Geiger-Nuttall Law for Nuclei in Strong Electromagnetic Fields

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We investigate the influence of a strong laser electromagnetic field on the α-decay rate by using the Hennenberger frame of reference. We introduce an adimensional parameter $D = S_0/R_0$, where R_0 is the geometrical nuclear radius and $S_0 \sim \sqrt{I}/\omega^2$ is a length parameter depending on the laser intensity I and frequency ω. We show that the barrier penetrability has a strong increase for intensities corresponding to $D > D_{\rm crit} = 1$, due to the fact that the resulting Coulomb potential becomes strongly anisotropic even for spherical nuclei. As a consequence, the contribution of the monopole term increases the barrier penetrability by 2 orders of magnitude, while the total contribution has an effect of 6 orders of magnitude at $D \sim 3D_{\rm crit}$. In the case of deformed nuclei, the electromagnetic field increases the penetrability by an additional order of magnitude for a quadrupole deformation $\beta_2 \sim 0.3$. The influence of the electromagnetic field can be expressed in terms of a shifted Geiger-Nuttal law by a term depending on S_0 and deformation.

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The α-decay process is governed by the Geiger-Nuttall law [1] expressing a linear dependence,

$$\log_{10} T_{1/2} = a\chi + b(Z), \qquad (1)$$

between the logarithm of the half life and the Coulomb-Sommerfeld parameter

$$\chi = \frac{4Ze^2}{\hbar v_\alpha}, \qquad (2)$$

where Z is the charge of the daughter nucleus, and $v_\alpha = \sqrt{2Q_\alpha/M_\alpha}$ the asymptotic α-daughter relative velocity, expressed in terms of the emission energy (Q value) Q_α and reduced mass M_α. This dependence was explained a long time ago in terms of the quantum-mechanical penetration through the Coulomb barrier surrounding the nuclear field by a preformed α-particle [2,3]. Later on, different models tried to explain the process of the α-particle formation on the nuclear surface within the R-matrix theory [4] or the fissionlike approach [5]. Let us mention that α clustering plays an important role in light nuclei [6]. A useful tool to investigate the structure of heavy and superheavy nuclei is the α decay to excited states [7]. Modern laser facilities allow the probing of subatomic structures by using strong electromagnetic fields [8–12]. Several papers have investigated the role of laser pulses on the Coulomb barrier governing the α-decay process [13–16]. In Ref. [17], a semiclassical correction to the α-decay rate in an oscillating electromagnetic field is obtained. The relative change in the α-decay rate is calculated as a function of the nuclear charge, Q value, and the laser-radiation intensity. In Refs. [18,19], the discussion treats the manner in which the α-decay dynamics in a spherical nucleus is modified by a linearly polarized ultraintense laser field by using a quantum time-dependent formalism. The wave-packet dynamics was determined for various laser intensities for continuous waves and for sequences of pulses, leading to an enhancement of the tunneling probability.

The purpose of this Letter is to demonstrate that the standard Geiger-Nuttall law (1) becomes shifted by a term depending on intensity and frequency of the laser field, as well as on the nuclear deformation.

The time-dependent Schrödinger equation describing the relative motion of an α-particle inside the Coulomb barrier is given by

$$i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t} = \left[\frac{1}{2M_\alpha}\left(\mathbf{P} - \frac{e_{\rm eff}}{c}\mathbf{A}(t)\right)^2 - V(\mathbf{r})\right]\Psi(\mathbf{r},t), \qquad (3)$$

where $\mathbf{A}(t)$ is the time-dependent magnetic vector potential, $e_{\rm eff} = eZ_{\rm eff} = e(2A - 4Z)/(A+4)$ is the effective charge [18], and $V(\mathbf{r})$ is the Coulomb potential. By using the unitary Hennenberger transformation [13]

$$\Omega = \exp\left[\frac{i}{\hbar}\int_{-\infty}^{t} H_{\rm int}(\tau)d\tau\right], \qquad (4)$$

with

$$H_{\rm int}(t) = -\frac{e_{\rm eff}}{M_\alpha c}\mathbf{A}\mathbf{P} + \frac{e_{\rm eff}^2}{2M_\alpha c^2}\mathbf{A}^2, \qquad (5)$$

