Heavy-Quark Symmetry Implies Stable Heavy Tetraquark Mesons $Q_iQ_j\bar{q}_k\bar{q}_l$

Estia J. Eichten* and Chris Quigg†
Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510, USA
(Received 8 August 2017; published 15 November 2017)

For very heavy quarks Q, relations derived from heavy-quark symmetry predict the existence of novel narrow doubly heavy tetraquark states of the form $Q_iQ_j\bar{q}_k\bar{q}_l$ (subscripts label flavors), where q designates a light quark. By evaluating finite-mass corrections, we predict that double-beauty states composed of $bb\bar{u}\bar{d}$, $bb\bar{u}\bar{s}$, and $bb\bar{d}\bar{s}$ will be stable against strong decays, whereas the double-charm states $cc\bar{q}_k\bar{q}_l$, mixed beauty + charm states $bc\bar{q}_k\bar{q}_l$, and heavier $bb\bar{q}_k\bar{q}_l$ states will dissociate into pairs of heavy-light mesons. Observation of a new double-beauty state through its weak decays would establish the existence of tetraquarks and illuminate the role of heavy color-antitriplet diquarks as hadron constituents.

DOI: 10.1103/PhysRevLett.119.202002

Following the discovery of the charmonium-associated state $X(3872)$ by the Belle Collaboration [1], experiments have led a renaissance in hadron spectroscopy [2].

Many of the newly observed states invite identification with compositions less spare than the traditional quark–antiquark meson and three-quark baryon schemes [3]. Tetraquark states composed of a heavy quark and antiquark plus a light quark and antiquark have attracted much attention. The observed candidates all fit the form $c\bar{c}q_k\bar{q}_l$, where the light quarks q may be u, d, or s. No such states are observed significantly below threshold for strong decays into two heavy-light meson states $\bar{c}q_l + c\bar{q}_k$; all have strong decays to $c\bar{c}$ charmonium + light mesons.

In this Letter we examine the possibility of tetraquark configurations for which all strong decays are kinematically forbidden. We show that, in the heavy-quark limit, stable—hence, exceedingly narrow—$Q_iQ_j\bar{q}_k\bar{q}_l$ mesons must exist. To apply this insight, we take into account corrections for finite heavy-quark masses to deduce which tetraquark states containing b or c quarks should be stable. The most promising example is a $J^P = 1^+$ isoscalar double-b meson, $\mathcal{T}^{\{bb\}-}_{[\bar{u}\bar{d}]}$.

In the heavy-quark limit, the lowest-lying tetraquark configurations resemble the helium atom, a factorized system with separate dynamics for the compact heavy color-$\bar{\mathbf{3}}$ Q_iQ_j "nucleus" and for the light quarks bound to the stationary color charge. (We recall that the one-gluon-exchange interaction is attractive for two quarks forming a color antitriplet, with half the strength of the attraction between a quark and antiquark bound in a color singlet.) At large $Q_i - Q_j$ separations, which become increasingly important as the heavy-quark masses decrease, the light $\bar{q}_k\bar{q}_l$ cloud screens the Q_iQ_j interaction, so that the $Q_iQ_j\bar{q}_k\bar{q}_l$ complex may rearrange into a pair of heavy-light mesons [4]. For heavy quarks Q_iQ_j bound in a color $\bar{\mathbf{3}}$ by an effective potential of the "Cornell" Coulomb + linear form at half strength for both components [5], the rms core radii are $\langle r^2\rangle^{1/2} = 0.28$ fm(cc), 0.24 fm(bc), and 0.19 fm(bb), all considerably smaller than the size of the associated tetraquark states. Hence the core-plus-light (anti)quarks idealization should be a reliable guide to the masses of ground-state tetraquarks containing charms and bottoms.

The ground state of the attractive $\bar{\mathbf{3}}$ Q_iQ_j configuration may have total spin $S_{Q_iQ_j} = 1$ for identical quarks ($i = j$) or for quarks of different flavors ($i \neq j$) in a symmetric flavor configuration $\{Q_iQ_j\}$ or total spin $S_{Q_iQ_j} = 0$ for quarks of different flavors ($i \neq j$) in an antisymmetric flavor configuration $[Q_iQ_j]$. To construct a color-singlet $Q_iQ_j\bar{q}_k\bar{q}_l$ state, the light $\bar{q}_k\bar{q}_l$ must be in a color-$\mathbf{3}$. For the tetraquark ground state, both the heavy Q_iQ_j and light $\bar{q}_k\bar{q}_l$ pairs must be in ($\ell = 0$) s waves. To satisfy the Pauli principle, the flavor-symmetric $\{\bar{q}_k\bar{q}_l\}$ state must have total (light-quark) spin $j_\ell = 1$, whereas the flavor-antisymmetric $[\bar{q}_k\bar{q}_l]$ must have $j_\ell = 0$.

