Cavity QED Engineering of Spin Dynamics and Squeezing in a Spinor Gas

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We propose a method for engineering spin dynamics in ensembles of integer-spin atoms confined within a high-finesse optical cavity. Our proposal uses cavity-assisted Raman transitions to engineer a Dicke model for integer-spin atoms, which, in a dispersive limit, reduces to effective atom-atom interactions within the ensemble. This scheme offers a promising and flexible new avenue for the exploration of a wide range of spinor many-body physics. As an example of this, we present results showing that this method can be used to generate spin-nematic squeezing in an ensemble of spin-1 atoms. With realistic parameters, the scheme should enable substantial squeezing on time scales much shorter than current experiments with spin-1 Bose-Einstein condensates.

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Gases of ultracold Bose atoms possessing internal spin degrees of freedom—spinor Bose gases—offer a remarkable variety of possibilities for the investigation of quantum fluids, in contexts that include, for example, magnetism, superfluidity, and many-body quantum dynamics [1,2]. In this latter context, tremendous experimental progress has occurred in recent years based upon collision-induced spin-mixing dynamics in spinor Bose-Einstein condensates (BECs) [3–24]. Such systems have allowed for the generation of quantum spin squeezing and entangled states [14–19,22–26] following a range of proposals [27–32], as well as the study of quantum phase transitions [7–9,20,21,24] and the parametric amplification of quantum spin fluctuations [10–12,19,33].

Spinor BECs in which atoms in all magnetic sublevels of a single hyperfine ground state (e.g., the $F=1$ ground state of ^{87}Rb) are condensed correspond to ensembles of integer-spin particles. For small, tightly confined condensates, one may assume that the different atomic states have the same spatial wave function—the single-mode approximation—after which one can show that the collisional spin dynamics is described by a Hamiltonian of the form $\lambda\hat{\mathbf{S}}^2$, where $\hat{\mathbf{S}}=(\hat{S}_x,\hat{S}_y,\hat{S}_z)$ is the total spin vector (operator) and λ is the collisional spin interaction energy per particle integrated over the condensate [34,35]. The spinor dynamical rate is $c=2N\lambda$, where N is the number of atoms, and is typically on the order of 10 Hz for 40 000 ^{87}Rb atoms [16]. If the longitudinal magnetization $\langle\hat{S}_z\rangle$ is a constant of the motion (e.g., zero), then this Hamiltonian can be reduced further to $\lambda(\hat{S}_x^2+\hat{S}_y^2)$. With the addition of a magnetic field, the Hamiltonian gains a linear Zeeman shift $p\hat{S}_z$ (which can also be assumed to be a constant of the motion) and a quadratic Zeeman shift $q\hat{N}_0$, where $\hat{N}_0$ is the population in the $m=0$ state. The ratio q/c describes a rich phase diagram, with highly entangled ground states in several limits [21,24,31,32]. In particular, if $|q|\ll c>0$, the ground state is the spin-singlet state, which has fundamental interest due to its high entanglement but also applications ranging from precision measurements [36] to no-classical solution quantum information processing [37]. In addition, the transitions between these different phases are of interest with respect to the Kibble-Zurek mechanism [20,38,39].

In this Letter, we propose an alternative scheme to producing spin-mixing dynamics in a gas of integer-spin atoms that uses cavity-mediated Raman transitions to engineer the required spinor dynamics. Our proposal borrows from earlier schemes for engineering effective Dicke models of collective two-level-atom ensembles coupled strongly to a quantized cavity mode [40–42] but considers an arguably simpler configuration and limit, which yields a Dicke model for integer-spin (alkali) atoms. This approach has in fact been demonstrated very recently in a study of nonequilibrium phase transitions in this model [43]. In the dispersive limit of this model, where the cavity mode is only virtually excited, the resulting Hamiltonian mimics collisional interactions in a spinor BEC.

