State-of-the-Art Calculation of the Decay Rate of Electroweak Vacuum in the Standard Model

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The decay rate of the electroweak (EW) vacuum is calculated in the framework of the standard model (SM) of particle physics, using the recent progress in the understanding of the decay rate of metastable vacuum in gauge theories. We give a manifestly gauge-invariant expression of the decay rate. We also perform a detailed numerical calculation of the decay rate. With the best-fit values of the SM parameters, we find that the decay rate of the EW vacuum per unit volume is about 10^{-554} $\mathrm{Gyr}^{-1}\,\mathrm{Gpc}^{-3}$; with the uncertainty in the top mass, the decay rate is estimated as 10^{-284}–10^{-1371} $\mathrm{Gyr}^{-1}\,\mathrm{Gpc}^{-3}$.

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Introduction.—It is highly nontrivial whether the vacuum we are living in, which we call the electroweak (EW) vacuum, is absolutely stable or not. If there exists a vacuum which has lower energy density than that of the EW vacuum, which is the case in a large class of particle-physics models, the EW vacuum decays via the quantum tunneling effect. If the decay rate is too large, the universe should have experienced a phase transition before the present epoch, with which the universe would show completely different aspects than the present one. From the particle-physics and cosmology points of view, the stability of the EW vacuum is of particular interest to have deep insight into particle-physics models and the nature of the universe.

Even in the standard model (SM) of particle physics, which is extremely successful in explaining particle interactions, the EW vacuum may be metastable [1–7]. In particular, the discovery of the Higgs boson by the LHC experiments [8,9] shed light on the stability of the EW vacuum. The observed value of the Higgs mass suggests that the Higgs quartic coupling becomes negative via the renormalization group (RG) effects at energy scale much higher than the EW scale. This fact implies that the Higgs potential becomes negative and that the EW vacuum is not absolutely stable if the SM is valid up to a scale much higher than the EW scale.

The decay rate of the EW vacuum has been estimated in the past, mostly using the method given in Refs. [10–12]. The decay rate of the metastable vacuum (i.e., false vacuum) per unit volume, which we call γ, is given in the following form:

$$\gamma = \mathcal{A} e^{-\mathcal{B}}, \tag{1}$$

where $\mathcal{B}$ is the action of the so-called bounce, which is the solution of the four-dimensional (4D) Euclidean equation of motion, while $\mathcal{A}$ takes account of the fluctuation around the bounce. The bounce action $\mathcal{B}$ can be evaluated relatively easily, while the calculation of the prefactor $\mathcal{A}$ is complicated both conceptually and numerically. In particular, if the bounce is coupled to gauge fields, which is the case when considering the decay of the EW vacuum in the SM, gauge-invariant calculation of $\mathcal{A}$ has not have been performed. In addition, in the calculation of the decay rate of the EW vacuum, the path integral of the zero mode in association with the (approximate) classical conformal invariance was not properly performed. The calculation of γ in the past could not avoid some or all of these difficulties, resulting in ambiguities in the final result.

Recently, however, a new formalism has been developed to calculate γ, which can give a manifestly gauge-invariant expression of $\mathcal{A}$ [13,14]. By using the method given there, a more unambiguous calculation of the decay rate of the EW vacuum has become possible.

The main purpose of this Letter is to perform a state-of-the-art calculation of the decay rate of the EW vacuum in the framework of the SM, using the recent progress to calculate the decay rate of metastable vacuum. We give a gauge-invariant expression of the decay rate of the EW vacuum. We also give a prescription to properly take care of the zero mode in association with the (classical) conformal invariance, which shows up in the limit of large Higgs amplitude. Then, we perform numerical calculations to estimate the decay rate, and show that the decay rate for the size of the present Hubble volume is much smaller than the inverse of the present age of the Universe.