being the perturbation Hamiltonian, the new wave function $\Phi = \Omega\Psi$ satisfies the following equation:

$$i\hbar\frac{\partial\Phi(\mathbf{r},t)}{\partial t} = \left[\frac{1}{2M_\alpha}\mathbf{P}^2 - V(\mathbf{r}-\mathbf{S}(t))\right]\Phi(\mathbf{r},t), \quad (6)$$

where we introduced the classical trajectory

$$\mathbf{S}(t) = \frac{e_{\mathrm{eff}}}{M_\alpha c}\int_{-\infty}^{t}\mathbf{A}(\tau)d\tau. \quad (7)$$

This time-dependent potential can be expanded in a Fourier basis [15]. For the present discussion the static component (the Kramers-Hennenberger approach) is the most important part due to the fact that $\hbar\omega \sim 10^{-4}$ MeV $\ll Q_\alpha \sim 5$ MeV. Although the new system of reference is noninertial, its energy (< 1 eV) is much less than the Q value. Moreover, one can show that the decay width remains unchanged in the static approach after this unitary transformation. The static component

$$V_0(\mathbf{r}) = \frac{1}{T}\int_0^T V(\mathbf{r}-\mathbf{S}(t))dt \quad (8)$$

for a spherical Coulomb potential

$$V(\mathbf{r}-\mathbf{S}(t)) = \frac{2Ze^2}{|\mathbf{r}-\mathbf{S}(t)|}, \quad (9)$$

can be written as follows:

$$V_0(r,\Theta,S_0) = \frac{2Ze^2}{r}\xi(r,\Theta,S_0), \quad (10)$$

where Θ is the angle between the α-emission direction and incidence direction of the beam and S_0 is the amplitude of a linearly polarized beam,

$$\mathbf{S}(\omega t) = \mathbf{e}_z S_0 \sin\omega t. \quad (11)$$

Using the integration variable $x = \omega t$, we obtain for the screening function the following expression:

$$\xi(r,\Theta,S_0) = \frac{1}{2\pi}\int_0^{2\pi}\frac{dx}{\left(1 - 2\frac{S(x)}{r}\cos\Theta + \frac{\mathbf{S}^2(x)}{r^2}\right)^{1/2}}, \quad (12)$$

leading to a deformation of the Coulomb field, and therefore to an anisotropic character of the α-emission process [20]. By using the relation connecting the intensity of the beam and the electric field magnitude [21], one gets

$$S_0 = Z_{\mathrm{eff}}\frac{\sqrt{4\pi\hbar\alpha I}}{M_\alpha\omega^2} \sim 8\sqrt{\frac{I}{10^{20}}}\left(\frac{100}{\hbar\omega}\right)^2 \text{ fm}, \quad (13)$$

where α is the fine-structure constant, I is given in W/cm^2, and $\hbar\omega$ in eV. This approach was used to predict a change of the ionization potential for H atom in a strong laser field when the length amplitude S_0 becomes comparable to the Bohr radius [13]. Similarly, we expect a relevant change of the decay width in our case as soon as S_0 becomes comparable to the nuclear radius R_0. By defining the adimensional parameter $D \equiv S_0/R_0 \sim \sqrt{I}/\omega^2$, one obtains for the threshold intensity, where $D \sim 1$, a value $I \sim 10^{20}$ W/cm^2 for $\hbar\omega = 100$ eV and $I \sim 10^{22}$ W/cm^2 for $\hbar\omega = 300$ eV. Let us mention that the results become analytical only if the monopole contribution is taken into account,

$$\begin{aligned}\xi(r,S_0) &= \xi\left(\frac{r}{S_0}\right) = 1, \quad r \geq S_0\\ &= \frac{1}{4\pi}\left\{8\arcsin\frac{r}{S_0} - 8\frac{r}{S_0}\log_{10}\left[\tan\left(\frac{1}{2}\arcsin\frac{r}{S_0}\right)\right]\right\},\\ &\quad r < S_0. \end{aligned} \quad (14)$$