We consider an ensemble of alkali atoms tightly confined (e.g., by a three-dimensional optical lattice) inside a high-finesse optical cavity. The atomic ensemble is considered dilute enough to exclude direct atom-atom interactions, while the atoms are coupled uniformly to cavity and laser fields. As illustrated in Fig. 1, we consider a scheme of cavity-assisted Raman transitions in which the fields are very far detuned from the relevant excited state manifold. Here, instead of isolating effective spin-1/2 systems [44], we consider transitions within a complete hyperfine ground state, in this instance, the $F=1$ ground state of ^{87}Rb. Adiabatic elimination of the atomic excited states then creates an effective model for an ensemble of spin-1

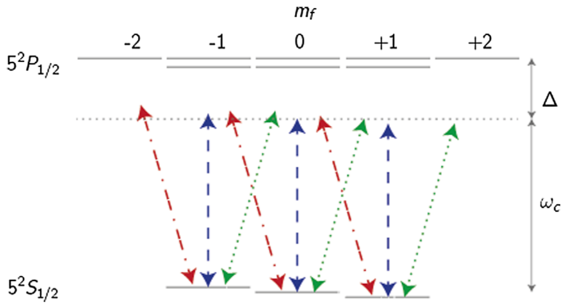

FIG. 1. Level diagram for the implementation of an effective Dicke model using the $F = 1$ ground state of ^{87}Rb. Interactions are engineered via Raman transitions on the D_1 line mediated by a cavity mode (blue dashed line) and σ_+ (green dotted line) or σ_- (red dot-dashed line) polarized laser fields.

atoms coupled to a cavity mode (see Supplemental Material [45]).

In the limit that the detunings of the fields are very large—in particular, much larger than the energy separations of the excited state hyperfine levels (e.g., as in [43], where the detuning is 127 GHz)—then the internal structure of the excited state manifold becomes unimportant, and symmetries in the dipole operator cause the effective Hamiltonian to simplify greatly. Considering an open quantum system, with the cavity field decay rate given by κ (but atomic spontaneous emission neglected due to the large detuning), this model is described by the master equation for the atom-field density operator ρ:

$$\dot{\rho} = -i[\hat{H}, \rho] + \kappa\mathcal{D}[\hat{a}]\rho, \tag{1}$$

where $\hat{a}$ is the cavity mode annihilation operator, $\mathcal{D}[\hat{a}]\rho = 2\hat{a}\rho\hat{a}^\dagger - \rho\hat{a}^\dagger\hat{a} - \hat{a}^\dagger\hat{a}\rho$, and

$$\hat{H} = \omega\hat{a}^\dagger\hat{a} + \omega_0\hat{S}_z + \frac{\lambda_-}{\sqrt{2N}}(\hat{a}\hat{S}_+ + \hat{a}^\dagger\hat{S}_-) + \frac{\lambda_+}{\sqrt{2N}}(\hat{a}\hat{S}_- + \hat{a}^\dagger\hat{S}_+). \tag{2}$$

Here, $\hat{S}_\pm$ are the spin-1 collective raising and lowering operators, while the coefficients of the various terms (for the $F = 1$ manifold in ^{87}Rb coupled via the D_1 line) are given by

$$\omega = \omega_c - \frac{\omega_- + \omega_+}{2} + \frac{Ng^2}{3\Delta}, \tag{3}$$

$$\omega_0 = \omega_z - \frac{\omega_- - \omega_+}{2} + \frac{\Omega_-^2 - \Omega_+^2}{24\Delta}, \tag{4}$$

$$\lambda_\pm = \frac{\sqrt{N}g\Omega_\pm}{12\Delta}. \tag{5}$$

Here ω_c is the frequency of the cavity mode, $\omega_\pm$ ($\Omega_\pm$) are the bare frequencies (Rabi frequencies) of the $\sigma_\pm$ polarized laser fields, ω_z is the Zeeman splitting of the $F = 1$ levels (due to an applied magnetic field, if present), g is the single-atom-cavity coupling strength (for the ^{87}Rb D_2 line cycling transition), and Δ is the detuning of the fields from the atomic resonance.