Stability in the heavy-quark limit.—For very heavy quarks, a hadron mass receives negligible contributions from the motion of the heavy quarks and spin interactions. Accordingly, the following relations hold among the masses of heavy-light mesons and heavy-heavy-light baryons [6]:

$$\begin{aligned}
m(\{Q_iQ_j\}\{\bar{q}_k\bar{q}_l\}) - m(\{Q_iQ_j\}q_y) &= m(Q_x\{q_kq_l\}) - m(Q_x\bar{q}_y),\\
m(\{Q_iQ_j\}[\bar{q}_k\bar{q}_l]) - m(\{Q_iQ_j\}q_y) &= m(Q_x[q_kq_l]) - m(Q_x\bar{q}_y),\\
m([Q_iQ_j]\{\bar{q}_k\bar{q}_l\}) - m([Q_iQ_j]q_y) &= m(Q_x\{q_kq_l\}) - m(Q_x\bar{q}_y),\\
m([Q_iQ_j][\bar{q}_k\bar{q}_l]) - m([Q_iQ_j]q_y) &= m(Q_x[q_kq_l]) - m(Q_x\bar{q}_y).
\end{aligned} \tag{1}$$

(In the limit, a heavy core is a heavy core.)

It is easy to see that the dissociation of $Q_iQ_j\bar{q}_k\bar{q}_l$ into two heavy-light mesons is kinematically forbidden, for sufficiently heavy quarks. The $\mathcal{Q}$ value for the decay is

$$\mathcal{Q} \equiv m(Q_iQ_j\bar{q}_k\bar{q}_l) - [m(Q_i\bar{q}_k) + m(Q_j\bar{q}_l)]$$
$$= \Delta(q_k, q_l) - \frac{1}{2}\left(\frac{2}{3}\alpha_s\right)^2[1 + O(v^2)]\bar{M} + O(1/\bar{M}), \quad (2)$$

where $\Delta(q_k, q_l)$, the contribution due to light dynamics, becomes independent of the heavy-quark masses, $\bar{M} \equiv (1/m_{Q_i} + 1/m_{Q_j})^{-1}$ is the reduced mass of Q_i and Q_j, and α_s is the strong coupling. The velocity-dependent hyperfine corrections, here negligible, are calculable in the nonrelativistic QCD formalism [7]. For large enough values of $\bar{M}$, the middle term dominates, so that the tetraquark is stable against decay into two heavy-light mesons.

The other possible decay channel is to a doubly heavy baryon and a light antibaryon,

$$(Q_iQ_j\bar{q}_k\bar{q}_l) \to (Q_iQ_jq_m) + (\bar{q}_k\bar{q}_l\bar{q}_m). \quad (3)$$

By Eq. (1), we have

$$m(Q_iQ_j\bar{q}_k\bar{q}_l) - m(Q_iQ_jq_m) = m(Q_xq_kq_l) - m(Q_x\bar{q}_m). \quad (4)$$

In the heavy-quark regime, the flavored-baryon–flavored-meson mass difference on the right-hand side of Eq. (4) has the generic form $\Delta_0 + \Delta_1/M_{Q_x}$. Using the observed mass differences, $m(\Lambda_c) - m(D) = 416.87$ MeV and $m(\Lambda_b) - m(B) = 340.26$ MeV, and choosing effective quark masses $m_c \equiv m(J/\psi)/2 = 1.55$ GeV, $m_b \equiv m(\Upsilon)/2 = 4.73$ GeV, we find $\Delta_1 = 176.6\ \mathrm{MeV}^2$ and $\Delta_0 = 303$ MeV; hence, the mass difference in the heavy-quark limit is 303 MeV. All of these mass differences are smaller than the mass of the lightest antibaryon, $m(\bar{p}) = 938.27$ MeV, so we conclude that no decay to a doubly heavy baryon and a light antibaryon is kinematically allowed. This completes the demonstration that, in the heavy-quark limit, stable $Q_iQ_j\bar{q}_k\bar{q}_l$ mesons must exist.

