Unconventional Chiral Fermions and Large Topological Fermi Arcs in RhSi

Guoqing Chang,^{1,2} Su-Yang Xu,^{3,*} Benjamin J. Wieder,⁴ Daniel S. Sanchez,³ Shin-Ming Huang,⁵ Ilya Belopolski,³ Tay-Rong Chang,⁶ Songtian Zhang,³ Arun Bansil,⁷ Hsin Lin,^{1,2,†} and M. Zahid Hasan^{3,8,‡}

¹Centre for Advanced 2D Materials and Graphene Research Centre,

National University of Singapore, 6 Science Drive 2, Singapore 117546, Singapore

²Department of Physics, National University of Singapore, 2 Science Drive 3, Singapore 117542, Singapore

Department of Physics, Princeton University, Princeton, New Jersey 08544, USA

⁴Nordita, Center for Quantum Materials, KTH Royal Institute of Technology and Stockholm University,

Roslagstullsbacken 23, SE-106 91 Stockholm, Sweden

⁵Department of Physics, National Sun Yat-sen University, Kaohsiung 804, Taiwan

⁶Department of Physics, National Cheng Kung University, Tainan 701, Taiwan

⁷Department of Physics, Northeastern University, Boston, Massachusetts 02115, USA

⁸Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA

(Received 11 May 2017; revised manuscript received 21 July 2017; published 17 November 2017)

The theoretical proposal of chiral fermions in topological semimetals has led to a significant effort towards their experimental realization. In particular, the Fermi surfaces of chiral semimetals carry quantized Chern numbers, making them an attractive platform for the observation of exotic transport and optical phenomena. While the simplest example of a chiral fermion in condensed matter is a conventional |C| = 1 Weyl fermion, recent theoretical works have proposed a number of unconventional chiral fermions beyond the standard model which are protected by unique combinations of topology and crystalline symmetries. However, materials candidates for experimentally probing the transport and response signatures of these unconventional fermions have thus far remained elusive. In this Letter, we propose the RhSi family in space group No. 198 as the ideal platform for the experimental examination of unconventional chiral fermions. We find that RhSi is a filling-enforced semimetal that features near its Fermi surface a chiral double sixfolddegenerate spin-1 Weyl node at R and a previously uncharacterized fourfold-degenerate chiral fermion at Γ . Each unconventional fermion displays Chern number ± 4 at the Fermi level. We also show that RhSi displays the largest possible momentum separation of compensative chiral fermions, the largest proposed topologically nontrivial energy window, and the longest possible Fermi arcs on its surface. We conclude by proposing signatures of an exotic bulk photogalvanic response in RhSi.

DOI: 10.1103/PhysRevLett.119.206401

The allowed band crossings in condensed matter have, until recently, been considered closely linked to elementary particles in high-energy physics [1–4]. In three-dimensional (3D) systems without spatial inversion (\mathcal{I}) or time-reversal (\mathcal{T}) symmetry, twofold-degenerate band crossings are permitted, resulting in condensed matter realizations of Weyl fermions with quantized Chern numbers [1-24]. Since the experimental realization of the Weyl semimetal state in TaAs [18,19], recent theoretical efforts have become focused on finding unconventional condensed matter quasiparticle excitations beyond the Dirac and Weyl paradigm described by the standard model [4,25]. These efforts have rapidly expanded the set of known nodal features, which now additionally include symmorphic threefold nexus fermions [26-28], eightfold-degenerate double Dirac fermions [25], and, as detailed by Bradlyn *et al.* in Ref. [4], threefold-degenerate single and sixfold-degenerate double spin-1 Weyl points. Unconventional chiral fermions, in particular, hold great promise for experimental applications, as they broaden beyond conventional Weyl semimetals the search for materials candidates for the observation of topological surface states, bulk chiral transport, and exotic circular photogalvanic effects [29–36].