Higgs potential and the bounce.—In the following, we consider the situation where the Higgs potential becomes negative due to the RG running of the quartic coupling of the Higgs potential. The instability of the potential occurs when the Higgs amplitude becomes much larger than the EW scale; in the rest of this letter, we concentrate on such a large Higgs amplitude. Then, denoting the Higgs doublet as Φ, the Higgs potential is well approximated by the quartic one [1]:

$$V(\Phi) = \lambda(\Phi^\dagger \Phi)^2. \quad (2)$$

When the renormalization scale is relatively large, $\lambda < 0$ is realized.

For the study of the decay of the false vacuum, we first consider the bounce, which corresponds to the classical path connecting the false and true vacua. In the present case, by using SU(2) and U(1) transformations, we can take the following bounce configuration:

$$\Phi|_{\rm bounce} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \bar{\phi}(r) \end{pmatrix}, \quad (3)$$

with vanishing gauge fields. Here, $\bar{\phi}$ is a real function of r (with r being the 4D radius in the Euclidean space) which obeys

$$\partial_r^2 \bar{\phi} + \frac{3}{r} \partial_r \bar{\phi} - \lambda \bar{\phi}^3 = 0, \quad (4)$$

with

$$\partial_r \bar{\phi}(r=0) = 0, \qquad \bar{\phi}(r=\infty) = 0. \quad (5)$$

Assuming that $\lambda < 0$, the solution of the above equation is given by

$$\bar{\phi} = \bar{\phi}_C \left(1 + \frac{|\lambda|}{8} \bar{\phi}_C^2 r^2\right)^{-1}, \quad (6)$$

where $\bar{\phi}_C$ is a constant which corresponds to the bounce amplitude at the center of the bounce configuration. Notice that the bounce contains a free parameter $\bar{\phi}_C$. The bounce action is given by

$$\mathcal{B} = \frac{8\pi^2}{3|\lambda|}. \quad (7)$$

Decay rate.—Now we are at the position to calculate the decay rate of the EW vacuum. As we have shown, we already have the analytic expression of the bounce action $\mathcal{B}$. On the other hand, the calculation of the prefactor $\mathcal{A}$ is highly nontrivial. The prefactor $\mathcal{A}$ is obtained by calculating the functional determinants of the fluctuation operators of the fields that couple to the bounce field [11].

First, let us consider the (physical) Higgs field h, which is embedded into the Higgs doublet as

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi^1 + i\varphi^2 \\ \bar{\phi}(r) + h + i\varphi^3 \end{pmatrix}, \quad (8)$$

where φ^a are Nambu-Goldstone (NG) modes. After the decomposition with respect to the 4D angular momentum, the fluctuation operator of h is given by

$$\mathcal{M}_J^{(h)} = -\Delta_J - 3|\lambda|\bar{\phi}^2, \quad (9)$$

where J characterizes the eigenvalues of 4D angular-momentum operators, and it takes $J = 0, \frac{1}{2}, 1, \ldots,$ with which ${\rm Det}\mathcal{M}^{(h)} = \prod_J [{\rm Det}\mathcal{M}_J^{(h)}]^{(2J+1)^2}$. In addition,

$$\Delta_J \equiv \partial_r^2 + \frac{3}{r}\partial_r - \frac{4J(J+1)}{r^2}. \quad (10)$$

The ratio of the functional determinants of the fluctuation operators relevant for the calculation of $\mathcal{A}$ can be performed with the method given in Refs. [12,15–17]. For the calculation, we first limit the region as $0 \le r \le r_\infty$, where r_∞ is a (large) radius which is taken to be infinity at the end of calculation, and impose relevant boundary conditions for the mode functions at $r = 0$ and $r = r_\infty$. By using analytic properties of the functional determinants, we obtain

$$\frac{{\rm Det}\mathcal{M}_J^{(h)}}{{\rm Det}\widehat{\mathcal{M}}_J^{(h)}} = \frac{f_J^{(h)}(r_\infty)}{r_\infty^{2J}}, \quad (11)$$

where $\widehat{\mathcal{M}}_J^{(h)} = [\mathcal{M}_J^{(h)}]_{\bar{\phi}\to 0}$ is the fluctuation operator around the false vacuum, and $f_J^{(h)}$ obeys

$$\mathcal{M}_J^{(h)} f_J^{(h)} = 0, \quad (12)$$

with the boundary condition $f_J^{(h)}(r \to 0) \simeq r^{2J}$.

