Spin-Wave Chirality and Its Manifestations in Antiferromagnets

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As first demonstrated by Tang and Cohen in chiral optics, the asymmetry in the rate of electromagnetic energy absorption between left and right enantiomers is determined by an optical chirality density. Here, we demonstrate that this effect can exist in magnetic spin systems. By constructing a formal analogy with electrodynamics, we show that in antiferromagnets with broken chiral symmetry, the asymmetry in local spin-wave energy absorption is proportional to a spin-wave chirality density, which is a direct counterpart of optical zilch. We propose that injection of a pure spin current into an antiferromagnet may serve as a chiral symmetry breaking mechanism, since its effect in the spin-wave approximation can be expressed in terms of additional Lifshitz invariants. We use linear response theory to show that the spin current induces a nonequilibrium spin-wave chirality density.

Introduction.—Chirality describes mirror image symmetry or the lack thereof [1,2]. Circularly polarized light provides a simple example. It has been known for a long time that when circularly polarized light interacts with a chiral molecule, the excitation rate is different for left and right polarizations, leading to remarkable effects such as natural optical activity and circular dichroism [3]. After Lipkin's discovery of a chirality conservation law for the electromagnetic Maxwell equations [4], it was realized that the electromagnetic field can be characterized by a locally conserving chirality density that is odd under spatial inversion ($\mathcal{P}$) and even under time reversal ($\mathcal{T}$) transformations. These symmetry properties are consistent with Barron's proposal of *true chirality* [5] that generalized the original definition by Kelvin [2].

Tang and Cohen realized that in local light-matter interactions of structured electromagnetic fields inside materials with broken chiral symmetry, electromagnetic chirality determines the asymmetry in the electromagnetic energy absorption rate [1]. Later, Bliokh and Nori demonstrated that chirality density in Ref. [1] is directly related to polarization helicity and energy density [6]. Further progress in understanding mutual relations between optical helicity, duality symmetry, and spin angular momentum of light was developed in Refs. [7–10]. These discoveries paved the way for possible practical applications of chiral electromagnetic fields in optics and plasmonics [11–14]. The purpose of this Letter is to demonstrate that this effect can be found in some magnetic spin systems. We consider the example of an antiferromagnetic material whose magnetic excitations—known as spin waves—can display some key properties analogous to optical light [15].

Spin dynamics in antiferromagnets attracted considerable attention recently [16–32] from the perspective of spintronics [33]. In this respect, noncentrosymmetric antiferromagnets are especially interesting. Lack of the inversion symmetry lifts the degeneracy between left- and right-polarized spin waves inside such materials making possible the observation of magnonic Nernst effects [34,35] or development of spin-wave field effect transistor devices [36]. Recently, an antiferromagnetic version of a chiral magnetic effect was proposed [37], thus, establishing a link between antiferromagnets and Weyl semimetals [38].

In this Letter, we examine the dynamics of antiferromagnetic spin-wave excitations and draw analogy with electrodynamics. This allows us to generalize the method of nongeometric symmetries, originally developed for the free electromagnetic field [39], to antiferromagnetic spin waves. Using this method, we find a conserving pseudoscalar, which is equivalent to Lipkin's zilch [4] in antiferromagnetic materials and which we propose as a measure of chirality for spin-wave excitations.

In order to observe spin-wave-chirality-related effects, the chiral symmetry inside the material itself must be broken. One possibility for such symmetry breaking is to consider antiferromagnets with nonzero Lifshitz invariants [40–42]. Another way proposed in this Letter is to inject a pure spin current, which lifts the $\mathcal{P}$ symmetry, at the same time, keeping the $\mathcal{T}$ symmetry unbroken. As we discuss below, the effect of spin current in the linear regime can be effectively expressed in terms of induced Lifshitz invariants in the spin-wave energy. We demonstrate that in such antiferromagnets with spin-current-driven chirality, spin-wave chirality plays a role similar to electromagnetic chirality [1] determining the asymmetry in the spin-wave energy absorption rate with respect to spin-current direction. We also show that on a quantum level, our spin-wave chirality is proportional to the difference between left- and right- polarized magnon

numbers and propose a linear response theory for the spin-current-induced nonequilibrium magnon chirality density.