Although the fundamental theory for these unconventional fermions has been established, one outstanding issue has been the relative lack of ideal material candidates for their experimental examination. In the band structures of previously proposed materials, the unconventional fermions have typically sat away from the Fermi energy or have in the cases of unconventional chiral fermions coexisted with additional, trivial bands. In these systems, while the unconventional fermions may be experimentally observable by photoemission, their topological properties are still prohibitively difficult to detect and utilize for transport and optical response. For example, in MoP, a threefold nexus fermion is observed 1 eV below the Fermi level, but the Fermi surface itself is unrelated to the threefold fermion and carries no net Chern number [37].

In this Letter, we identify the RhSi materials family of structurally chiral cubic crystals in space group (SG) 198

0031-9007/17/119(20)/206401(6)

³Laboratory for Topological Quantum Matter and Spectroscopy (B7),

PHYSICAL REVIEW LETTERS



FIG. 1. Lattice and electronic structure of RhSi in SG 198. (a) Crystal structure of RhSi. Each unit cell contains four Rh and four Si atoms lying at Wyckoff positions with the minimum multiplicity of SG 198. (b) The cubic bulk BZ of RhSi. (c) Band structure of RhSi in the absence of spin-orbit coupling (SOC). The highest valance and lowest conduction bands are colored in blue and red, respectively. (d) Band structure in the presence of SOC. A chiral double spin-1 Weyl point sits ~0.4 eV below the Fermi energy at *R*, and a previously uncharacterized fourfolddegenerate chiral fermion lies at the Fermi energy at Γ .

 $P2_13$ [38] [Fig. 1(a)] as the first ideal materials candidates for the experimental study of the novel transport and response effects of unconventional chiral fermions. Using first-principles calculations detailed in Sec. A of Supplemental Material [39], we find that the Fermi surface of RhSi consists of only two well-isolated pieces which carry equal and opposite quantized Chern numbers. The bulk bands near the Fermi energy feature a chiral sixfolddegenerate double spin-1 Weyl at the Brillouin zone (BZ) corner R and a fourfold-degenerate chiral fermion at the zone center Γ [Figs. 1(c), 1(d), 2(a), and 2(b)], which is uncharacterized by previous studies on unconventional fermions [4,25–28]. RhSi therefore displays the *largest* possible separation of chiral fermions allowed in crystals. With an otherwise large band gap, RhSi also therefore features the largest topologically nontrivial energy window proposed thus far [Fig. 1(d) and Supplemental Material, Sec. E [39]). Furthermore, as these two chiral fermions lie at time-reversal-invariant momenta (TRIMs), they are unrelated by symmetry and free to exhibit an energy offset; here, the fourfold fermion at Γ lies roughly 400 meV above the sixfold fermion at R. This offset allows for the possibility of unique optical transport, such as the quantized circular photogalvanic effect [33]. Among all known chiral semimetals, both conventional Weyl and unconventional higherfold fermions, RhSi therefore stands as possibly the most electronically ideal material yet proposed.

To understand the unusual high-fold-degenerate nodes displayed in the minimal band connectivity of SG 198, we construct an eight-band tight-binding (TB) model (Supplemental Material, Sec. C [39]). SG 198 is



week ending

FIG. 2. Energy dispersions and chiral character of the fourfoldand sixfold-degenerate unconventional fermions in RhSi. (a)–(c) 3D energy dispersions of the degeneracies at Γ , R, and *M*, respectively. (d)–(f) Band structures in the vicinities of Γ , *R*, and M, respectively. Because of the local Kramers theorem enforced under the combined operation of $(s_{2x,y,z} \times T)^2 = -1$, bands along $k_{x,y,z} = \pi$ are twofold-degenerate (e),(f). The absence of rotoinversion symmetries in SG 198 allows for nodes at TRIMs to have nontrivial Chern numbers; nodes with multiple finite-qgaps can exhibit different Chern numbers occupying bands up to each gap (Supplemental Material, Sec. F [39]) (d)-(f). At the Fermi energy, the fourfold-degenerate fermion at Γ has Chern number +4, and the double spin-1 Weyl at R has Chern number -4. The quadratic fourfold-degenerate crossing at M (f) also exhibits a Chern number, but the bands dispersing from it are almost entirely covered by the Fermi energy. The fourfold-degenerate unconventional fermion at Γ (d) can be considered the combination of two J = 1/2 and two J = 3/2 states pinned together by time-reversal and screw symmetries. The analogous angular momentum eigenvalues for each band can then be deduced by observing the band eigenvalues of $C_{3,111}$ and considering the symmetry-allowed term $\vec{k} \cdot \vec{J}$ (Supplemental Material, Sec. C2 [39]). (g),(h) The Berry curvature $\vec{\Omega}$ on the $k_x = k_y$ plane flows almost directly from Γ to R with minimal out-of-plane deviations. Measuring the intensity of the xy (g) and z (h) components of $\vec{\Omega}$, we verify that Γ and R exhibit the local vector fields of $C = \pm 4$ hedgehog defects.