This configuration provides a "clean" and tuneable system with which to study the Dicke model, as demonstrated in Ref. [43]. In particular, it has the independence of couplings $\lambda_\pm$ not present in BEC formulations of the Dicke model [49,50], while it also avoids a nonlinear coupling term of the form $\hat{S}_z\hat{a}^\dagger\hat{a}$ that features in all the current spin-1/2 versions of the Dicke model [40,44,49–53]. We note that it can also be applied to other hyperfine ground states in alkali atoms, enabling, e.g., tunable interactions for ensembles of effective spin-2 (^{87}Rb or ^{85}Rb), -3 (^{85}Rb, ^{133}Cs), or -4 (^{133}Cs) atoms.

While most previous work has considered many-body cavity QED with two-level systems [54,55], the generalization to integer-spin ensembles offers a range of interesting physics not available to spin-1/2 systems. Integer spins have more degrees of freedom, which means that there are different ways to manipulate excitations and constrain the state. In particular, a coherent ensemble of integer spins is not limited to the surface of the angular momentum Bloch sphere. This allows for novel entangled states such as the spin-singlet, two-mode squeezed spin states or, as discussed in more detail below, the redistribution of quantum noise into degrees of freedom that are simply not present in two-level systems [56–58].

We now consider the dispersive limit in which the Raman transitions are themselves off resonant, i.e., $\omega \gg \omega_0, \lambda_\pm$, in which case we can also adiabatically eliminate the cavity mode to yield the reduced master equation

$$\dot{\rho} = -i[\hat{H}, \rho] + \frac{\kappa}{2N(\omega^2 + \kappa^2)}\mathcal{D}[\lambda_-\hat{S}_- + \lambda_+\hat{S}_+]\rho, \tag{6}$$

with

$$\hat{H} = \left(\omega_0 - \frac{\omega(\lambda_-^2 - \lambda_+^2)}{2N(\omega^2 + \kappa^2)}\right)\hat{S}_z - \frac{\omega}{2N(\omega^2 + \kappa^2)}[(\lambda_- + \lambda_+)^2\hat{S}_x^2 + (\lambda_- - \lambda_+)^2\hat{S}_y^2]. \tag{7}$$

If we set $\lambda_+ = 0$ and $\lambda_- = \lambda$, then (6) becomes

$$\dot{\rho} = -i[\hat{H}, \rho] + \frac{\Gamma}{2N}\mathcal{D}[\hat{S}_-]\rho, \tag{8}$$

where

$$\hat{H} = \omega_0'\hat{S}_z + \frac{\Lambda}{2N}(\hat{S}_x^2 + \hat{S}_y^2), \tag{9}$$

with parameters given by

$$\omega_0' = \omega_0 + \frac{\Lambda}{2N}, \quad \Lambda = -\frac{\omega\lambda^2}{\omega^2 + \kappa^2}, \quad \Gamma = -\frac{\kappa}{\omega}\Lambda. \tag{10}$$

Note that, by choosing the sign of ω, it is possible to produce ferromagnetic or antiferromagnetic behavior with the same atomic species. An artificial quadratic Zeeman shift could also be added to the system by, for example, a weak π-polarized laser field acting near the $F' = 1$ line in the excited manifold. Then, in the limit that $\Gamma/\Lambda \ll 1$, the atoms will undergo spin-mixing interactions with dynamics of the sort found in spinor BECs. However, here the relevant dynamical rate is set by Raman transition rates, light shifts, and detunings and can therefore be orders of magnitude larger than in spinor BECs. Consider, e.g., the feasible experimental parameters $\{g, \kappa, \gamma\}/(2\pi) = \{10, 0.2, 6\}$ MHz (see, e.g., [43,59,60]), where γ is the atomic spontaneous emission linewidth. With $N = 10^4$ atoms, values of $\lambda/(2\pi) \simeq 200$ kHz are then readily achievable, which, with $\omega/(2\pi) \simeq 4$ MHz, lead to $\Lambda/(2\pi) \simeq 10$ kHz and $\Gamma = 0.05\Lambda$. This means that such a system can emulate the dynamics of a spinor BEC but orders of magnitude faster.

This Hamiltonian gives the opportunity to study a range of models, such as the Lipkin-Meshkov-Glick model, where, unlike in the spin-$1/2$ case, the spin-1 case features quantum chaotic behavior [61]. Similar studies have shown that spin-2 models can offer very different dynamics again [62].