Nongeometric symmetries.—We consider dissipative magnetization dynamics in a uniaxial antiferromagnet described by the semiclassical Landau-Lifshitz-Gilbert equation

$$\dot{M}_i = \gamma M_i \times H_i^{\text{eff}} - \eta M_i \times \dot{M}_i, \qquad (1)$$

where γ is a gyromagnetic ratio, M_i denotes the magnetization for the ith sublattice ($i = 1, 2$), the effective fields $H_i^{\text{eff}} = -\delta W / \delta M_i$ are determined by the magnetic energy functional W, and η is the Gilbert damping coefficient. The energy dissipation is described by the Rayleigh dissipation function

$$\dot{W} = -\frac{\eta}{\gamma} \int d^3 r (\dot{M}_1^2 + \dot{M}_2^2), \qquad (2)$$

where $\eta/\gamma > 0$ [15].

In what follows, we consider a general form of the magnetic energy

$$W = \int d^3 r \left(w_a + \frac{\delta}{2} M_1 \cdot M_2 + \frac{\alpha_{ij}}{2} \nabla M_i \cdot \nabla M_j \right), \quad (3)$$

where δ and α_{ij} are the exchange parameters, and $w_a = -(\beta/2)[(M_1 \cdot n)^2 + (M_2 \cdot n)^2]$ corresponds to the uniaxial anisotropy energy density, where n is the unit vector along the anisotropy axis. In what follows, we take $\alpha_{ij} = \alpha$ for $i = j$, and $\alpha_{ij} = \alpha'$ otherwise. For $\beta > 0$, w_a stabilizes uniform antiferromagnetic ordering with $M_1 = -M_2$ parallel to n [15].

In the spin-wave approximation, the equations of motion are linearized by taking $M_i(t,r) = (-1)^{i+1} M_s n + m_i(t,r)$, where M_s is the saturation magnetization. Transforming to the momentum space $m_i(t,r) = \int d^3 p \exp(ipr) \tilde{m}_i(t,p)$ and keeping only linear terms in the complex $\tilde{m} = \tilde{m}_1 + \tilde{m}_2$ and $\tilde{l} = \tilde{m}_1 - \tilde{m}_2$, we express the equations of motion in the following form,

$$\dot{\tilde{m}} = -\varepsilon_l(p) n \times \tilde{l} + \eta n \times \dot{\tilde{l}},$$
$$\dot{\tilde{l}} = -\varepsilon_m(p) n \times \tilde{m} + \eta n \times \dot{\tilde{m}}, \qquad (4)$$

where $\varepsilon_m(p) = \gamma M_s (\delta + \beta + (\alpha + \alpha')p^2)$, $\varepsilon_l(p) = \gamma M_s (\beta + (\alpha - \alpha')p^2)$, and p is the spin-wave wave vector.

For symmetry analysis of Eqs. (4), it is convenient to use an analogue of the Silberstein-Bateman representation of Maxwell's equations [43]. For this purpose, we combine $\tilde{m}$ and $\tilde{l}$ into the six-component vector $\phi(t,p) = (\tilde{m}(t,p), \tilde{l}(t,p))^T$. The equation of motion for $\phi(t,p)$ can be written in the matrix form $i\partial_t \phi(t,p) = \mathcal{H}\phi(t,p)$ with

$$\mathcal{H} = \begin{pmatrix} 0 & -\varepsilon_l(p)(\hat{S} \cdot n) \\ -\varepsilon_m(p)(\hat{S} \cdot n) & 0 \end{pmatrix}, \qquad (5)$$

