ii) on average are as close to the primary data as possible. This corresponds to a linear program:

$$S \equiv \max_{\{w\}} \frac{1}{6}\sum_{x_0=0}^{2}\sum_{x=0}^{1} w_{x_0,x}^{x_0,x},$$
$$\text{such that}\sum_{x_0}\mathbf{P}'^{x_0 0} = \sum_{x_0}\mathbf{P}'^{x_0 1}. \tag{10}$$

We find $S \approx 0.9938$, indicating that the secondary data are close to the primary data. Using the secondary data to compute $A_3$, we obtain $A_3^{\mathrm{sec}} \approx 0.7118 \pm 0.0365$. This is only marginally smaller than $A_3^{\mathrm{pri}}$. It is in good agreement with the theoretical prediction of the quantum theory and strictly satisfies the obliviousness constraint.

*Conclusions.*—We have established relations between operational statistics in a class of communication games and tested preparation contextuality. We showed close relations between quantum nonlocality and quantum correlations in such communication games and also shown all mixed quantum states of finite dimension to be preparation contextual. Furthermore, we provided an experimental demonstration of a quantum communication game showing a large violation of a preparation noncontextuality inequality.

We conclude with some open problems: (i) Do communication games without obliviousness constraints admit a connection to some operational physical assumption in the same spirit as presented here for games respecting an obliviousness constraint? (ii) Are generalizations of the presented framework to more than two players possible? (iii) Can the considered communication games be used in one-sided device-independent cryptography protocols?

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[19] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.119.220402 for (i) the theory part of SM shows a family of preparation noncontextuality inequalities based on parity-obliviousness and the details of the quantum strategy realized by the experiment, (ii) the experimental part of SM shows details for the experimental settings and the results obtain by our implementation.
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