Strongly Correlated Metal Built from Sachdev-Ye-Kitaev Models

Xue-Yang Song,[1,2] Chao-Ming Jian,[2,3] and Leon Balents[2]
[1]International Center for Quantum Materials, School of Physics, Peking University, Beijing 100871, China
[2]Kavli Institute of Theoretical Physics, University of California, Santa Barbara, California 93106, USA
[3]Station Q, Microsoft Research, Santa Barbara, California 93106-6105, USA
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Prominent systems like the high-T_c cuprates and heavy fermions display intriguing features going beyond the quasiparticle description. The Sachdev-Ye-Kitaev (SYK) model describes a $(0+1)$D quantum cluster with random all-to-all *four*-fermion interactions among N fermion modes which becomes exactly solvable as $N\to\infty$, exhibiting a zero-dimensional non-Fermi-liquid with emergent conformal symmetry and complete absence of quasiparticles. Here we study a lattice of complex-fermion SYK dots with random intersite *quadratic* hopping. Combining the imaginary time path integral with *real* time path integral formulation, we obtain a heavy Fermi liquid to incoherent metal crossover in full detail, including thermodynamics, low temperature Landau quasiparticle interactions, and both electrical and thermal conductivity at all scales. We find linear in temperature resistivity in the incoherent regime, and a Lorentz ratio $L\equiv(\kappa\rho/T)$ varies between two universal values as a function of temperature. Our work exemplifies an analytically controlled study of a strongly correlated metal.

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Introduction.—Strongly correlated metals comprise an enduring puzzle at the heart of condensed matter physics. Commonly a highly renormalized heavy Fermi liquid occurs below a small coherence energy scale, while at higher temperatures a broad incoherent regime pertains in which quasiparticle description fails [1–9]. Despite the ubiquity of this phenomenology, strong correlations and quantum fluctuations make it challenging to study. The exactly soluble SYK models, which systematize and extend early ideas of random interaction models [10–13], provide a powerful framework to study such physics. The most-studied SYK_4 model, a $(0+1)$D quantum cluster of N Majorana fermion modes with random all-to-all four-fermion interactions [14–22] has been generalized to SYK_q models with q-fermion interactions. Subsequent works [23,24] extended the SYK model to higher spatial dimensions by coupling a lattice of SYK_4 quantum clusters by additional four-fermion "pair hopping" interactions. They obtained electrical and thermal conductivities completely governed by diffusive modes and nearly temperature-independent behavior owing to the identical scaling of the interdot and intradot couplings.

Here, we take one step closer to realism by considering a lattice of complex-fermion SYK clusters with SYK_4 intracluster interaction of strength U_0 and random intercluster "SYK_2" two-fermion hopping of strength t_0 [25–30]. Unlike the previous higher dimensional SYK models where local quantum criticality governs the entire low temperature physics, here as we vary the temperature, two distinctive metallic behaviors appear, resembling the previously mentioned heavy fermion systems. We assume $t_0\ll U_0$, which implies strong interactions, and focus on the correlated regime $T\ll U_0$. We show the system has a coherence temperature scale $E_c\equiv t_0^2/U_0$ [25,31,32] between a heavy Fermi liquid and an incoherent metal. For $T<E_c$, the SYK_2 induces a Fermi liquid, which is, however, highly renormalized by the strong interactions. For $T>E_c$, the system enters the incoherent metal regime and the resistivity ρ depends linearly on temperature. These results are strikingly similar to those of Parcollet and Georges [33], who studied a variant SYK model obtained in a double limit of infinite dimension and large N. Our model is simpler, and does not require infinite dimensions. We also obtain further results on the thermal conductivity κ, entropy density, and Lorentz ratio [34,35] in this crossover. This work bridges traditional Fermi-liquid theory and the hydrodynamical description of an incoherent metallic system.

SYK model and imaginary-time formulation.—We consider a d-dimensional array of quantum dots, each with N species of fermions labeled by $i,j,k,\ldots$,

$$H=\sum_x\sum_{i<j,k<l}U_{ijkl,x}c_{ix}^\dagger c_{jx}^\dagger c_{kx}c_{lx}+\sum_{\langle xx'\rangle}\sum_{i,j}t_{ij,xx'}c_{i,x}^\dagger c_{j,x'}, \tag{1}$$

where $U_{ijkl,x}=U_{klij,x}^*$ and $t_{ij,xx'}=t_{ji,x'x}^*$ are random zero mean complex variables drawn from Gaussian distribution whose variances $\overline{|U_{ijkl,x}|^2}=2U_0^2/N^3$ and $\overline{|t_{ij,x,x'}|^2}=t_0^2/N$.