where for the symmetry analysis we omitted the damping terms. We introduce the spin-1 matrices $(\hat{S}_\alpha)_{\beta\gamma} = -i\epsilon_{\alpha\beta\gamma}$ where $\epsilon_{\alpha\beta\gamma}$ is the Levi-Cività symbol ($\alpha, \beta, \gamma = x, y, z$). Although $\mathcal{H}$ is not Hermitian, it can be easily symmetrized by applying the momentum-dependent variable change $\phi = \mathcal{N}\bar{\phi}$, where $\mathcal{N} = \text{diag}(\varepsilon_m^{-1/2}, \varepsilon_l^{-1/2})$ [44,45]. After this transformation, the equation of motion acquires a Schrödinger-like form

$$i\partial_t \bar{\phi}(t,p) = \mathcal{H}_0 \bar{\phi}(t,p), \qquad (6)$$

where the Hermitian matrix $\mathcal{H}_0$ is given by the Cartesian product $\mathcal{H}_0 = -\sqrt{\varepsilon_m \varepsilon_l} \sigma_1 \otimes (\hat{S} \cdot n)$, where σ_1 is the Pauli matrix.

Equation (6) has the form similar to the Silberstein-Bateman representation of Maxwell's equations in dispersive medium [43]. Transformation to the electrodynamics is reached by replacing ϕ with $\phi_{\text{em}} = (E, B)^T$ composed from the electric and magnetic field, and $\mathcal{H}_0$ with $\mathcal{H}_{\text{em}} = -(\sqrt{\varepsilon\mu})^{-1}\sigma_2 \otimes (\hat{S} \cdot p)$, where $\varepsilon(p)$ and $\mu(p)$ are the permittivity and permeability of the medium. Notably, $\mathcal{H}_0$ and $\mathcal{H}_{\text{em}}$ share similar algebraic structure. The difference between them is related to their transformation properties under $\mathcal{T}$ and $\mathcal{P}$ symmetries [46].

The analogy between spin-wave dynamics and electrodynamics allows us to generalize the symmetry analysis of Maxwell's equations to antiferromagnetic spin waves. Similar to electrodynamics [39], the equations of motion (6) are invariant under the eight-dimensional algebra of *nongeometric symmetries* [39]. The basis elements of this algebra are given by $\mathcal{Q}_1 = i\sigma_2 \otimes (\hat{S} \cdot n)\hat{D}$, $\mathcal{Q}_2 = \sigma_1 \otimes \hat{I}$, $\mathcal{Q}_3 = \sigma_3 \otimes (\hat{S} \cdot n)\hat{D}$, $\mathcal{Q}_4 = i\sigma_2 \otimes \hat{D}$, $\mathcal{Q}_5 = \sigma_0 \otimes (\hat{S} \cdot n)$, $\mathcal{Q}_6 = \sigma_3 \otimes \hat{D}$, $\mathcal{Q}_7 = \sigma_0 \otimes \hat{I}$, and $\mathcal{Q}_8 = \sigma_1 \otimes (\hat{S} \cdot n)$, where $\hat{D} = 2[(\hat{S} \cdot n_\perp)^2 - \hat{I}_3 n_\perp^2]/n_\perp^2 - (\hat{S} \cdot n)^2$, $n_\perp = (n_1, n_2, 0)$, $\hat{I}_3 = \text{diag}(0,0,1)$, and σ_0 and $\hat{I}$ denote two- and three-dimensional unit matrices, respectively [47].

Some basis elements have clear interpretation. For example, $\mathcal{Q}_8$, which is proportional to $\mathcal{H} \equiv i\partial_t$, represents the symmetry with respect to taking the time derivative. $\mathcal{Q}_2$ plays a role similar to the duality transformations of the electromagnetic field [48,49]. It generates a continuous symmetry transformation $\tilde{m} \to \tilde{m} \cosh\theta + \sqrt{\varepsilon_l/\varepsilon_m}\tilde{l} \sinh\theta$ and $\tilde{l} \to \tilde{l} \cosh\theta + \sqrt{\varepsilon_m/\varepsilon_l}\tilde{m} \sinh\theta$ for any real parameter θ.