For $J \ge 1$, the functional determinants necessary for the calculation of $\mathcal{A}$ are obtained by using Eq. (11). On the contrary, for $J = 0$ and $J = \frac{1}{2}$, special care is needed because of the existence of zero modes; $\mathcal{A}$ diverges if one naively uses Eq. (11) for those cases.

The zero mode for $J = 0$ is related to the conformal invariance; in the present analysis, we approximate that the Higgs potential is quartic, and, hence, the theory has a conformal invariance at the classical level. Consequently, the bounce configuration is not uniquely determined and its continuous deformation with respect to the parameter $\bar{\phi}_C$ is possible. This is easily understood from the expression of the mode function of the conformal zero mode, which is given by

$$\psi^{(\rm conf)} \equiv \mathcal{N}_{\rm conf} \left(1 - \frac{|\lambda|}{8}\bar{\phi}_C^2 r^2\right)\left(1 + \frac{|\lambda|}{8}\bar{\phi}_C^2 r^2\right)^{-2} = \mathcal{N}_{\rm conf} \frac{\partial \bar{\phi}}{\partial \bar{\phi}_C}, \quad (13)$$

where $\mathcal{N}_{\rm conf}$ is the normalization factor. Indeed, one can see that $\mathcal{M}_0^{(h)} \psi^{(\rm conf)} = 0$. The normalization factor is given by

$$\mathcal{N}_{\rm conf}^{-2} = \frac{1}{2\pi}\int d^4 r \left(\frac{\partial\bar{\phi}}{\partial\bar{\phi}_C}\right)^2 \simeq \frac{64\pi}{|\lambda|^2\bar{\phi}_C^4}\ln r_\infty. \quad (14)$$

We comment here that $\mathcal{N}_{\rm conf}^{-2}$ diverges when r_∞ is taken to infinity. As we will see below, however, $\mathcal{N}_{\rm conf}$ disappears from the final expression by properly taking into account the measure of the path integral of the conformal zero mode.

Because the zero-mode wave function in association with the conformal invariance is given by the derivative of $\bar{\phi}$ with respect to $\bar{\phi}_C$, the path integral of the conformal zero mode should be regarded as the integration over all the possible deformation of the bounce configuration with the change of $\bar{\phi}_C$:

$$\int \mathcal{D}h^{(\rm conf)} \to \int \frac{d\bar{\phi}_C}{\mathcal{N}_{\rm conf}}. \quad (15)$$

Then, remembering that the functional determinants originate from the path integral of the fields coupled to the bounce, the functional determinant of $\mathcal{M}_0^{(h)}$ should be understood as

$$[{\rm Det}\mathcal{M}_0^{(h)}]^{-1/2} \to \int \frac{d\bar{\phi}_C}{\mathcal{N}_{\rm conf}}[{\rm Det}'\mathcal{M}_0^{(h)}]^{-1/2}, \quad (16)$$

where the prime indicates that the zero eigenvalue is omitted from the functional determinant. In order to omit the zero eigenvalue, we use the following technique [14]:

$$\frac{{\rm Det}'\mathcal{M}_0^{(h)}}{{\rm Det}\hat{\mathcal{M}}_0^{(h)}} = \lim_{\nu\to 0}\nu^{-1}\frac{{\rm Det}(\mathcal{M}_0^{(h)}+\nu)}{{\rm Det}\hat{\mathcal{M}}_0^{(h)}} = \check{f}_0^{(h)}(r_\infty), \quad (17)$$