Beyond the heavy-quark limit.—To ascertain whether stable tetraquark mesons might be observed, we must estimate masses of the candidate configurations. Numerous model calculations exist in the literature [8], but it is informative to make estimates in the spirit of heavy-quark symmetry.

The leading-order corrections for finite heavy-quark mass correspond to hyperfine spin-dependent terms and a kinetic energy shift that depends only on the light degrees of freedom,

$$\delta m = \mathcal{S}\frac{\vec{S}\cdot\vec{j}_\ell}{2\mathcal{M}} + \frac{\mathcal{K}}{2\mathcal{M}}, \quad (5)$$

where $\mathcal{M} = m_{Q_i}$ or $m_{Q_i} + m_{Q_j}$ denotes the mass of the heavy-quark core for hadrons containing one or two heavy quarks and the coefficients $\mathcal{S}$ and $\mathcal{K}$ are to be determined from experimental data summarized in Table I. The spin splittings lead directly to the coefficients $\mathcal{S}$ tabulated in the last column. The pattern of the spin coefficients is entirely consistent with the expectations of heavy-quark symmetry.

TABLE I. Representative masses [9], in MeV, and derived quantities for ground-state hadrons containing heavy quarks.

State[a]	j_ℓ	Mass $(j_\ell + \frac{1}{2})$	Mass $(j_\ell - \frac{1}{2})$	Centroid	Spin splitting	$\mathcal{S}$ (GeV^2)
$D^{(*)}$ $(c\bar{d})$	$\frac{1}{2}$	2010.26	1869.59	1975.09	140.7	0.436
$D_s^{(*)}$ $(c\bar{s})$	$\frac{1}{2}$	2112.1	1968.28	2076.15	143.8	0.446
Λ_c $(cud)_{\bar{\mathbf{3}}}$	0	2286.46	⋯	⋯		⋯
Σ_c $(cud)_{\mathbf{6}}$	1	2518.41	2453.97	2496.93	64.44	0.132
Ξ_c $(cus)_{\bar{\mathbf{3}}}$	0	2467.87	⋯	⋯		⋯
Ξ_c' $(cus)_{\mathbf{6}}$	1	2645.53	2577.4	2622.82	68.13	0.141
Ω_c $(css)_{\mathbf{6}}$	1	2765.9	2695.2	2742.33	70.7	0.146
Ξ_{cc} $(ccu)_{\bar{\mathbf{3}}}$	0	3621.40[b]	⋯		⋯	
$B^{(*)}$ $(b\bar{d})$	$\frac{1}{2}$	5324.65	5279.32	5313.32	45.33	0.427
$B_s^{(*)}$ $(b\bar{s})$	$\frac{1}{2}$	5415.4	5366.89	5403.3	48.5	0.459
Λ_b $(bud)_{\bar{\mathbf{3}}}$	0	5619.58	⋯		⋯	
Σ_b $(bud)_{\mathbf{6}}$	1	5832.1	5811.3	5825.2	20.8	0.131
Ξ_b $(bds)_{\bar{\mathbf{3}}}$	0	5794.5	⋯		⋯	
Ξ_b' $(bds)_{\mathbf{6}}$	1	5955.33	5935.02	5948.56	20.31	0.128
Ω_b $(bss)_{\mathbf{6}}$	1		6046.1			
B_c $(b\bar{c})$	$\frac{1}{2}$	6329[c]	6274.9	6315.4[c]	54[c]	0.340[c]

[a]Subscripts denote flavor-SU(3) representations for heavy baryons.
[b]From the LHCb observation, Ref. [10].
[c]Inferred from the lattice QCD calculation of Ref. [11].