In the imaginary time formalism, one studies the partition function $Z=\mathrm{Tr}e^{-\beta(\mathcal{H}-\mu\mathcal{N})}$, with $\mathcal{N}=\sum_{i,x}c_{i,x}^\dagger c_{i,x}$, written as a path integral over Grassman fields $c_{ix\tau}$, $\bar{c}_{ix\tau}$. Owing to the self-averaging established for the SYK model at large N, it is sufficient to study $\bar{Z}=\int[d\bar{c}][dc]e^{-S_c}$, with (repeated species indices are summed over)

$$S_c = \sum_x \int_0^\beta d\tau \bar{c}_{ix\tau}(\partial_\tau - \mu) c_{ix\tau} - \int_0^\beta d\tau_1 d\tau_2 \left(\sum_x \frac{U_0^2}{4N^3} \bar{c}_{ix\tau_1} \bar{c}_{jx\tau_1} c_{kx\tau_1} c_{lx\tau_1} \bar{c}_{lx\tau_2} \bar{c}_{kx\tau_2} c_{jx\tau_2} c_{ix\tau_2} + \sum_{\langle xx' \rangle} \frac{t_0^2}{N} \bar{c}_{ix\tau_1} c_{jx'\tau_1} \bar{c}_{jx'\tau_2} c_{ix\tau_2} \right). \quad (2)$$

The basic features can be determined by a simple power counting. Considering for simplicity $\mu = 0$, starting from $t_0 = 0$, the U_0^2 term is invariant under $\tau \to b\tau$ and $c \to b^{-1/4} c$, $\bar{c} \to b^{-1/4} \bar{c}$, fixing the scaling dimension $\Delta = 1/4$ of the fermion fields. Under this scaling $\bar{c}\partial_\tau c$ term is irrelevant. Yet upon addition of two-fermion coupling, under rescaling, $t_0^2 \to b t_0^2$, so two-fermion coupling is a *relevant* perturbation. By standard reasoning, this implies a crossover from the SYK_4-like model to another regime at the energy scale where the hopping perturbation becomes dominant, which is $E_c = t_0^2/U_0$. Assuming no intermediate fixed points, we expect the renormalization flow is to the SYK_2 regime, i.e., to a Fermi liquid. Indeed, keeping the SYK_2 term invariant fixes $\Delta = 1/2$, and $U_0^2 \to b^{-1} U_0^2$ is irrelevant. Since the SYK_2 Hamiltonian (i.e., $U_0 = 0$) is quadratic, the disordered free fermion model supports quasiparticles and defines a Fermi-liquid limit. For $t_0 \ll U_0$, E_c defines a crossover scale between SYK_4-like non-Fermi-liquid and the low temperature Fermi liquid. The crossover behavior studied below will justify our previous assumption of the absence of intermediate fixed points between the SYK_2 and SYK_4 regimes.

At the level of thermodynamics, this crossover can be rigorously established using imaginary time formalism. Introducing a composite field $G_x(\tau,\tau') = (-1/N)\sum_i c_{ix\tau}\bar{c}_{ix\tau'}$ and a Lagrange multiplier $\Sigma_x(\tau,\tau')$ enforcing the previous identity, one obtains $\bar{Z} = \int [dG][d\Sigma] e^{-NS}$, with the action

$$\begin{aligned} S = &-\sum_x \ln\det\left[(\partial_\tau - \mu)\delta(\tau_1 - \tau_2) + \Sigma_x(\tau_1, \tau_2)\right] \\ &+ \int_0^\beta d\tau_1 d\tau_2 \Bigg\{ -\sum_x \left[\frac{U_0^2}{4} G_x(\tau_1,\tau_2)^2 G_x(\tau_2,\tau_1)^2 \right. \\ &\left. + \Sigma_x(\tau_1,\tau_2) G_x(\tau_2,\tau_1) \right] + t_0^2 \sum_{\langle xx' \rangle} G_{x'}(\tau_1,\tau_2) G_x(\tau_2,\tau_1) \Bigg\}. \end{aligned} \quad (3)$$