where the function $\check{f}_0^{(h)}$ satisfies

$$\left(\partial_r^2 + \frac{3}{r}\partial_r + 3|\lambda|\bar{\phi}^2\right)\check{f}_0^{(h)} = \frac{\partial\bar{\phi}}{\partial\bar{\phi}_C}, \quad (18)$$

and $\check{f}_0^{(h)}(r\to 0) = 0$, resulting in

$$\check{f}_0^{(h)}(r_\infty) = \int_0^{r_\infty} dr_1 r_1^{-3}\int_0^{r_1} dr_2 r_2^3 \frac{\partial\bar{\phi}}{\partial\bar{\phi}_C} \simeq -\frac{4}{|\lambda|\bar{\phi}_C^2}\ln r_\infty. \quad (19)$$

Consequently,

$$\left|\frac{{\rm Det}\mathcal{M}_0^{(h)}}{{\rm Det}\hat{\mathcal{M}}_0^{(h)}}\right|^{-1/2} \to \int \frac{d\bar{\phi}_C}{\bar{\phi}_C}\left(\frac{16\pi}{|\lambda|}\right)^{1/2}. \quad (20)$$

The zero modes for $J = \frac{1}{2}$ are related to the translational invariance; they can be taken care of as [11]

$$\frac{{\rm Det}\mathcal{M}_{1/2}^{(h)}}{{\rm Det}\widehat{\mathcal{M}}_{1/2}^{(h)}} \to \mathcal{V}_{4D}^{-1/2}\left(\frac{\mathcal{B}}{2\pi}\right)^{-1}\frac{\check{f}_{1/2}^{(h)}(r_\infty)}{r_\infty}, \quad (21)$$

where $\mathcal{V}_{4D}$ is the volume of the 4D Euclidean space, and the function $\check{f}_{1/2}^{(h)}$ obeys

$$\mathcal{M}_{1/2}^{(h)}\check{f}_{1/2}^{(h)} = -r\left(1+\frac{|\lambda|}{8}\bar{\phi}_C^2 r^2\right)^{-2}, \quad (22)$$

with $\check{f}_{1/2}^{(h)}(r\to 0) = 0$. Notice that $\check{f}_{1/2}^{(h)}(r_\infty) \propto \bar{\phi}_C^{-2}$.

For the effects of the gauge and NG bosons, a new technique has been recently developed in Refs. [13,14], which gives a simple and manifestly gauge-invariant formula for the gauge- and NG-boson contributions. In Refs. [13,14], the scalar potential was assumed to be quadratic around the false vacuum, while it is quartic in the present case. Based on Ref. [14], we derive the formula relevant for the present case. [The result is given in Eq. (25); a more detailed derivation of the following formulas will be given elsewhere [18].]

Combining the contributions of particles which have sizable couplings with the bounce, the decay rate of the EW vacuum is expressed as

$$\gamma = \int d\ln\bar{\phi}_C [I^{(H)}I^{(W,Z,{\rm NG})}I^{(t)}e^{-\delta\mathcal{S}_{\overline{\rm MS}}}e^{-\mathcal{B}}]_{\mu(\bar{\phi}_C)}, \quad (23)$$

where $\delta\mathcal{S}_{\overline{\rm MS}}$ is the effect of the so-called divergent part [14] (which is calculated with the $\overline{\rm MS}$ scheme), and μ is the renormalization scale at which the SM coupling constants for the calculation of the integrand are evaluated. The Higgs contribution as well as the gauge- and NG-boson contribution are given by

$$I^{(H)} = \frac{\mathcal{B}^2}{4\pi^2}\left(\frac{16\pi}{|\lambda|}\right)^{1/2}\left[\frac{\check{f}_{1/2}^{(h)}(r_\infty)}{r_\infty}\right]^{-2} \times e^{s_0^{(h)}+s_{1/2}^{(h)}}\prod_{J\geq 1}e^{s_J^{(h)}}\left[\frac{f_J^{(h)}(r_\infty)}{r_\infty^{2J}}\right]^{-(2J+1)^2/2}, \quad (24)$$