The kinetic energy shift due to light quarks will be different in $Q\bar{q}$ mesons and Qqq baryons. By comparing the centroid [or center-of-gravity (c.g.)] masses for the charm and bottom systems we can extract the difference of the kinetic-energy coefficients $\mathcal{K}$ for states that contain one or two light quarks, viz., $\delta\mathcal{K} \equiv \mathcal{K}_{(ud)} - \mathcal{K}_d$. For example,

$$\{(m[(cud)_{\bar{\mathbf{3}}}] - m(c\bar{d})\} - \{m[(bud)_{\bar{\mathbf{3}}}] - m(b\bar{d})\} = \delta\mathcal{K}\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right) = 5.11\ \text{MeV}, \tag{6}$$

from which we extract $\delta\mathcal{K} = 0.0235\ \text{GeV}^2$. The resulting mass shifts are

$$\begin{aligned} m[\{cc\}(\bar{u}\,\bar{d})] - m(\{cc\}d)&:\ \frac{\delta\mathcal{K}}{4m_c} = 2.80\ \text{MeV},\\ m[(bc)(\bar{u}\,\bar{d})] - m(\{bc\}d)&:\ \frac{\delta\mathcal{K}}{2(m_c+m_b)} = 1.87\ \text{MeV},\\ m[\{bb\}(\bar{u}\,\bar{d})] - m(\{bb\}d)&:\ \frac{\delta\mathcal{K}}{4m_b} = 1.24\ \text{MeV}. \end{aligned} \tag{7}$$

These values are small—only slightly larger than the isospin breaking effects that we neglect as too small to affect the question of stability [12].

Combining the heavy-quark-symmetry relations of Eq. (1) with the leading-order corrections, we obtain the masses of ground-state $Q_iQ_j\bar{q}_k\bar{q}_l$ tetraquarks summarized in Table II [13]. As inputs for the doubly heavy baryons not yet experimentally measured, we use the model calculations of Karliner and Rosner [14].

Narrow tetraquark states.—As we explained in the discussion surrounding Eq. (4), strong decays of $Q_iQ_j\bar{q}_k\bar{q}_l$ tetraquarks to a doubly heavy baryon and a light antibaryon are kinematically forbidden for all the ground states. Strong decay to a pair of heavy-light mesons will occur if the tetraquark state lies above the threshold. For $J^P = 0^+$ or 2^+, a $Q_iQ_j\bar{q}_k\bar{q}_l$ meson might decay to a pair of heavy-light pseudoscalar mesons, while for $J^P = 1^+$ the allowed decay channel would be a pseudoscalar plus a vector meson. According to our mass estimates, the only tetraquark mesons below threshold are the axial vector $\{bb\}[\bar{u}\,\bar{d}]$ meson, $\mathcal{T}^{\{bb\}-}_{[\bar{u}\,\bar{d}]}$, which is bound by 121 MeV, and the axial vector $\{bb\}[\bar{u}\,\bar{s}]$ and $\{bb\}[\bar{d}\,\bar{s}]$ mesons bound by 48 MeV. We expect all the other $Q_iQ_j\bar{q}_k\bar{q}_l$ tetraquarks to lie at least 78 MeV above the corresponding thresholds for strong decay [16]. Promising final states include $\mathcal{T}^{\{bb\}-}_{[\bar{u}\bar{d}]} \to \Xi^0_{bc}\bar{p}$, $B^-D^+\pi^-$, and $B^-D^+\ell^-\bar{\nu}$ (which establishes a weak decay), $\mathcal{T}^{\{bb\}-}_{[\bar{u}\bar{s}]} \to \Xi^0_{bc}\bar{\Sigma}^-$, $\mathcal{T}^{\{bb\}0}_{[\bar{d}\bar{s}]} \to \Xi^0_{bc}(\bar{\Lambda},\bar{\Sigma}^0)$, and so on.

As others have noted [8,17], unstable doubly heavy tetraquarks might be reconstructed as resonances in the "wrong-sign" combinations of DD, DB, and BB. The doubly charged $\mathcal{T}^{\{cc\}++}_{[\bar{d}\,\bar{s}]} \to D^+D_s^+$, etc., would stand out as *prima facie* evidence for a non-$q\bar{q}$ level.

While the production of $Q_iQ_j\bar{q}_k\bar{q}_l$ mesons is undoubtedly a rare event, we draw some encouragement for near-term searches from the large yield of B_c mesons recorded in the LHCb experiment [18] and the

TABLE II. Expectations for the ground-state tetraquark masses, in MeV.[a] The column labeled "HQS relation" is the result of our heavy-quark symmetry relations and is explicitly given by the sum of the right-hand side of Eq. (1) and the kinetic-energy mass shifts of Eq. (7). Here q denotes an up or down quark. For stable tetraquark states the $\mathcal{Q}$ value is highlighted in a box.