The large N limit is controlled by the saddle point conditions $\delta S/\delta G = \delta S/\delta\Sigma = 0$, satisfied by $G_x(\tau,\tau') = G(\tau - \tau')$, $\Sigma_x(\tau,\tau') = \Sigma_4(\tau - \tau') + z t_0^2 G(\tau - \tau')$ (z is the coordination number of the lattice of SYK dots), which obey

$$\begin{aligned} G(i\omega_n)^{-1} &= i\omega_n + \mu - \Sigma_4(i\omega_n) - z t_0^2 G(i\omega_n), \\ \Sigma_4(\tau) &= -U_0^2 G(\tau)^2 G(-\tau), \end{aligned} \quad (4)$$

where $\omega_n = (2n+1)\pi/\beta$ is the Matsubara frequency. We solve them numerically and reinsert into Eq. (3) to obtain the free energy, hence the full thermodynamics [24,36,37]. Consider the entropy S. A key feature of the SYK_4 solution is an extensive ($\propto N$) entropy [17] in the $T \to 0$ limit, an extreme non-Fermi-liquid feature. This entropy must be removed over the narrow temperature window set by the coherence energy E_c. Consequently, we expect that $\mathrm{S}/N = \mathcal{S}(T/E_c)$ for T, $E_c \ll U_0$, where the universal function $\mathcal{S}(\mathcal{T} = 0) = 0$ indicating no zero temperature entropy in a Fermi liquid, and $\mathcal{S}(\mathcal{T} \to \infty) = 0.4648\ldots$, recovering the zero temperature entropy of the SYK_4 model. The universal scaling collapse is confirmed by numerical solution, as shown in Fig. 1. This implies also that the specific heat $NC = (T/E_c)\mathcal{S}'(T/E_c)$, and, hence, the low-temperature Sommerfeld coefficient

$$\gamma \equiv \lim_{T \to 0} \frac{C}{T} = \frac{\mathcal{S}'(0)}{E_c} \quad (5)$$

is *large* due to the smallness of E_c. Specifically, compared with the Sommerfeld coefficient in the weak interaction limit $t_0 \gg U_0$, which is of order t_0^{-1}, there is an "effective mass enhancement" of $m^*/m \sim t_0/E_c \sim U_0/t_0$. Thus, the low temperature state is a heavy Fermi liquid.

To establish that the low temperature state is truly a strongly renormalized Fermi liquid with large Fermi-liquid parameters, we compute the compressibility, $NK = \partial\mathcal{N}/\partial\mu|_T$. Because the compressibility has a smooth low temperature limit in the SYK_4 model, we expect that K is only weakly perturbed by small t_0. For $t_0 \ll U_0$, we indeed have $K \approx K|_{t_0=0} = c/U_0$ with the constant $c \approx 1.04$ regardless of T/E_c. For free fermions, the compressibility and Sommerfeld coefficient are both proportional to the single-particle density of states (DOS), and in particular $\gamma/K = \pi^2/3$ for free fermions. Here we find $\gamma/K = [\mathcal{S}'(0)/c]U_0/E_c \sim (U_0/t_0)^2 \gg 1$. This can only be reconciled with Fermi-liquid theory by introducing a large Landau interaction parameter. In Fermi-liquid

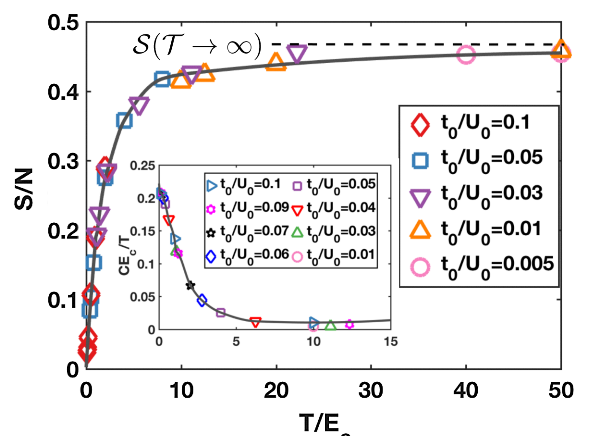

FIG. 1. The entropy and specific heat(inset) collapse to universal functions of (T/E_c), given t_0, $T \ll U_0 (z = 2)$. $C \to \mathcal{S}'(0)T/E_c$ as $T/E_c \to 0$. Solid curves are guides to the eyes.