$$I^{(W,Z,{\rm NG})} = \mathcal{V}_{SU(2)}\left(\frac{16\pi}{|\lambda|}\right)^{3/2}\prod_{V=W^1,W^2,Z}e^{s_0^{(V,{\rm NG})}}\prod_{J\geq 1/2}e^{s_J^{(V,{\rm NG})}} \times\left[\frac{|\lambda|J\bar{\phi}_C^2 f_J^{(\eta^V)}(r_\infty)}{8(J+1)r_\infty^{2J-2}}\right]^{-(2J+1)^2/2}\left[\frac{f_J^{(T^V)}(r_\infty)}{r_\infty^{2J}}\right]^{-(2J+1)^2}, \quad (25)$$

where $\mathcal{V}_{SU(2)} = 2\pi^2$ is the volume of the SU(2) group parametrizing the possible deformation of the bounce configuration. Here, $s_J^{(h)}$ and $s_J^{(V,{\rm NG})}$ are the effects of counterterms to subtract divergences; the calculations of these quantities are found in Refs. [1,14]. The functions $f_J^{(\eta^V)}$ and $f_J^{(T^V)}$ satisfy

$$(\Delta_J - g_V^2 \bar{\phi}^2) f_J^{(\eta^V)} - \frac{2\bar{\phi}'}{r^2 \bar{\phi}} \partial_r (r^2 f_J^{(\eta^V)}) = 0, \tag{26}$$

$$(\Delta_J - g_V^2 \bar{\phi}^2) f_J^{(T^V)} = 0, \tag{27}$$

and $f_J^{(\eta^V)}(r \to 0) \simeq f_J^{(T^V)}(r \to 0) \simeq r^{2J}$, where

$$g_V = \frac{1}{2} \begin{cases} g_2, & V = W^1, W^2 \\ \sqrt{g_2^2 + g_1^2}, & V = Z \end{cases} \tag{28}$$

with g_2 and g_1 being the gauge coupling constants of $SU(2)_L$ and $U(1)_Y$, respectively. Expression of the top contribution $I^{(t)}$ can be found in Ref. [1]. We emphasize that the above expressions for the decay rate is manifestly gauge invariant; they hold irrespective of the choice of the gauge parameter (which is often called ξ). Furthermore, $\delta \mathcal{S}_{\overline{\rm MS}}$ is given by the sum of the Higgs and top contributions as well as the gauge and NG contributions: $\delta \mathcal{S}_{\overline{\rm MS}} = \delta \mathcal{S}_{\overline{\rm MS}}^{(h)} + \delta \mathcal{S}_{\overline{\rm MS}}^{(t)} + \sum_{V=W^1,W^2,Z} \delta \mathcal{S}_{\overline{\rm MS}}^{(V,{\rm NG})}$. The Higgs and top contributions are given in Ref. [1], while $\delta \mathcal{S}_{\overline{\rm MS}}^{(V,{\rm NG})}$ is obtained with the prescription given in Ref. [14]:

$$\delta \mathcal{S}_{\overline{\rm MS}}^{(V,{\rm NG})} = -\left(\frac{1}{3} + \frac{2g_V^2}{|\lambda|} + \frac{g_V^4}{|\lambda|^2}\right) \times \left[\frac{5}{6} + \gamma_E + \ln\left(\sqrt{\frac{2}{|\lambda|}} \frac{\mu}{\bar{\phi}_C}\right)\right] - \frac{2g_V^2}{3|\lambda|}, \tag{29}$$

with γ_E being Euler's constant.