State	J^P	j_ℓ	$m(Q_iQ_jq_m)$ (c.g.)	HQS relation	$m(Q_iQ_j\bar{q}_k\bar{q}_l)$	Decay channel	$\mathcal{Q}$ (MeV)
$\{cc\}[\bar{u}\,\bar{d}]$	1^+	0	3663[b]	$m(\{cc\}u) + 315$	3978	D^+D^{*0} 3876	102
$\{cc\}[\bar{q}_k\bar{s}]$	1^+	0	3764[c]	$m(\{cc\}s) + 392$	4156	$D^+D_s^{*-}$ 3977	179
$\{cc\}\{\bar{q}_k\bar{q}_l\}$	$0^+, 1^+, 2^+$	1	3663	$m(\{cc\}u) + 526$	4146,4167,4210	D^+D^0, D^+D^{*0} 3734,3876	412,292,476
$[bc][\bar{u}\,\bar{d}]$	0^+	0	6914	$m([bc]u) + 315$	7229	B^-D^+/B^0D^0 7146	83
$[bc][\bar{q}_k\bar{s}]$	0^+	0	7010[d]	$m([bc]s) + 392$	7406	B_sD 7236	170
$[bc]\{\bar{q}_k\bar{q}_l\}$	1^+	1	6914	$m([bc]u) + 526$	7439	B^*D/BD^* 7190/7290	249
$\{bc\}[\bar{u}\,\bar{d}]$	1^+	0	6957	$m(\{bc\}u) + 315$	7272	B^*D/BD^* 7190/7290	82
$\{bc\}[\bar{q}_k\bar{s}]$	1^+	0	7053[d]	$m(\{bc\}s) + 392$	7445	DB_s^* 7282	163
$\{bc\}\{\bar{q}_k\bar{q}_l\}$	$0^+, 1^+, 2^+$	1	6957	$m(\{bc\}u) + 526$	7461,7472,7493	BD/B^*D 7146/7190	317,282,349
$\{bb\}[\bar{u}\,\bar{d}]$	1^+	0	10 176	$m(\{bb\}u) + 306$	10 482	$B^-\bar{B}^{*0}$ 10 603	$\boxed{-121}$
$\{bb\}[\bar{q}_k\bar{s}]$	1^+	0	10 252[c]	$m(\{bb\}s) + 391$	10 643	$\bar{B}\bar{B}_s^*/\bar{B}_s\bar{B}^*$ 10 695/10 691	$\boxed{-48}$
$\{bb\}\{\bar{q}_k\bar{q}_l\}$	$0^+, 1^+, 2^+$	1	10 176	$m(\{bb\}u) + 512$	10 674,10 681,10 695	B^-B^0, B^-B^{*0} 10 559,10 603	115,78,136

[a]Masses of the unobserved doubly heavy baryons are taken from Ref. [14]; for lattice evaluations of b-baryon masses, see Ref. [15].
[b]Based on the mass of the LHCb Ξ_{cc}^{++} candidate, 3621.40 MeV, Ref. [10].
[c]Using the s/d mass differences of the corresponding heavy-light mesons.
[d]Evaluated as $\frac{1}{2}[m(c\bar{s}) - m(c\bar{d}) + m(b\bar{s}) - m(b\bar{d})] + m(bcd)$.

not-inconsiderable rate of double-Υ production observed in 8-TeV pp collisions by the CMS experiment, $\sigma(pp \to \Upsilon\Upsilon + \text{anything}) = 68 \pm 15$ pb [19]. The ultimate search instrument might be a future electron-positron tera-Z factory, for which the branching fractions [9] $Z \to b\bar{b} = 15.12 \pm 0.05\%$ and $Z \to b\bar{b}b\bar{b} = (3.6 \pm 1.3) \times 10^{-4}$ offer hope of many events containing multiple heavy quarks.

Concluding remarks.—We have shown that, in the heavy-quark limit, stable $Q_iQ_j\bar{q}_k\bar{q}_l$ tetraquarks must exist. Our estimates of tetraquark masses lead us to expect that strong decays of the $J^P = 1^+$ $\{bb\}[\bar{u}\,\bar{d}]$, $\{bb\}[\bar{u}\,\bar{s}]$, and $\{bb\}[\bar{d}\,\bar{s}]$ states are kinematically forbidden, so that these states should be exceedingly narrow, decaying only through the charged-current weak interaction. Observation of any of these states would signal the existence of a new form of stable matter, in which the doubly heavy color-$\bar{\mathbf{3}}$ Q_iQ_j diquark is a basic building block. The unstable $Q_iQ_j\bar{q}_k\bar{q}_l$ tetraquarks—particularly those with small Q values—may be observable as resonances decaying into pairs of heavy-light mesons, if they are not too broad to stand out above backgrounds.