The α-decay half-life is inversely proportional to the barrier penetrability P and reduced width γ_α^2,

$$T_{1/2} = \frac{\hbar ln2}{2P(R)\gamma_\alpha^2(R)}. \quad (15)$$

The product in the denominator does not depend on the radius R. The reduced width at a given radius has a smooth behavior within 1 order of magnitude along the periodic table. Therefore, the main contribution in the dependence between the half-life and Coulomb-Sommerfeld parameter is given by the penetrability [5]. The penetrability can be expressed within the semiclassical approach in terms of the action integral,

$$P(\Theta) = \exp\left(-2\int_{R_c}^{R_1(\Theta)}\sqrt{\frac{2M_\alpha}{\hbar^2}[V_0(r,\Theta) - Q_\alpha]dr}\right), \quad (16)$$

where $R_c = R_0 + R_\alpha = 1.2(A^{1/3} + 4^{1/3})$ is the geometrical touching radius and $R_1(\Theta)$ gives the external turning point. The total penetrability is then given by [5]

$$P = \frac{1}{2}\int_0^\pi P(\Theta)\sin\Theta d\Theta. \quad (17)$$

In the case of an axially deformed nucleus, the important contribution aside the monopole one is given by the quadrupole term

$$V(\mathbf{r}) = V(r,\theta) = \frac{2Ze^2}{r}\left[1 + \frac{1}{10r^4}(a^2 - c^2)(3z^2 - r^2)\right], \quad (18)$$

where a and c are the two different semiaxes of the spheroid and θ is the angle of $\mathbf{r}$ with respect to the nuclear symmetry axis. It turns out that the relation (10) can be generalized for a deformed potential,

$$V(r,\theta,\Theta,S_0) = \frac{2Ze^2}{r}\xi_d, \tag{19}$$

in terms of a deformed screening function

$$\xi_d = \left[\xi + \frac{1}{10r^4}(a^2-c^2)(\xi_q^{(1)} - r^2\xi_q^{(2)})\right], \tag{20}$$

where ξ is defined by Eq. (12) (depending now on $\theta - \Theta$) and

$$\xi_q^{(1)} = \frac{1}{2\pi}\int_0^{2\pi} \frac{3r^2[\cos\theta - S_0(x,r)\cos\Theta]^2 dx}{[1-2S_0(x,r)\cos(\theta-\Theta)+S_0^2(x,r)]^{5/2}}$$
$$\xi_q^{(2)} = \frac{1}{2\pi}\int_0^{2\pi} \frac{dx}{[1-2S_0(x,r)\cos(\theta-\Theta)+S_0^2(x,r)]^{3/2}}$$
$$S_0(x,r) \equiv \frac{S_0 \sin x}{r}. \tag{21}$$

The angular penetrability is dependent on both the $\mathbf{r}(\theta)$ and $\mathbf{S}(\Theta)$ directions,

$$P(\theta,\Theta) = \exp\left(-2\int_{R_c(\theta)}^{R_1(\Theta)} \sqrt{\frac{2M_\alpha}{\hbar^2}[V(r,\theta,\Theta) - Q_\alpha]dr}\right), \tag{22}$$

where $R_c(\theta) = R_0[1+\beta_2 Y_{20}(\theta)] + R_\alpha$. The total penetrability becomes

$$P = \frac{1}{4}\int_0^\pi \sin\theta d\theta \int_0^\pi P(\theta,\Theta)\sin\Theta d\Theta. \tag{23}$$

In Fig. 1, we plotted the angular penetrability given by Eq. (16) normalized to its maximal value (in the logarithmic scale) versus the angle between the α particle and laser beam Θ for $D = 1/\sqrt{10}$ (stars), $D = 1$ (triangles), and $D = \sqrt{10}$ (circles). In spite of the fact that we considered the spherical emitter ^{212}Po, the anisotropy strongly increases from a factor less than 2 for $D = 1/\sqrt{10}$ up to more than 6 orders of magnitude for $D = \sqrt{10}$, with α emission being practically concentrated in the equatorial direction $\Theta \sim 90°$.