Spin-wave chirality conservation law.—From the existence of symmetry transformations, we can establish various conservation laws, which can be conveniently written in terms of bilinear forms

$$C_A = \frac{1}{2} \int d^3 p \, \phi^\dagger(t,p) \rho \mathcal{Q}_A \phi(t,p), \qquad (7)$$

where $\rho = (\mathcal{N}^{-1})^\dagger \mathcal{N}^{-1}$ is the measure that takes into account the non-Hermitian character of $\mathcal{H}$ [50].

Since the rotation symmetry with respect to the $\boldsymbol{n}$ direction is unbroken, we can introduce spin-wave chirality conservation associated with conservation of the operator $p_n \mathcal{Q}_5$ in Eq. (7), where $p_n = \boldsymbol{p} \cdot \boldsymbol{n}$ is the spin-wave momentum component along $\boldsymbol{n}$. The explicit form of this conservation law in the momentum space is given by

$$C_\chi = \frac{i}{2} \int d^3 p [\varepsilon_m(\boldsymbol{p}) \tilde{\boldsymbol{m}}^* \cdot (\boldsymbol{p}_n \times \tilde{\boldsymbol{m}}) + \varepsilon_l(\boldsymbol{p}) \tilde{\boldsymbol{l}}^* \cdot (\boldsymbol{p}_n \times \tilde{\boldsymbol{l}})]. \quad (8)$$

This relation is an analogue of Lipkin's zilch [4]. The corresponding real space spin-wave chirality density can be written as

$$\rho_\chi(t, \boldsymbol{r}) = \frac{1}{2}(\dot{\boldsymbol{l}} \cdot \nabla_n \boldsymbol{m} + \dot{\boldsymbol{m}} \cdot \nabla_n \boldsymbol{l}), \quad (9)$$

where $\nabla_n = \boldsymbol{n} \cdot \nabla$. In this case, the total chirality is obtained by taking the volume integral $C_\chi = \int d^3 r \rho_\chi(t, \boldsymbol{r})$.

In order to clarify the physical meaning of the spin-wave chirality in Eqs. (8) and (9), we rewrite these equations in terms of magnon operators. By applying the Holstein-Primakoff transformation [51] for sublattice magnetizations $M_1^{(+)} = \sqrt{2M_s}a$, $M_1^{(-)} = \sqrt{2M_s}a^\dagger$, $M_1^z = M_s - a^\dagger a$ and $M_2^{(+)} = \sqrt{2M_s}b^\dagger$, $M_2^{(-)} = \sqrt{2M_s}b$, $M_2^z = -M_s + b^\dagger b$, where a and b are bosonic operators combined with Bogolyubov's rotation $a_{\boldsymbol{p}} = a_{L\boldsymbol{p}} \cosh\theta - a_{R-\boldsymbol{p}}^\dagger \sinh\theta$, and $b_{-\boldsymbol{p}}^\dagger = a_{R-\boldsymbol{p}}^\dagger \cosh\theta - a_{L\boldsymbol{p}} \sinh\theta$ with $\tanh\theta = (\varepsilon_m - \varepsilon_l)/(\varepsilon_m + \varepsilon_l + 2\sqrt{\varepsilon_m \varepsilon_l})$, the total magnon Hamiltonian can be written in terms of left- (L) and right- (R) polarized magnon number operators

$$\hat{H} = \frac{1}{2}\sum_{\boldsymbol{p}} \omega_{\boldsymbol{p}}(a_{L\boldsymbol{p}}^\dagger a_{L\boldsymbol{p}} + a_{R\boldsymbol{p}}^\dagger a_{R\boldsymbol{p}}), \quad (10)$$