characterized by three nonintersecting twofold screw rotations $s_{2x,y,z}$, related by diagonal cubic threefold rotation $C_{3,111}$ [47]:

$$s_{2x} = \left\{ C_{2x} | \frac{1}{2} \frac{1}{2} 0 \right\}, \qquad s_{2y} = \left\{ C_{2y} | 0 \frac{1}{2} \frac{1}{2} \right\},$$
$$s_{2z} = \left\{ C_{2z} | \frac{1}{2} 0 \frac{1}{2} \right\}, \qquad C_{3,111} = \left\{ C_{3,111} | 000 \right\}.$$
(1)

Without the threefold rotation, this combination of screws and \mathcal{T} symmetry characterizes orthorhombic SG 19 and has been shown to force groups of eight or more bands to tangle together [48-52]. The additional cubic threefold rotation $C_{3,111}$ in SG 198 serves to increase the band degeneracy at TRIMs while still preserving this eight-band connectivity. At an electron filling of $\nu \in 8\mathbb{Z} + 4$, RhSi is gapless due to the combination of time-reversal and nonsymmorphic symmetries and is therefore a "filling-enforced" semimetal (Supplemental Material, Sec. B [39]) [48,50,53]. We find that our minimal TB model of SG 198 captures all of the degeneracy structure and topological character of RhSi. We list in Supplemental Material, Sec. C [39] the irreducible corepresentations of the high-fold degeneracies at Γ and R in the language of Refs. [4,25,47] and describe our results for the full BZ in detail; here we focus on characterizing the structure of the chiral nodes at Γ and R.

We begin by examining the band splitting and *previously* uncharacterized fourfold-degenerate unconventional chiral fermion at Γ . In the absence of SOC, our eight-band model permits only a single mass term at Γ which splits bands into a 3×2 -fold-degenerate fermion and a doubly degenerate quadratic band, which in RhSi lies more than 2 eV above the Fermi energy [Fig. 1(c)]. Upon the introduction of SOC, this quadratic crossing opens into a Kramers Weyl [24], and the 3×2 -fold-degenerate node splits into a fourfold-degenerate unconventional fermion and a second Kramers Weyl [Figs. 2(a) and 2(d)]. This fourfolddegenerate fermion is distinct from the spin-3/2 chiral fermion introduced in Ref. [4]: Whereas that fermion is described by a corepresentation equivalent to the fourdimensional irreducible representation \overline{F} of chiral point group 432 (O), the fourfold-degenerate fermion in RhSi is described by the \mathcal{T} -symmetric corepresentation formed by pairing the two-dimensional irreducible representations ${}^1\bar{F}$ and ${}^{2}\bar{F}$ of chiral point group 23 (T) [47]. In the language of atomic orbitals, this fourfold degeneracy can be understood by modeling the six degenerate states without SOC by three p orbitals and an electron spin in the 111 direction. Calling z' the 111 direction and x', y' as orthonormal axes spanning the plane normal to z', we group the p orbitals into a $p_{z'}$, $m_l = 0$ orbital and $p_{x'} \pm i p_{y'}$, $m_l = \pm 1$ orbitals. When coupled to the spin-1/2 electron, the six total states split into four J = 1/2 and two J = 3/2 states. Time-reversal pairs states with the same J and opposite m_i , and s_{2x} flips m_s without affecting m_l , such that under the SG 198 generators two J = 3/2 states pair with two J = 1/2 states and the remaining two J = 1/2 states split off and form the second Kramers Weyl (Supplemental Material, Sec. C 2 [39]). By numerically calculating the eigenvalues of $C_{3,111}$ and considering the symmetry-allowed term $\vec{k} \cdot \vec{J}$, each band near Γ can be assigned J and m_i eigenvalues, a structure we confirm explicitly with a symmetry-generated four-band $k \cdot p$ model in Supplemental Material, Sec. C 2 [39]. As the irreducible representations at Γ are reflective of the position-space atomic orbitals [49], this analogy should also provide physical insight into the bonding character of RhSi. By integrating the Berry curvature between bands with J = 1/2, $m_j = \pm 1/2$ over a k-space sphere in the vicinity of Γ [3], we find that this unconventional fermion exhibits Chern number +4 at the Fermi level in RhSi [Fig. 2(d)].