In Fig. 2, we plotted the logarithm of the penetrability (23) versus $D = S_0/R_0$ for the deformed emitter ^{232}Pu. We mention a behavior similar to that in Fig. 1, namely, that the penetrability has a weak dependence on the laser intensity for $D < 1$ and a strong increase in the region $D > 1$. In the same plot we considered several values of the quadrupole deformation. Notice that for $\beta_2 = 0.3$ one obtains an overall increase of about 1 order of magnitude. Let us mention that the influence of the preformation probability $\gamma_\alpha^2(R)$ on the classical trajectory (7), and therefore on the half-life (15), still remains an open question. The real experimental dependence of the half-life on the laser intensity will provide information on this effect.

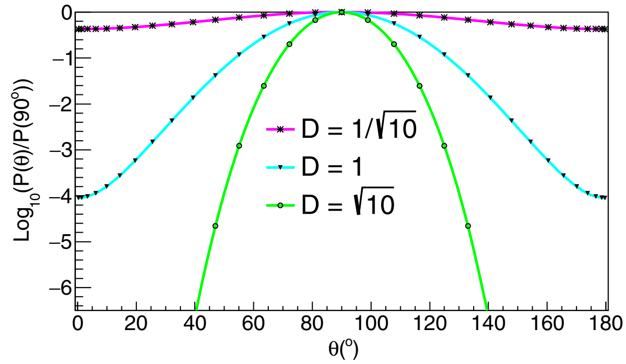

FIG. 1. Logarithm of the penetrability given by Eq. (16) normalized to its maximal value versus the angle between the emitted particle and laser beam Θ for $D = S_0/R_0 = 1/\sqrt{10}$ (stars), $D = 1$ (triangles), and $D = \sqrt{10}$ (circles) from ^{212}Po.

We analyzed the penetrability along two isotope chains, one for spherical and the other for deformed nuclei. In Fig. 3, we investigated the chain of quasispherical Po isotopes with $A = 192$–218. With crosses, we plotted the standard Geiger-Nuttall dependence between the logarithm of the penetrability and Coulomb-Sommerfeld parameter (2), corresponding to the absence of the external field. With circles we plotted the same dependence, but considered only the spherical part of the laser field with $D = \sqrt{10}$, thus giving an increase of about 2 orders of magnitude. With triangles we considered the full electromagnetic field. We notice a very strong influence of about 6 orders of magnitude. In all cases, the fitting red lines in terms of the standard Coulomb-Sommerfed parameter are practically parallel.

In Fig. 4, we investigated the Geiger-Nuttall law for deformed Pu isotopes with $A = 232$–244 and quadrupole deformations $\beta_2 = 0.21$–0.23 [22], in the absence of the

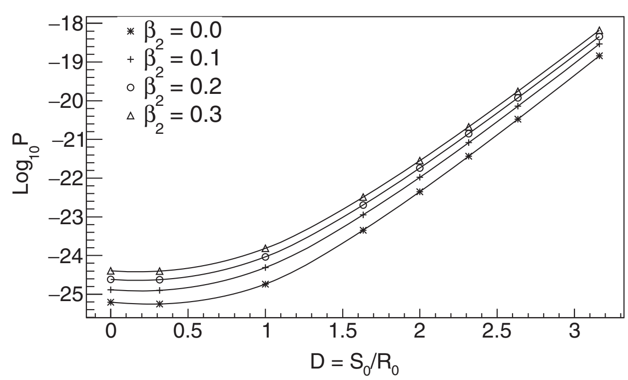

FIG. 2. Logarithm of the total penetrability versus $D = S_0/R_0$ for various quadrupole deformations of ^{232}Pu.

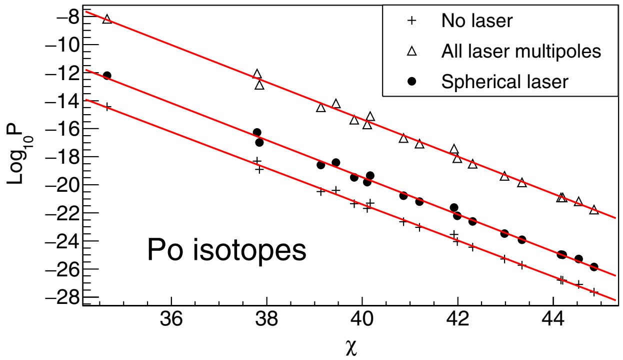

FIG. 3. Logarithm of penetrability versus the Coulomb-Sommerfeld parameter for Po istopes in the absence of the laser field (crosses), by considering only the monopole component (circles) and the full contribution (triangles) for $D = \sqrt{10}$. The corresponding linear fits, expressing the Geiger-Nuttall law, are plotted by red lines.

laser field (crosses) and by considering the full contribution of the external field with $D = \sqrt{10}$ (triangles). One sees that the laser field induces a correction of about 6 orders of magnitude with respect to the dependence in the absence of the laser field, which corresponds to deformed nuclei.