In Eq. (23), the renormalization scale μ is taken to be $\bar{\phi}_C$ dependent in the following reason. For fixed $\bar{\phi}_C$, the typical mass scale of the fields which have sizable couplings to the bounce is $O(\bar{\phi}_C)$, and only the scales in the calculation are $\bar{\phi}_C$ and μ. Thus, one-loop effects give terms proportional to $\ln(\bar{\phi}_C/\mu)$ to the integrand; the μ dependence from such terms should be canceled by the μ dependence of the coupling constants [19]. The two- and higher-loop effects are expected to introduce terms proportional to $\ln^p(\bar{\phi}_C/\mu)$ (with $p \geq 1$) which are, on the contrary, not included in the present result. In order to minimize the higher order effects, we set $\mu(\bar{\phi}_C) \sim \bar{\phi}_C$; hereafter, we take $\mu(\bar{\phi}_C) = \bar{\phi}_C$ unless otherwise stated. In fact, a proper choice of μ is important for the convergence of the integral over $\ln \bar{\phi}_C$. In the SM, λ is minimized at $\mu \sim O(10^{17})$ GeV, and it increases above such a scale. (The runnings of the SM coupling constants are precisely included in our numerical calculation; see the discussion below.) Then, with taking $\mu = \bar{\phi}_C$, because $\mathcal{B}$ is inversely proportional to $|\lambda|$, the integrand of Eq. (23) is maximized when $\bar{\phi}_C \sim O(10^{18})$ GeV and is significantly suppressed when $\bar{\phi}_C \gg O(10^{18})$ GeV. Based on this observation, we expect that the integration over $\bar{\phi}_C$ converges.

Numerical results.—Now we apply our formula for the estimation of the decay rate of the EW vacuum. We evaluate $f_J^{(h)}$, $\check{f}_{1/2}^{(h)}$, $f_J^{(\eta^V)}$, and $f_J^{(T^V)}$ (as well as other functions necessary to calculate $I^{(t)}$ and counterterms) by numerically solving differential equations. The renormalization-scale dependence of the SM coupling constants are evaluated by using the method given in Ref. [20], which partially takes into account three- and four-loop effects. Then, with performing the integration over $\ln \bar{\phi}_C$ numerically, the decay rate of the EW vacuum is obtained. We use the following Higgs and top masses [21]:

$$m_H = 125.09 \pm 0.24 \text{ GeV}, \tag{30}$$

$$m_t^{(\rm pole)} = 173.1 \pm 1.1 \text{ GeV}, \tag{31}$$

while the strong coupling constant is

$$\alpha_s(m_Z) = 0.1181 \pm 0.0011. \tag{32}$$

For the best-fit values of the Higgs mass, top mass, and strong coupling constant given above, we find $\gamma \simeq 10^{-718}$ GeV$^4 \simeq 10^{-554}$ Gyr^{-1} Gpc^{-3}. Taking account of the uncertainties, we obtain

$$\log_{10}[\gamma({\rm Gyr}^{-1}\,{\rm Gpc}^{-3})] \simeq -554^{+38+270+137}_{-41-817-204}, \tag{33}$$

where the first, second, and third errors are due to those in the Higgs mass, top mass, and the strong coupling constant given in Eqs. (30), (31), and (32), respectively. Thus, the decay rate is extremely sensitive to the top mass. So far, we have chosen the renormalization scale to be $\mu(\bar{\phi}_C) = \bar{\phi}_C$. Varying the renormalization scale from $\mu(\bar{\phi}_C) = \frac{1}{2}\bar{\phi}_C$ to $2\bar{\phi}_C$, for example, the change of the decay rate is $\delta \log_{10} \gamma \sim 6$. In Fig. 1, we show the contours of constant γ on Higgs mass vs top mass plane.