Our examinations of the unconventional fermions at R with and without SOC (Supplemental Material, Sec. C 2 [39]) confirm the results of previous analyses of SGs 19 and 198 [4,25,50–52]. When SOC is taken into consideration, RhSi displays at R a sixfold-degenerate chiral double spin-1 Weyl at ~0.4 eV below the Fermi level, which at the finite-q gap spanned by the Fermi level, which at the finite-q gap spanned by the Fermi energy exhibits Chern number -4 [Figs. 2(b) and 2(e)]. Projecting out of our TB model the six-band subspace of this chiral fermion results in a $k \cdot p$ theory related by a unitary transformation to that presented in Ref. [4] of two coupled spin-1 fermions with individual $\vec{k} \cdot \vec{S}$ dispersion.

Calculating the surface states of RhSi [Fig. 3(a)] through surface Green's functions (Supplemental Material, Sec. A [39]), we find that the (001) surface displays four topological Fermi arcs connecting the projections of the bulk chiral fermions at $\overline{\Gamma}$ to those at \overline{M} across the entire surface BZ. Unlike the recently observed trivial arcs in WTe₂ [23,54], the long Fermi arcs in RhSi are guaranteed by bulk topology and should therefore be robust against changes in surface chemical potential and disorder (Supplemental Material, Sec. E [39]). Though the arcs in our calculations demonstrate a particularly elaborate connectivity [Fig. 3(c)], a much simpler direct connectivity is also allowed [Fig. 3(d)]. We also find the Fermi arcs to have ~80% spin polarization [55] [Fig. 3(b)]. Therefore, RhSi is also an attractive platform for spintronic applications [56,57].

To summarize our analysis of the electronic structure of RhSi, we find that it is a remarkably ideal candidate for the observation of chiral transport and optical phenomena and for the direct examination of unconventional fermions. Bands within the $k \cdot p$ regime of the unconventional fermions at Γ and R cleanly characterize the *entire* Fermi surface, such that the separation between Fermi pockets of opposite Chern number is the entire length of the 3D diagonal of the BZ cube. The remaining bulk band manifolds are otherwise separated by a gap of $\sim 1.2 \text{ eV}$ [Fig. 1(d)], such that RhSi has by far the largest topologically nontrivial energy window of any previously proposed or experimentally realized chiral semimetal (Supplemental Material, Sec. E [39]). RhSi also therefore displays on its surface topologically guaranteed Fermi arcs that span the entire surface BZ and uniquely come in time-reversed pairs (Fig. 3). Finally, unlike in previous band-inversion Weyl semimetals where pairs of Weyl points have been related by mirror symmetry, the chiral fermions in RhSi are free to sit with an energy offset, enabling chiral photogalvanic transport [33].