theory, one introduces the interaction f_{ab} via $\delta\varepsilon_a = \sum_b f_{ab}\delta n_b$, where a, b label quasiparticle states. For a diffusive disordered Fermi liquid, we take $f_{ab} = F/g(0)$, where $g(0)$ is the quasiparticle DOS, and F is the dimensionless Fermi-liquid interaction parameter. The standard result of Fermi-liquid theory [37] is that γ is unaffected by F but K is renormalized, leading to $\gamma/K = (\pi^2/3)(1+F)$. We see that $F \sim (U_0/t_0)^2 \gg 1$, so that the Fermi liquid is extremely strongly interacting. Comparing to the effective mass, one has $F \sim (m^*/m)^2$.

Real time formulation.—While imaginary time formulation is adequate for thermodynamics, it encounters difficulties in addressing transport due to the difficulty of analytic continuation to zero *real* frequency in the presence of the emergent low energy scale E_c. Instead, we reformulate the problem in real time using the Keldysh path integral. The Keldysh formalism calculates the partition function $Z = (\mathrm{Tr}[\rho U]/\mathrm{Tr}[\rho])$ with $\rho = e^{-\beta(H-\mu N)}$ and U the *identity* evolution operator $U = e^{-i(\mathcal{H}-\mu\mathcal{N})(t_0-t_f)}e^{-i(\mathcal{H}-\mu\mathcal{N})(t_f-t_0)}$ describing evolving forward from $t_0 \to t_f$ (with Keldysh label $+$) and backward (Keldysh label $-$) identically. Paralleling the imaginary-time development, we introduce collective variables $G_{x,ss'}(t,t') = (-i/N)\sum_i c^s_{ixt}\bar{c}^{s'}_{ixt'}$ and $\Sigma_{x,ss'}$ with s, $s' = \pm$ labeling the Keldysh contour, and integrate out the fermionic fields to obtain $\bar{Z} = \int[dG][d\Sigma]e^{iNS_K}$ [37], with the Keldysh action,

$$\begin{aligned} iS_K = &\sum_x \ln\det[\sigma^z(i\partial_t+\mu)\delta(t-t') - \Sigma_x(t,t')] \\ &- \sum_{ss'}\int_{t_0}^{t_f} dt dt'\Bigg[\sum_x \frac{U_0^2}{4}ss'G_{x,ss'}(t,t')^2 G_{x,s's}(t',t)^2 \\ &- \sum_x \Sigma_{x,ss'}(t,t')G_{x,s's}(t',t) \\ &+ \sum_{\langle x'x\rangle} t_0^2 ss' G_{x,ss'}(t,t')G_{x',s's}(t',t)\Bigg], \end{aligned} \quad (6)$$

where Σ_x in the determinant is to be understood as the matrix $[\Sigma_{x,ss'}]$ and σ^z acts in Keldysh space. We obtain the numerical solution to the Green's functions [37] by solving for the saddle point of S_K. We plot in Fig. 2 the spectral weight $A(\omega) \equiv (-1/\pi)\mathrm{Im}G_R(\omega)$ (G_R is the retarded Green function) at fixed $U_0/T = 10^4$ for $E_c/T = 0$, 0.09, 1, 9, which illustrates the crossover between the SYK$_4$ and Fermi-liquid regimes. For $\omega \gg E_c$, we observe the quantum critical form of the SYK$_4$ model, which displays ω/T scaling, evident in the figure from the collapse onto a single curve at large ω/T. At low frequency, the SYK$_4$ model has $A(\omega \ll T) \sim 1/\sqrt{U_0 T}$, whose divergence as $T \to 0$ is cut-off when $T \lesssim E_c$. This is seen in the reduction of the peak height in Fig. 2, $\sqrt{U_0 T}A(\omega = 0)$, with increasing E_c/T. On a larger frequency scale (inset), the narrow "coherence peak," associated with the small spectral weight of heavy quasiparticles, is clearly visible.