This manuscript has been authored by employees of Fermi Research Alliance, LLC, under Contract No. DE-AC02-07CH11359 with the U.S. Department of Energy, Office of Science, Office of High Energy Physics. The U.S. Government retains and the publisher, by accepting the article for publication, acknowledges that the U.S. Government retains a nonexclusive, paid-up, irrevocable, worldwide license to publish or reproduce the published form of this manuscript, or allow others to do so, for U.S. Government purposes.

Note added.—We recently learned of interesting calculations of tetraquark masses that also highlight the likelihood of a stable doubly heavy tetraquark [20].

[*]eichten@fnal.gov
[†]quigg@fnal.gov

[1] S. K. Choi *et al.* (Belle Collaboration), Observation of a Narrow Charmoniumlike State in Exclusive $B^\pm \to K^\pm\pi^+\pi^-J/\psi$ Decays, Phys. Rev. Lett. **91**, 262001 (2003).

[2] For recent surveys of new states and candidate interpretations, see S. L. Olsen, A new hadron spectroscopy, Front. Phys. **10**, 121 (2015); R. F. Lebed, R. E. Mitchell, and E. S. Swanson, Heavy-quark QCD exotica, Prog. Part. Nucl. Phys. **93**, 143 (2017); A. Esposito, A. Pilloni, and A. D. Polosa, Multiquark resonances, Phys. Rep. **668**, 1 (2017); A. Ali, J. S. Lange, and S. Stone, Exotics: Heavy pentaquarks and tetraquarks, arXiv:1706.00610.

[3] The possibility of nonminimal configurations was foreseen in the foundational papers by G. Zweig, Report No. CERN-TH-401, 1964; Report No. CERN-TH-412, 1964; M. Gell-Mann, A schematic model of baryons and mesons, Phys. Lett. **8**, 214 (1964); For an early survey of the emerging exotic spectroscopy, emphasizing the role of color couplings, see R. L. Jaffe, Exotica, Phys. Rep. **409**, 1 (2005).

[4] This behavior is exhibited in exploratory lattice QCD calculations reported in A. Peters, P. Bicudo, K. Cichy, B. Wagenbach, and M. Wagner, Exploring possibly existing $qq\bar{b}\,\bar{b}$ tetraquark states with $qq = ud, ss, cc$, *Proc. Sci.*, LATTICE2015 (2016) 095 [arXiv:1508.00343]; A. Peters, P. Bicudo, L. Leskovec, S. Meinel, and M. Wagner, Lattice QCD study of heavy-heavy-light-light tetraquark candidates, *Proc. Sci.*, LATTICE2016 (2016) 104 [arXiv:1609.00181]; It emerged in analytic calculations by A. V. Manohar and M. B. Wise, Exotic $QQ\bar{q}\,\bar{q}$ states in QCD, Nucl. Phys. **B399**, 17 (1993), who anticipated that, for infinitely heavy quarks Q, $QQ\bar{q}\,\bar{q}$ states would be stable against dissociation into two heavy-light mesons.

[5] The strength of the Coulomb contribution is fixed by the color Casimir. Lattice studies indicate that the string tension for the color $\bar{\mathbf{3}}$ is half that for the singlet configuration: A. Nakamura and T. Saito, QCD color interactions between two quarks, Phys. Lett. B **621**, 171 (2005).

[6] The essence of this argument is developed in E. Eichten, Heavy quarks on the lattice, Nucl. Phys. B, Proc. Suppl. **4**, 170 (1988); G. P. Lepage and B. A. Thacker, Effective Lagrangians for simulating heavy quark systems, Nucl. Phys. B, Proc. Suppl. **4**, 199 (1988); For a comprehensive review, see A. V. Manohar and M. B. Wise, *Heavy Quark Physics* (Cambridge University Press, Cambridge, England, 2000).

[7] W. E. Caswell and G. P. Lepage, Effective Lagrangians for bound state problems in QED, QCD, and other field theories, Phys. Lett. **167B**, 437 (1986).