Comparing the decay rate with $H_0^{-4} \sim 10^3$ Gyr Gpc^3 (with H_0 being the Hubble constant), the probability of having a phase transition within the present Hubble volume for the present cosmic time scale is enormously small for the best-fit values of the SM parameters. [Even if we vary m_H, $m_t^{(\rm pole)}$, and $\alpha_s(m_Z)$ within 2σ uncertainties, γ is at most 10^{-68} Gyr^{-1} Gpc^{-3} which is still much smaller than H_0^4.] If the top mass were much larger than the observed value, γ would be larger than $\sim H_0^4$ so that the EW vacuum would decay before the present epoch; such an instability bound derived from our formula is consistent with that given in previous work [2]. In the future, the universe will be dominated by the dark energy, assuming that it is a cosmological constant. Using the observed energy density

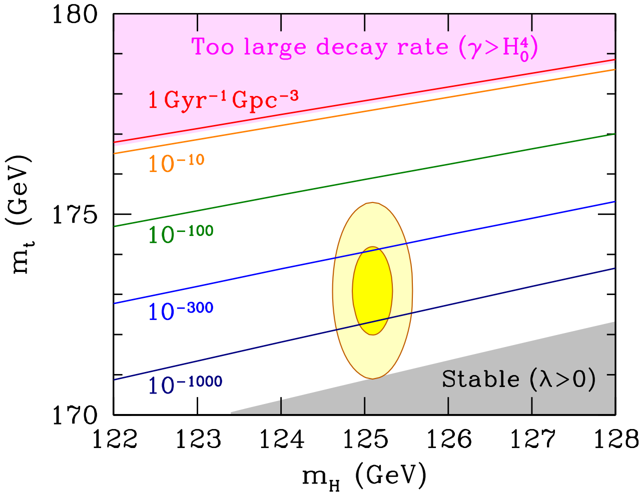

FIG. 1. The contours of constant γ on Higgs mass vs top mass plane with $\alpha_s(m_Z) = 0.1181$. The contours are $\gamma = 1$, 10^{-10}, 10^{-100}, 10^{-300}, and 10^{-1000} $\mathrm{Gyr}^{-1}\,\mathrm{Gpc}^{-3}$ from above. In the upper shaded region (pink), γ becomes larger than H_0^4. In the lower shaded region (gray), the EW vacuum is stable because λ is always positive. We also show the constraint on the Higgs and top masses (yellow-shaded regions) adding their 1σ (inside) or 2σ (outside) uncertainties in quadrature.

of the dark energy, the expansion rate will eventually become $H_\infty \simeq 56.3$ km/sec Mpc [22]. Then, the phase transition rate within the horizon scale of such a de Sitter universe is about 10^{-552} $\mathrm{Gyr}^{-1} \simeq 10^{-551} H_\infty$, which we regard as the decay rate of the EW vacuum. Uncertainty in this estimation can be obtained from Eq. (33).

Summary.—We have calculated the decay rate of the EW vacuum, assuming that the SM is valid up to high energy scale. We have derived a gauge-invariant expression of the decay rate, properly performing the path integral of the zero mode in association with the conformal invariance. With the best-fit values of the Higgs and top masses and $\alpha_s(m_Z)$, the decay rate of the EW vacuum per unit volume is given by 10^{-554} $\mathrm{Gyr}^{-1}\,\mathrm{Gpc}^{-3}$. The probability of the phase transition within the present horizon scale is found to be enormously small. This is a good news for us all because we can safely live in the EW vacuum unless a new physics beyond the SM significantly alters this conclusion.

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Note added.—Recently, Ref. [23] appeared, which has significant overlap with our work. We found, however, several disagreements between the results in Ref. [23] and ours, which are in (i) the counterterms based on the angular-momentum decomposition (corresponding to $s_J^{(V,\mathrm{NG})}$ in our calculation), (ii) $\delta S_{\overline{\mathrm{MS}}}^{(V,\mathrm{NG})}$, and (iii) the volume of the SU(2) group. Because of these, $\log_{10}\gamma$ based on Ref. [23] becomes larger than ours by ∼65. In addition, the method of the path integral over the conformal mode and the choice of the renormalization scale are different; they result in the shift of $\log_{10}\gamma$ by ~ -33, which should be regarded as a theoretical uncertainty.

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