where the energy dispersion $\omega_{\boldsymbol{p}} = \sqrt{\varepsilon_l(\boldsymbol{p})\varepsilon_m(\boldsymbol{p})}$ is doubly degenerated with respect to polarization directions [15]. In terms of $a_{L\boldsymbol{p}}$ and $a_{R\boldsymbol{p}}$, C_χ is determined by the difference in numbers of L- and R-polarized magnons

$$\hat{C}_\chi = 2\sum_{\boldsymbol{p}} p_n \omega_{\boldsymbol{p}}(a_{L\boldsymbol{p}}^\dagger a_{L\boldsymbol{p}} - a_{R\boldsymbol{p}}^\dagger a_{R\boldsymbol{p}}). \quad (11)$$

A similar expression for optical helicity and Lipkin's zilch in terms of the photon numbers has been known for a long time [7,48,49,52–54].

Chiral symmetry breaking.—At this point, we have established a chirality conservation law for spin waves in antiferromagnets. We now discuss the consequences and potential for observation and application.

We note that C_χ is odd under both transformations, $\mathcal{P}$ and exchange of sublattices $\boldsymbol{m}_1 \leftrightarrow \boldsymbol{m}_2$ ($\mathcal{M}$). Therefore, to observe spin-wave-chirality-related effects, these symmetries should be broken inside the material. To break the inversion symmetry, we may try to exploit the Doppler shift of spin waves, which is formally reached by replacement $\partial_t \to \partial_t - \boldsymbol{v}_s \cdot \nabla$ in the equations of motion, where $\boldsymbol{v}_s$
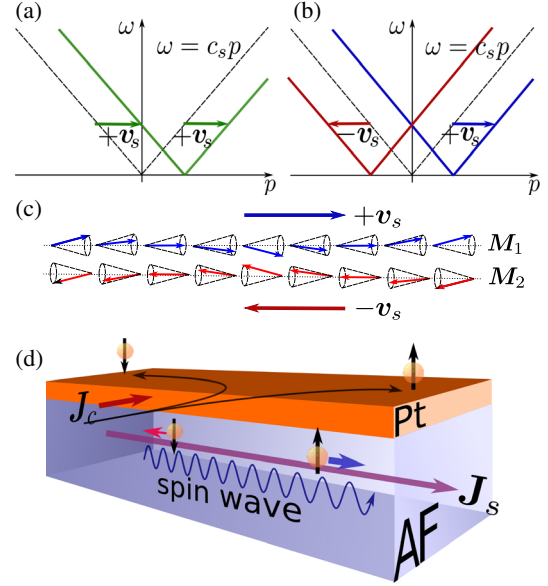

FIG. 1. (a) Schematic picture of the Doppler shift for spin waves with the energy dispersion $\omega = c_s p$. Both L- and R-polarized magnon modes shift in the same direction. (b) Antiparallel Doppler shifts for L and R modes lift the degeneracy with respect to polarization. (c) Magnetizations $\boldsymbol{M}_1$ and $\boldsymbol{M}_2$ boosted in the opposite directions by spin-current injection along $\boldsymbol{n}$. (d) Possible experimental realization: charge current J_c is converted into pure spin current J_s in the material with large θ_{SH} (Pt) and injected into the antiferromagnet (AF) across the interface.

is the velocity of the moving frame [20]. This effect was observed in ferromagnetic metals under applied spin-polarized current [55] and was proposed for antiferromagnets [20]. However, pure Doppler shift does not lift the degeneracy between L- and R-polarized modes [28] and, therefore, cannot induce chirality [see Fig. 1(a)].

To create chirality, we propose to realize two different Doppler shifts for L- and R-polarized magnons in the *opposite directions*, as schematically shown in Fig. 1(b), which also breaks $\mathcal{M}$ symmetry. Below, we consider how this situation can be experimentally realized. Here, we only note that, formally, this can be achieved by two antiparallel Galilean boosts for $\boldsymbol{M}_1$ and $\boldsymbol{M}_2$ sublattice magnetizations [see Fig. 1(c)], which correspond to the transformation $\partial_t \to \partial_t \mp v_s \nabla_n$ in Eq. (1), where the upper (lower) sign is for $\boldsymbol{M}_1$ ($\boldsymbol{M}_2$), and we take v_s parallel to $\boldsymbol{n}$.