[8] A useful compilation appears in Table IX of S.-Q. Luo, K. Chen, X. Liu, Y.-R. Liu, and S.-L. Zhu, Exotic tetraquark states with the $qq\bar{Q}\,\bar{Q}$ configuration, arXiv:1707.01180.

[9] Except as noted, masses are taken from C. Patrignani *et al.* (Particle Data Group), Review of particle physics, Chin. Phys. C **40**, 100001 (2016) and 2017 update, pdg.lbl.gov.

[10] R. Aaij *et al.* (LHCb Collaboration), Observation of the doubly charmed baryon Ξ_{cc}^{++}, arXiv:1707.01621.

[11] R. J. Dowdall, C. T. H. Davies, T. C. Hammant, and R. R. Horgan, Precise heavy-light meson masses and hyperfine splittings from lattice QCD including charm quarks in the sea, Phys. Rev. D **86**, 094510 (2012).

[12] Compare M. Karliner and J. L. Rosner, Isospin splittings in baryons with two heavy quarks, Phys. Rev. D **96**, 033004 (2017).

[13] Communication with decay channels tends to push the bound-state levels deeper. Open channels would induce mixing between the color-$\bar{3}$ (core)-$\mathbf{3}$ (light-quark pair) configuration and meson–meson configurations.

[14] M. Karliner and J. L. Rosner, Baryons with two heavy quarks: Masses, production, decays, and detection, Phys. Rev. D **90**, 094007 (2014).

[15] Z. S. Brown, W. Detmold, S. Meinel, and K. Orginos, Charmed bottom baryon spectroscopy from lattice QCD, Phys. Rev. D **90**, 094507 (2014).

[16] Note that if we took the SELEX value for the Ξ_{cc}^{+} mass, 3519 MeV, rather than the LHCb Ξ_{cc}^{++} mass of Ref. [10],

[M. Mattson *et al.* (SELEX Collaboration), First Observation of the Doubly Charmed Baryon Ξ_{cc}^{+}, Phys. Rev. Lett. **89**, 112001 (2002)], we would find $m(\{cc\}[\bar{u}\bar{d}])=3876\,\mathrm{MeV}$, coincident with the 3876-MeV threshold for dissociation into a heavy-light pseudoscalar and heavy-light vector. Signatures for weak decay would include $D^{+}K^{-}\ell^{+}\nu$ and $\Xi_{c}^{+}\bar{n}$.

[17] A. Esposito, M. Papinutto, A. Pilloni, A. D. Polosa, and N. Tantalo, Doubly charmed tetraquarks in B_c and Ξ_{bc} decays, Phys. Rev. D **88**, 054029 (2013).

[18] In 8-TeV pp collisions, R. Aaij *et al.* (LHCb Collaboration), Measurement of the B_c^{+} meson lifetime using $B_c^{+}\to J/\psi\mu^{+}\nu_{\mu}X$ decays, Eur. Phys. J. C **74**, 2839 (2014), reported 8995 ± 103 B_c candidates in 2 fb^{-1}.

[19] V. Khachatryan *et al.* (CMS Collaboration), Observation of $\Upsilon(1S)$ pair production in proton-proton collisions at $\sqrt{s}=8$ TeV, J. High Energy Phys. 05 (2017) 013.

[20] M. Karliner and J. L. Rosner, preceding Letter, Discovery of Doubly Charmed Ξ_{cc} Baryon Implies a Stable $bb\bar{u}\bar{d}$ Tetraquark, Phys. Rev. Lett. **119**, 202001 (2017), which contains an extensive list of references to early work; M.-L. Du, W. Chen, X.-L. Chen, and S.-L. Zhu, Exotic $QQ\bar{q}\bar{q}$, $QQ\bar{q}\bar{s}$ and $QQ\bar{s}\bar{s}$ states, Phys. Rev. D **87**, 014003 (2013); W. Chen, T. G. Steele, and S.-L. Zhu, Exotic open-flavor $bc\bar{q}\bar{q}$, $bc\bar{s}\bar{s}$ and $qc\bar{q}\bar{b}$, $sc\bar{s}\bar{b}$ tetraquark states, Phys. Rev. D **89**, 054037 (2014); A. Francis, R. J. Hudspith, R. Lewis, and K. Maltman, Lattice Prediction for Deeply Bound Doubly Heavy Tetraquarks, Phys. Rev. Lett. **118**, 142001 (2017).