The methods described above are not limited to emulations of spinor BEC physics. Since this is an open system, there is also flexibility to deliberately engineer a particular dissipative evolution or monitor the cavity output to gain information about the evolution without destroying the spinor gas.

It is also possible to produce Hamiltonians which do not naturally arise in spinor BECs. For example, by setting $\lambda_- = \lambda_+$ we obtain a Hamiltonian $\sim\hat{S}_x^2$, which produces squeezing via one-axis twisting in two-level systems [63–68]. By adding more cavity and laser modes, an even wider range of Hamiltonians is possible [42,69], with, for example, the possibility of a two-axis twisting Hamiltonian $\propto \hat{S}_x^2 - \hat{S}_y^2$ [63], which can offer Heisenberg limited metrology, but has yet to be implemented experimentally. Such systems with spin-1 (or higher) particles offer the same squeezing possibilities but should also allow further novel, many-body ground states and dynamical phenomena.

The principle behind our scheme could also be applied to the emerging field of quantum simulation with cold atoms coupled to a photonic crystal waveguide [70]. Atoms coupled to the waveguide, but with the atomic resonance frequency located within a photonic band gap, enable localized excitations at the atom trapping sites, while the tunneling of excitations between neighboring sites produces effective atom-atom interactions. While work to date has focused on spin-$1/2$ systems, the application of our approach should enable the generalization of this work to integer-spin lattice models with engineered interactions that could be tuned in form, strength, and range, allowing, for example, the exploration of Haldane physics [71].

Now, we consider an example of how our scheme can be used to emulate spinor BEC physics. In particular, we consider model (8) and the preparation of "spin-nematic squeezing" in an ensemble of spin-1 atoms that are initially prepared in the $m = 0$ sublevel [16,72]. With a suitable choice of microscopic parameters, it is possible to set $\omega_0' = 0$ (or at least approximately so). We note, however, that initially $\langle \hat{S}_z \rangle = 0$, and since $\hat{S}_z$ is conserved by the Hamiltonian, this term should not have any significant impact on the evolution, provided that the cavity-mediated damping of the spin (which does not conserve $\hat{S}_z$) is weak, i.e., $\kappa \ll \omega$, which means $\Gamma \ll \Lambda$. With this condition met, the Hamiltonian is an active generator of spin-nematic squeezing.

Spin squeezing is a well-established method to produce a metrological enhancement (for reviews, see [73,74]). In particular, atom interferometers can be used for precision measurements of acceleration, time, rotation, and, potentially, even gravitational waves [75]. If the input states to these interferometers are uncorrelated states of N atoms, then the precision of the measurement is limited by the standard quantum limit, which scales as $1/\sqrt{N}$. However, by generating suitable entanglement within the atomic ensemble, it is possible to exceed this and approach the Heisenberg limit, which scales like $1/N$.

Considering ensembles of N two-level, or spin-$1/2$, atoms with internal spin degrees of freedom, spin squeezing involves a redistribution of quantum noise on the angular momentum Bloch sphere in such a way as to produce reduced quantum fluctuations along one coordinate axis. Integer-spin systems possess additional degrees of freedom associated with the quadrupole or nematic tensor operator $\hat{Q}_{ij} = \sum_{n=1}^N \hat{S}_i^{(n)}\hat{S}_j^{(n)} + \hat{S}_j^{(n)}\hat{S}_i^{(n)} - (4/3)\delta_{ij}$, where $(\{i,j\} \in \{x,y,z\})$, $\hat{S}_i^{(n)}$ are spin-1 angular momentum operators for a single atom, and δ_{ij} is the Kronecker delta function. Spin-nematic squeezing involves the redistribution of quantum noise in the subspaces $\{\hat{S}_x, \hat{Q}_{yz}, \hat{Q}_{zz} - \hat{Q}_{yy}\}$ and $\{\hat{S}_y, \hat{Q}_{xz}, \hat{Q}_{zz} - \hat{Q}_{xx}\}$ [16]. Focusing on the first of these subspaces, the degree of spin-nematic squeezing can be characterized by a parameter ξ_x, which gives the metrological precision relative to the standard quantum limit for Ramsey interferometry and is calculated by minimizing the following expression over the angle θ:

$$\xi_x^2 = \frac{2\langle[\Delta(\hat{S}_x \cos\theta + \hat{Q}_{yz} \sin\theta)]^2\rangle}{|\langle \hat{Q}_{zz} - \hat{Q}_{yy} \rangle|}, \tag{11}$$

with $\xi_x^2 < 1$ indicating spin-nematic squeezing.