FIG. 3. Surface state texture of RhSi. (a) The (001)-surface states of RhSi calculated using surface Green's functions (Supplemental Material, Sec. A [39]). Four Fermi arcs radiate at $\overline{\Gamma}$ from the projection of the bulk fourfold-degenerate fermion at Γ , grouping into two time-reversed pairs and spiraling around the BZ (c) until they meet at \overline{M} at the projection of the bulk double spin-1 Weyl at R. (b) The surface states demonstrate $\sim 80\%$ spin polarization (Supplemental Material, Sec. A [39]). (d) An allowed simplified Fermi arc connectivity. For both possible connectivities (c),(d), plotting the surface bands along a clockwise loop surrounding $\overline{\Gamma}$ [red loop in (a), dashed loop in (d)], the surface bands (e) demonstrate a C = +4 spectral flow. (f) Conversely, taking a loop along the zone-spanning dashed line at $k_{\rm v} = 0.15$ results in a surface state texture with just C = +2spectral flow, as only two Fermi arcs cross each half of the surface BZ.

We therefore conclude with a numerical prediction of quantized optical transport in RhSi. In Ref. [33], the authors show that in a structurally chiral system for which only a single two-band Weyl fermion is partially unoccupied, such as a Kramers Weyl metal [24], the difference in the rate of current density resulting from exciting electrons with left- and right-handed circularly polarized light is quantized in terms of fundamental constants:

$$\frac{dj}{dt} = \frac{2I\beta_0}{c\epsilon_0}C, \qquad \beta_0 = \frac{\pi e^3}{h^2}, \tag{2}$$

where *I* is the intensity of applied light and *C* is the Chern number of the Weyl point [Figs. 4(a) and 4(b)]. In RhSi, the fourfold fermion at Γ sits just above the Fermi energy, while the chiral double spin-1 Weyl at *R* sits below and is fully occupied; the location of the chiral fermions in its band structure [Fig. 1(d)] is *practically identical* to the ideal case



FIG. 4. Quantized circular photogalvanic effect (CPGE) of the fourfold-degenerate unconventional fermion in RhSi. (a),(b) The CPGE of a single conventional Weyl node as proposed in Ref. [33]. (b) Calculations of the CPGE for the fourfold unconventional fermion at Γ in RhSi, tuned to half filling. The photocurrent rate saturates at 4 times the value it did for the conventional Weyl in (a), as the Chern number in this gap is 4 times as large. (c) The more realistic case of a partial occupation of this fourfold fermion; multiple transitions contribute to the photocurrent. (d) Contributions to the traced photocurrent rate from each transition in (c), calculated from the fitted TB model (Supplemental Material, Sec. C 3 [39]). Trend lines in (d) are labeled by the color of their contributing transition in (c), with pink representing the overall photocurrent rate.

proposed in Ref. [33]. We observe that the angular momentum selection rules for circularly polarized light appear to strongly constrain the allowed transitions in this fourfold fermion, such that only transitions between bands with $\Delta m_j = \pm 1$ contribute to the photocurrent [58]. Therefore, when weighting by Fermi occupation factors, the photocurrent rate calculated from the trace of the gyrotropic tensor (Supplemental Material, Sec. D [39]), though initially fluctuating, still saturates at the quantized value $(2I\beta_0/c\epsilon_0) \times 4$ with increasing incident photon energy E_p in the vicinity of Γ , or 4 times the value predicted for a conventional Weyl fermion [Figs. 4(b) and 4(d)]. Therefore, despite the multiband complexities of its unconventional chiral fermions, RhSi remains a plausible candidate for probing the quantized photogalvanic effect.

The authors thank Charles L. Kane, Barry Bradlyn Jennifer Cano, and B. A. Bernevig for discussions. The work at Princeton is supported by the National Science Foundation, Division of Materials Research, under Grants No. NSF-DMR-1507585 and No. NSF-DMR-1006492 and by the Gordon and Betty Moore Foundation through the EPIQS program Grant No. GBMF4547-HASAN. The work

at the National University of Singapore was supported by the National Research Foundation, Prime Minister's Office, Singapore under its NRF fellowship (NRF Award No. NRF-NRFF2013-03). B. J. W. was supported through Nordita under ERC DM 321031. The work at Northeastern University was supported by the US Department of Energy (DOE), Office of Science, Basic Energy Sciences Grant No. DE-FG02-07ER46352, and benefited from Northeastern University's Advanced Scientific Computation Center (ASCC) and the NERSC supercomputing center through DOE Grant No. DE-AC02-05CH11231. The work at the National Sun Yat-sen University was supported by the Ministry of Science and Technology in Taiwan under Grant No. MOST105-2112-M-110-014-MY3. T.-R.C. was supported by the Ministry of Science and Technology and National Cheng Kung University, Taiwan. T.-R. C. also thanks National Center for Theoretical Sciences (NCTS), Taiwan for technical support.