In this way, the Geiger-Nuttall law becomes modified in the presence of a strong electromagnetic field

$$\log_{10} T_{1/2} = a\chi + b(Z) + c(S_0, \beta_2), \qquad (24)$$

by a new shifting term depending on the parameter S_0 given by Eq. (13), as well as on the nuclear deformation.

In Ref. [17] the static Coulomb barrier was corrected by an oscillating dipole electric field instead of the exact Eq. (3). As a consequence, its amplitude $F = \sqrt{8\pi I/c}$ does not depend on laser frequency and therefore the threshold value of intensity cannot be estimated. The static component of the correction vanishes and therefore it was introduced an averaged angular penetrability instead. The decay width depends linearly on intensity, predicting a relative small decrease of the half-life at $I = 10^{28}/\mathrm{cm}^2$. On the other hand, in Refs. [18,19] a one-dimensional time-dependent numerical approach for a Woods-Saxon plus Coulomb potential was applied, based on the correct Eq. (3). The present authors are in disagreement with Ref. [17] and report an important increase of the penetrability corresponding to intensities larger than the threshold value $I = 10^{25}\ \mathrm{W/cm^2}$ at $\hbar\omega = 300$ eV for ^{106}Te. Our Eq. (13) predicts a smaller threshold intensity $I = 10^{22}\ \mathrm{W/cm^2}$ at this frequency, due to the fact that we used a deformed approach. Thus, in Fig. 3 we reported a difference about 4 orders of magnitude between the monopole and full approaches for ^{212}Po.

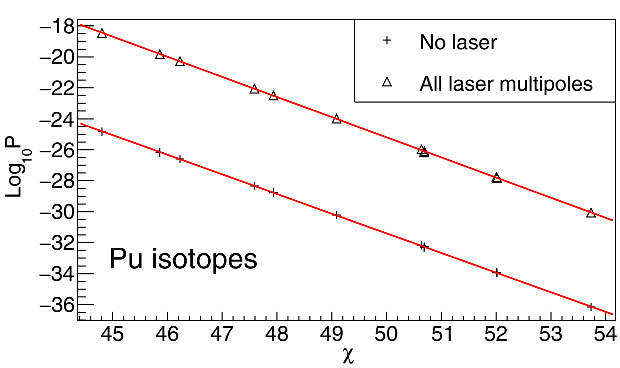

FIG. 4. Same as in Fig. 3 but for Pu isotopes.

Concluding, we have proposed an approach describing the α-decay process in a strong electromagnetic laser field based on the unitary Hennenberger transformation (4). One obtains a "standard" Schrodinger equation with the new Coulomb potential (9) expressed in terms of the time-dependent radial coordinate $|\mathbf{r} - \mathbf{S}_0 \sin \omega t|$, where S_0 is given by Eq. (13). We have shown that for $D \equiv S_0/R_0 > 1$ the oscillating electric field after the averaging procedure leads to a significant dynamical deformation of the Coulomb field in the new system of coordinates by decreasing the effective height of the Coulomb barrier in the equatorial direction. Therefore (i) the decay half-life significantly changes (as seen in Fig. 2) and (ii) the emission process becomes strongly anisotropic towards the equatorial direction (as seen in Fig. 1). We have shown that the contribution of the monopole laser term increases the barrier penetrability by 2 orders of magnitude, while the total contribution has an effect of about 6 orders of magnitude. The influence of the nuclear deformation leads to an additional increase of the barrier penetrability by 1 order of magnitude for a deformation $\beta_2 \sim 0.3$. The influence of the electromagnetic field can be expressed as a Geiger-Nuttal law modified by a new shifting parameter. This effect becomes important for the decontamination of radioactive waste resulting from nuclear power plants. A problem still open to be investigated is the role played by the reduced width γ_α^2.

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