Figure 2 shows the development of spin-nematic squeezing with and without damping for an ensemble of $N = 120$ atoms. These results are obtained from quantum trajectory simulations of the master equation (8), in which we make use of a representation in terms of bosonic mode operators, $\hat{a}_m$, for the three Zeeman states $m = 0, \pm 1$ [45]. In particular, $\hat{a}_m$ ($\hat{a}_m^\dagger$) annihilates (creates) an atom in state m, and, e.g., $\hat{S}_- = \sqrt{2}(\hat{a}_0^\dagger \hat{a}_1 + \hat{a}_{-1}^\dagger \hat{a}_0)$. Note that in this

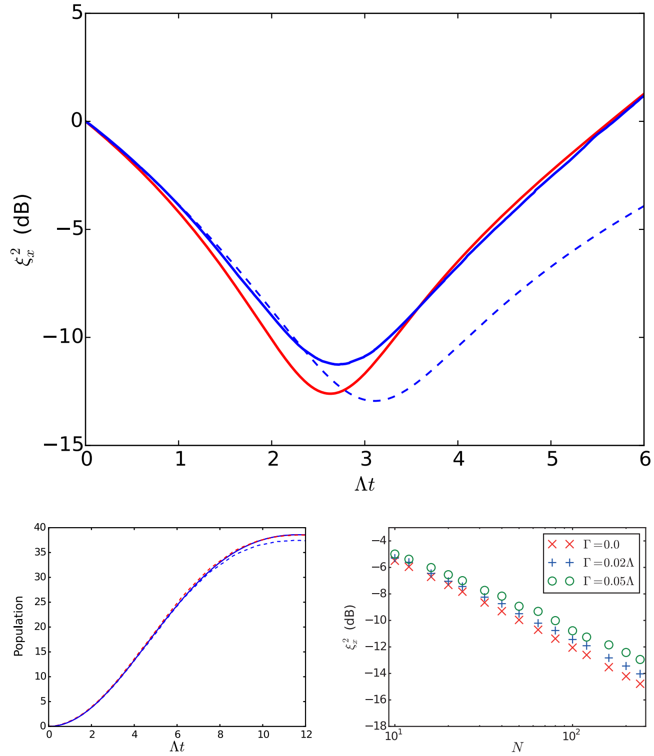

FIG. 2. (Top) Time evolution of the spin-nematic squeezing for $N = 120$ atoms without damping (red line) and, with damping rate $\Gamma = 0.05\Lambda$, an ensemble average of 1000 trajectories (dark blue line) and a single trajectory in which no jumps occur (dashed dark blue line). The phase angle in each case is just below $\theta = 170°$. (Bottom left) Populations in each of the states $m = \pm1$ for $\Gamma = 0$ (solid line) and $\Gamma = 0.05\Lambda$ (dashed lines). (Bottom right) Optimized squeezing scaling with atom number with and without damping. With damping, ensemble averages of 1000 trajectories were used to estimate the master equation result.

picture the Hamiltonian contains a term proportional to $\hat{a}_0^\dagger\hat{a}_0^\dagger\hat{a}_{-1}\hat{a}_{+1} + \hat{a}_{-1}^\dagger\hat{a}_{+1}^\dagger\hat{a}_0\hat{a}_0$, which highlights explicitly the link to squeezing via four-wave mixing in light [16,72].

With $\Gamma = 0$ the system simply follows the Hamiltonian evolution, and we see significant squeezing generated on a time scale $(\Lambda/2)^{-1}$. After this time, squeezing reduces and ultimately turns into antisqueezing. This turnaround correlates with a growing number of atoms in the $m = \pm1$ states, resulting in a reduction in $|\hat{Q}_{zz} - \hat{Q}_{yy}|$.