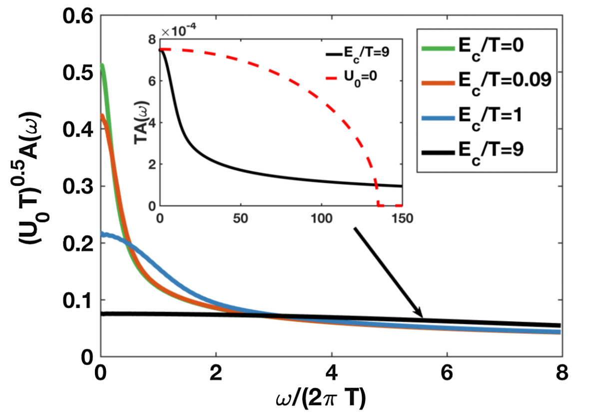

FIG. 2. The spectral weight $A(\omega)$ at fixed $U_0/T = 10^4$, $\mu = 0$, $z = 2$ for $E_c/T = 0$, 0.09, 1, 9, corresponding a crossover from SYK$_4$ limit to the "heavy-Fermi-liquid" regime. Inset shows the comparison of Green's function for $T/E_c = 9$ with the free fermion limit result.

We now turn to transport, and for simplicity focus on the particle-hole symmetric case hereafter. The strategy is to obtain electrical and heat conductivities from the fluctuations of charge and energy, respectively, using the Einstein relations. We first consider charge, and study the low-energy $U(1)$ phase fluctuation $\varphi(x,t)$, which is the conjugate variable to particle number density $\mathcal{N}(x,t)$, around the saddle point of the action S_K. Allowing for phase fluctuations around the saddle point solution amounts to taking

$$\begin{aligned} G_{x,ss'}(t,t') &\to G_{x,ss'}(t-t')e^{-i[\varphi_s(x,t)-\varphi_{s'}(x,t')]}, \\ \Sigma_{x,ss'}(t,t') &\to \Sigma_{x,ss'}(t-t')e^{-i[\varphi_s(x,t)-\varphi_{s'}(x,t')]}, \end{aligned} \quad (7)$$

where $G_{x,ss'}(t-t')$ and $\Sigma_{x,ss'}(t-t')$ are the saddle point solutions. Expanding (6) to quadratic order in φ_s, $S_K = S_K^{sp} + S_\varphi$, yields the lowest order effective action for the $U(1)$ fluctuations. This is most conveniently expressed in terms of the "classical" and "quantum" components of the phase fluctuations, defined as $\varphi_{c/q} = (\varphi_+ \pm \varphi_-)/2$ and in Fourier space:

$$\begin{aligned} iS_\varphi = \sum_{\mathbf{p}}\int_{t_0}^{t_f} dt dt'[&\Lambda_1(t-t')\partial_t\varphi_{c,\mathbf{p}}(t)\partial_t\varphi_{q,-\mathbf{p}}(t') \\ &- v(\mathbf{p})\Lambda_2(t-t')\varphi_{c,\mathbf{p}}(t)\varphi_{q,-\mathbf{p}}(t')] + \cdots. \end{aligned} \quad (8)$$

Here the first term arises from the $\ln\det[\cdot]$ and the second from the hopping (t_0^2) term in Eq. (6). The function $v(\mathbf{p})$ encodes the band structure for the two-fermion hopping term, dependent on lattice details, and the ellipses represent $O(\varphi_q^2)$ terms which do not contribute to the density correlations (and are omitted hereafter—see the Supplemental Material [37] for reasons). The coefficients $\Lambda_1(t)$ and $\Lambda_2(t)$ are expressed in terms of saddle point Green's functions in Ref. [37]. We remark here that any further approximations, e.g., conformal invariance, are not assumed to arrive at action (8), and hence this derivation applies in *all* regimes.

In the low frequency limit, the Fourier transforms of $\Lambda_1(t)$, $\Lambda_2(t)$ behave as $\Lambda_1(\omega) \approx -2iK$ and $\Lambda_2(\omega) \approx 2KD_\varphi \omega$, which *defines* the positive real parameters K and D_φ. At small momentum, for an isotropic Bravais lattice, $v(\mathbf{p}) = p^2$ (with unit lattice spacing), and the phase action becomes

$$iS_\varphi = -2K \sum_p \int_{-\infty}^{+\infty} d\omega \varphi_{c,\omega}(i\omega^2 - D_\varphi p^2 \omega)\varphi_{q,-\omega}. \qquad (9)$$