G. C., S.-Y. X., B. J. W., and D. S. S. contributed equally to this work.

Note added.—We noticed a related work that reports similar chiral fermions [46].

^{*}Corresponding author. suyangxu@princeton.edu [†]Corresponding author. nilnish@gmail.com [‡]Corresponding author. mzhasan@princeton.edu

- [1] H. Weyl, Z. Phys. 56, 330 (1929).
- [2] F. Wilczek, Phys. Today 51, No. 11 (1998).
- [3] G. E. Volovik, *The Universe in a Helium Droplet* (Clarendon, Oxford, 2003).
- [4] B. Bradlyn, J. Cano, Z. Wang, M. G. Vergniory, C. Felser, R. J. Cava, and B. Andrei Bernevig, Science 353, aaf5037 (2016).
- [5] C. L. Kane and E. J. Mele, Phys. Rev. Lett. 95, 226801 (2005).
- [6] C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 146802 (2005).
- [7] M.Z. Hasan and C.L. Kane, Rev. Mod. Phys. 82, 3045 (2010).
- [8] X.-L. Qi and S.-C. Zhang, Rev. Mod. Phys. 83, 1057 (2011).
- [9] B. A. Bernevig, T. L. Hughes, and S.-C. Zhang, Science 314, 1757 (2006).
- [10] A. Bansil, H. Lin, and T. Das, Rev. Mod. Phys. 88, 021004 (2016).
- [11] M. Z. Hasan, S.-Y. Xu, I. Belopolski, and S.-M. Huang, Annu. Rev. Condens. Matter Phys. 8, 289 (2017).
- [12] N. P. Armitage, E. J. Mele, and A. Vishwanath, arXiv: 1705.01111.
- [13] X. Wan, A. M. Turner, A. Vishwanath, and S. Y. Savrasov, Phys. Rev. B 83, 205101 (2011).
- [14] A. A. Burkov and L. Balents, Phys. Rev. Lett. 107, 127205 (2011).
- [15] S. Murakami, New J. Phys. 9, 356 (2007).