With the addition of a small rate of damping, which causes (infrequent) quantum jumps with $\hat{S}_-$, we find that the trajectories can be split into two categories. First are those that reach the point of peak squeezing without a jump having occurred. Interestingly, these have squeezing at a slightly higher level than with $\Gamma = 0$, meaning that the null measurement backaction (which essentially adds an imaginary element to the spin-nematic squeezing generator) actually improves the squeezing. Second, when there is a jump before that point, the squeezing is substantially reduced, and so, on average, the presence of damping does decrease the degree of squeezing. However, for sufficiently small κ/ω, such jumps should be rare. In addition, since these jumps are mediated by the cavity mode (i.e., they correspond to the emission of a photon from the cavity mode), then, by monitoring that output and postselecting based on the absence of a photon measurement, it would be possible to remove some of the runs with nonoptimal squeezing (allowing for finite detection efficiency).

Figure 2 also illustrates more clearly how the best achievable squeezing varies with the number of atoms, with results obtained from trajectory simulations of (8). For $\Gamma = 0$, we find that $\xi_x^2 \sim N^{-0.673}$ in the range of atom numbers that we consider. This indicates that this spin-nematic squeezing scales very similarly to one-axis twisting (where the squeezing scales at best as $N^{-2/3}$). Note that we have also considered spin-nematic squeezing in spin-2 particles, as would be relevant to the situation in which the present scheme is applied to the $F = 2$ ground states in either ^{85}Rb or ^{87}Rb, and find similar results.

In Fig. 3, we plot the squeezing parameter as a function of both the time and phase angle θ for $N = 120$ atoms (left) and in the limit of large N (right), where we assume that the $m = 0$ state is essentially undepleted and $\hat{a}_0$ can be replaced by $\sqrt{N}$. In this case, it is possible to derive the simple result [45]

$$\xi_x^2 = (\cos\theta + 2\Lambda t \sin\theta)^2 + (1 + 2\Gamma t)^2 \sin^2\theta. \tag{12}$$

This agrees quite well with the $N = 120$ results up to $\Lambda t \simeq 2$ but then predicts continued improvement in the degree of squeezing at longer evolution times and for phase angles that approach (slowly) 180°.

Finally, we note that the rate of atomic spontaneous emission due to off-resonant excitation of the $5^2P_{1/2}$ state is estimated, for our configuration, as $\Gamma_{sp} = \gamma(\Omega^2/12\Delta^2)$, which gives $\Gamma_{sp}/(\Lambda/2) \simeq 48\omega/(NC\kappa)$, where $C = 2g^2/(\kappa\gamma)$ is the single-atom cooperativity. For the parameters discussed above, this ratio is ~0.0006. With more atoms and/or increased cooperativity, this can evidently be reduced even further.

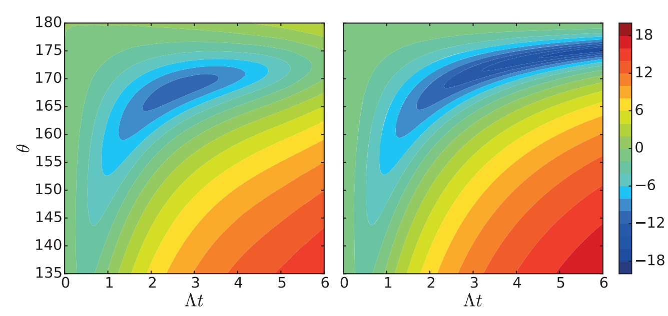

FIG. 3. Values of ξ_x^2 (in decibels) as a function of the time and phase angle for $\Gamma/\Lambda = 0.05$ with $N = 120$ atoms (left) and in the undepleted mode approximation ($N \to \infty$), Eq. (12) (right).

In conclusion, we have proposed a method for engineering spinor dynamics using cavity-mediated Raman transitions and demonstrated that such a scheme could be used to produce spin-nematic squeezing in an ensemble of spin-1 atoms. We believe this work opens up a range of exciting possibilities for emulating spinor BEC dynamics on much shorter time scales and extending this to explore a much wider range of spinor physics with significant flexibility.

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