The density-density correlator is expressed as

$$D_{Rn}(x,t;x',t') \equiv i\theta(t-t')\langle[\mathcal{N}(x,t),\mathcal{N}(x',t')]\rangle$$
$$= \frac{i}{2}\langle \mathcal{N}_c(x,t)\mathcal{N}_q(x',t')\rangle, \qquad (10)$$

where $\mathcal{N}_s \equiv \frac{sN\delta S_\varphi}{\delta \dot{\varphi}_s}$, $\mathcal{N}_{c/q} = \mathcal{N}_+ \pm \mathcal{N}_-$ (keeping momentum-independent components—see Supplemental Material [37]). Adding a contact term to ensure that $\lim_{p\to 0} D_{Rn}(p, \omega \neq 0) = 0$ [38], the action (9) yields the diffusive form [39]

$$D_{Rn}(p,\omega) = \frac{-iNK\omega}{i\omega - D_\varphi p^2} + NK = \frac{-NKD_\varphi p^2}{i\omega - D_\varphi p^2}. \qquad (11)$$

From this we identify NK and D_φ as the compressibility and charge diffusion constant, respectively. The electric conductivity is given by Einstein relation $\sigma \equiv 1/\rho = NKD_\varphi$, or, restoring all units, $\sigma = NKD_\varphi(e^2/\hbar)a^{2-d}$ (a is lattice spacing). Note the proportionality to N: in the standard nonlinear sigma model formulation, the dimensionless conductance is large, suppressing localization effects. This occurs because both U and t interactions scatter between all orbitals, destroying interference from closed loops.

The analysis of energy transport proceeds similarly. Since energy is the generator of time translations, one considers the time-reparametrization (TRP) modes induced by $t_s \to t_s + \epsilon_s(t)$ and defines $\epsilon_{c/q} = \frac{1}{2}(\epsilon_+ \pm \epsilon_-)$. The effective action for TRP modes to the lowest-order in p, ω reads [37]

$$iS_\epsilon = \sum_p \int_{-\infty}^{+\infty} d\omega \epsilon_{c,\omega}[2i\gamma\omega^2 T^2 - p^2\Lambda_3(\omega)]\epsilon_{q,-\omega} + \cdots, \qquad (12)$$

where the ellipses has the same meaning as in Eq. (8). At low frequency, the correlation function integral, given in the Supplemental Material [37], behaves as $\Lambda_3(\omega) \approx 2\gamma D_\epsilon T^2 \omega$, which defines the energy diffusion constant D_ϵ. This identification is seen from the correlator for energy density modes $\varepsilon_{c/q} \equiv (iN\delta S_\epsilon/\delta\dot{\epsilon}_{q/c})$,

$$D_{R\varepsilon}(p,\omega) = \frac{i}{2}\langle \varepsilon_c \varepsilon_q \rangle_{p,\omega} = \frac{-NT^2\gamma D_\epsilon p^2}{i\omega - D_\epsilon p^2}, \qquad (13)$$

where we add a contact term to ensure conservation of energy at $p = 0$. The thermal conductivity reads $\kappa = NT\gamma D_\epsilon$ ($k_B = 1$)–like σ, is $O(N)$.

Scaling collapse, Kadowaki-Woods, and Lorentz ratios.—Electric or thermal conductivities are obtained from $\lim_{\omega\to 0}\Lambda_{2/3}(\omega)/\omega$, expressed as integrals of real-time correlation functions, and can be evaluated numerically for any T, t_0, U_0. Introducing generalized resistivities, $\rho_\varphi = \rho$, $\rho_\varepsilon = T/\kappa$, we find remarkably that for t_0, $T \ll U_0$, they collapse to universal functions of one variable,

$$\rho_\zeta(t_0, T \ll U_0) = \frac{1}{N} R_\zeta\left(\frac{T}{E_c}\right), \qquad \zeta \in \{\varphi, \varepsilon\}, \qquad (14)$$

where $R_\varphi(\mathcal{T})$, $R_\varepsilon(\mathcal{T})$ are dimensionless universal functions. This scaling collapse is verified by direct numerical calculations shown in Fig. 3(a). From the scaling form (14), we see the *low temperature* resistivity obeys the usual Fermi-liquid form

$$\rho_\zeta(T \ll E_c) \approx \rho_\zeta(0) + A_\zeta T^2, \qquad (15)$$

where the temperature coefficient of resistivity $A_\zeta = (R''_\zeta(0)/2NE_c^2)$ is large due to small coherence scale in