- [16] S.-M. Huang et al., Nat. Commun. 6, 7373 (2015).
- [17] H. Weng, C. Fang, Z. Fang, B. A. Bernevig, and X. Dai, Phys. Rev. X 5, 011029 (2015).
- [18] S.-Y. Xu et al., Science 349, 613 (2015).
- [19] B. Q. Lv et al., Phys. Rev. X 5, 031013 (2015).
- [20] L. Lu, Z. Wang, D. Ye, L. Ran, L. Fu, J. D. Joannopoulos, and M. Soljačić, Science 349, 622 (2015).
- [21] A. A. Soluyanov, D. Gresch, Z. Wang, Q. Wu, M. Troyer, X. Dai, and B. Andrei Bernevig, Nature (London) 527, 495 (2015).
- [22] Y. Xu, F. Zhang, and C. Zhang, Phys. Rev. Lett. 115, 265304 (2015).
- [23] S.-Y. Xu et al., Sci. Adv. 3, e1603266 (2017).
- [24] G. Chang et al., arXiv:1611.07925.
- [25] B. J. Wieder, Y. Kim, A. M. Rappe, and C. L. Kane, Phys. Rev. Lett. **116**, 186402 (2016).
- [26] H. Weng, C. Fang, Z. Fang, and X. Dai, Phys. Rev. B 93, 241202 (2016).
- [27] Z. Zhu, G. W. Winkler, Q. S. Wu, J. Li, and A. A. Soluyanov, Phys. Rev. X 6, 031003 (2016).
- [28] G. Chang et al., Sci. Rep. 7, 1688 (2017).
- [29] S. Zhong, J. E. Moore, and I. Souza, Phys. Rev. Lett. 116, 077201 (2016).
- [30] J. Ma and D. A. Pesin, Phys. Rev. B 92, 235205 (2015).
- [31] C.-K. Chan, N. H. Lindner, G. Refael, and P. A. Lee, Phys. Rev. B 95, 041104(R) (2017).
- [32] Q. Ma et al., Nat. Phys. 13, 842 (2017).
- [33] F. de Juan, A. G. Grushin, T. Morimoto, and J. E. Moore, Nat. Commun. 8, 15995 (2017).
- [34] L. Balents, Physics 4, 36 (2011).
- [35] T. Ojanen, Phys. Rev. B 87, 245112 (2013).
- [36] C. Fang, L. Lu, J. Liu, and L. Fu, Nat. Phys. 12, 936 (2016).
- [37] B. Q. Lv et al., Nature (London) 546, 627 (2017).
- [38] I. Engström and J. Torsten, Acta Chem. Scand. **19**, 1508 (1965).
- [39] See Supplemental Material at http://link.aps.org/ supplemental/10.1103/PhysRevLett.119.206401 for the details of our first-principles electronic structure and photocurrent calculations, additional materials candidates in SG 198, our tight-binding model of SG 198 with and without SOC, and for our symmetry and group theoretic analysis of the band connectivity and degeneracy of RhSi, which includes Refs. [40–46].
- [40] P. Demchenko et al., Chem. Met. Alloys 1, 50 (2008).
- [41] G. Wenski and A. Mewis, Z. Anorg. Allg. Chem. 535, 110 (1986).
- [42] H. Takizawa, T. Sato, T. Endo, and M. Shimada, J. Solid State Chem. 73, 40 (1988).
- [43] V. Larchev and S. Popova, J. Less-Common Met. 87, 53 (1982).
- [44] M. Ettenberg, K. L. Komarek, and E. Miller, Metall. Trans.2, 1173 (1971).
- [45] K. Schubert, W. Burkhardt, P. Esslinger, E. Günzel, H. G. Meissner, W. Schütt, J. Wegst, and M. Wilkens, Naturwissenschaften 43, 248 (1956).
- [46] P. Tang *et al.*, following Letter, Phys. Rev. Lett. **119**, 206402 (2017).
- [47] C. J. Bradley and A. P. Cracknell, *The Mathematical Theory* of Symmetry in Solids (Clarendon, Oxford, 1972), ISBN 0199582580.

- [48] H. Watanabe, H. C. Po, A. Vishwanath, and M. Zaletel, Proc. Natl. Acad. Sci. U.S.A. 112, 14551 (2015).
- [49] B. Bradlyn, L. Elcoro, J. Cano, M. G. Vergniory, Z. Wang, C. Felser, M. I. Aroyo, and B. Andrei Bernevig, Nature (London) 547, 298 (2017).
- [50] B. J. Wieder and C. L. Kane, Phys. Rev. B 94, 155108 (2016).
- [51] A. Bouhon and A. M. Black-Schaffer, Phys. Rev. B 95, 241101(R) (2017).
- [52] R. M. Geilhufe, S. S. Borysov, A. Bouhon, and A. V. Balatsky, arXiv:1611.04316.
- [53] S. M. Young and B. J. Wieder, Phys. Rev. Lett. 118, 186401 (2017).
- [54] F. Y. Bruno *et al.*, Phys. Rev. B **94**, 121112(R) (2016).
- [55] S.-Y. Xu et al., Phys. Rev. Lett. 116, 096801 (2016).
- [56] I. Žutić, J. Fabian, and S. Das Sarma, Rev. Mod. Phys. 76, 323 (2004).
- [57] A. R. Mellnik *et al.*, Nature (London) **511**, 449 (2014).
- [58] C. L. Tang and H. Rabin, Phys. Rev. B 3, 4025 (1971).