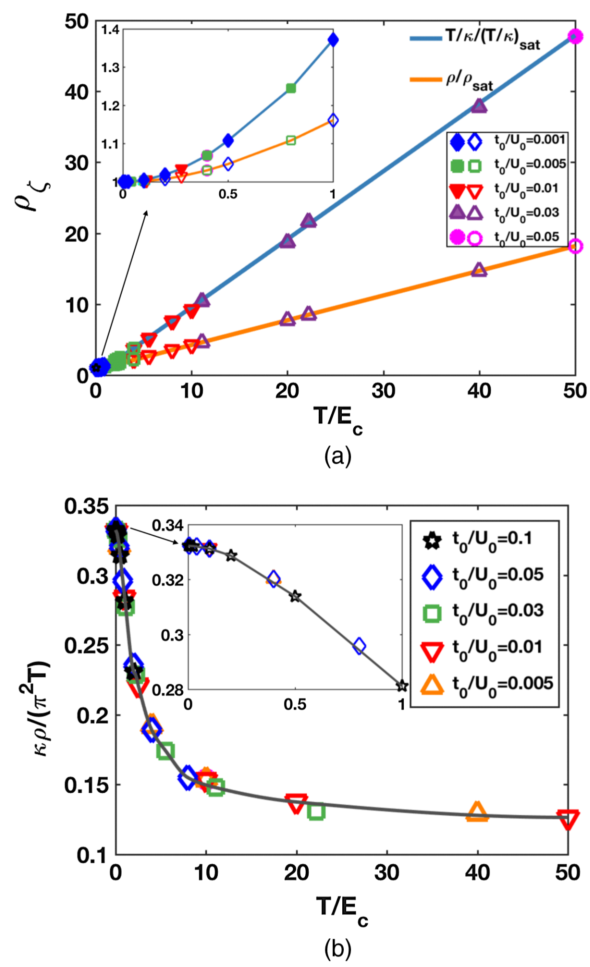

FIG. 3. (a) For t_0, $T \ll U_0$, $\rho_{\varphi/\varepsilon}$ "collapse" to $R_{\varphi/\varepsilon}(T/E_c)/N$. (b) The Lorentz ratio $(\kappa\rho/T)$ reaches two constants $(\pi^2/3)$, $(\pi^2/8)$, in the two regimes. The solid curves are guides to the eyes.

denominator, characteristic of a strongly correlated Fermi liquid. Famously, the Kadowaki-Woods ratio, $A_\varphi/(N\gamma)^2$, is approximately system independent for a wide range of correlated materials [40,41]. We find here $(A_\varphi/(N\gamma)^2) = (R''_\varphi(0)/2[\mathcal{S}'(0)]^2 N^3)$ is independent of t_0 and U_0!

Turning now to the incoherent metal regime, in the limit of large arguments, $\mathcal{T} \gg 1$, the generalized resistivities vary linearly with temperature: $R_\zeta(\mathcal{T}) \sim c_\zeta \mathcal{T}$. We analytically obtain $c_\varphi = (2/\sqrt{\pi})$ and $c_\varepsilon = (16/\pi^{5/2})$ [37], implying that the Lorenz number, characterizing the Wiedemann-Franz law, takes the unusual value $L = (\kappa/\sigma T) \to (\pi^2/8)$ for $E_c \ll T \ll U_0$. More generally, the scaling form (14) implies that L is a universal function of T/E_c, verified numerically as shown in Fig. 3(b). The Lorenz number increases with lower temperature, saturating at $T \ll E_c$ to the Fermi-liquid value $\pi^2/3$.

Conclusion.—We have shown that the SYK model provides a soluble source of strong local interactions which, when coupled into a higher-dimensional lattice by ordinary but random electron hopping, reproduces a remarkable number of features of strongly correlated metals, including heavy quasiparticles with small spectral weight, a largely system-independent Kadowaki-Woods ratio, T-linear high temperature resistivity, and an anomalous Lorenz number in the incoherent regime. The remarkable success of this simple soluble model suggests exciting prospects for extending the treatment to more realistic systems, and to shed light on the physical content of various numerical results from dynamical mean field theory [42], which shares significant mathematical similarity to basic equations of this work.

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