Applying the transformation $\partial_t \to \partial_t \mp v_s \nabla_n$ to the energy absorption rate in Eq. (2), we find that for spin waves traveling in such medium, $\dot{W}$ splits into symmetric and asymmetric parts under $\mathcal{P}$ and $\mathcal{M}$. The latter part is proportional to the spin-wave chirality

$$\dot{W}_\chi = \frac{2\eta v_s}{\gamma} \int d^3 r (\dot{\boldsymbol{m}}_1 \cdot \nabla_n \boldsymbol{m}_1 - \dot{\boldsymbol{m}}_2 \cdot \nabla_n \boldsymbol{m}_2) = \frac{2\eta v_s}{\gamma} C_\chi. \quad (12)$$

This result is the magnetic counterpart of the effect first demonstrated in optics by Tang and Cohen [1].

Experimental realization.— How do we experimentally realize the antiparallel Galilean boosts for M_1 and M_2? For this purpose, we invoke the spin-transfer torque (STT) mechanism [32].

We consider pure spin current injected into the antiferromagnet along the n direction [see Fig. 1(d)]. The existence of spin current means that a portion of $s_\uparrow$ electrons is flowing into the positive direction with the velocity v_s, while an equal amount of $s_\downarrow$ electrons drifts in the opposite directions with $-v_s$. If the entire antiferromagnet is in the *exchange-dominant* regime [28], intersublattice electron dynamics can be neglected, and $s_\uparrow$ ($s_\downarrow$) electrons couple only to the M_1 (M_2) sublattice. In this case, these two sorts of electrons are able to produce an adiabatic STT onto M_1 and M_2 pointing in the opposite directions via the Zhang-Li mechanism [56]. In particular, $s_\uparrow$-electron flow produces the torque $\mathfrak{T}_1 = M_s^{-2} M_1 \times M_1 \times (v_s \cdot \nabla) M_1$ acting on M_1, while at the same time, $s_\downarrow$ electrons produce $\mathfrak{T}_2 = -M_s^{-2} M_2 \times M_2 \times (v_s \cdot \nabla) M_2$ acting on M_2, where $v_s = \mu_B j_s/(e M_s)$ is proportional to the spin-current density j_s (in electric units).

The spin current can be either injected from the metal with large spin-Hall angle θ_{SH} (≈ 0.1 in Pt) or created inside a metallic antiferromagnet with intrinsic spin-Hall effect (e.g., $\theta_{SH} \approx 0.06$ in PtMn [57]). To estimate v_s, we take $j_s = \theta_{SH} j_c$ with $j_c = 10^{12}$ A/m^2 being the charge current density previously used to observe STT effects in ferromagnets [55,58]. For $\theta_{SH} = 0.1$ and $M_s = 3.5 \times 10^5$ A/m, we obtain $v_s = 33$ m/s. We use this value below.

The effect of $\mathfrak{T}_1$ and $\mathfrak{T}_2$ on the spin-wave spectrum is equivalent to the Doppler shifts of L- and R-polarized modes in the opposite directions, as schematically shown in Fig. 1(b) [47]. Spin-current injection lifts the degeneracy with respect to helicity and turns the antiferromagnet into chiral material with magnonic optical activity and circular dichroism. The characteristic length scale of the dichroism in a typical antiferromagnetic insulator with linearly dispersing spin waves $\omega_p = c_s p$ can be estimated as $\ell_{CD} = c_s^2/(\eta v_s \omega) \approx 5$ mm, where we take the spin-wave velocity $c_s = 10$ km/s, $\eta = 10^{-4}$, and frequency $\omega/2\pi = 1$ THz.

In the spin-wave approximation, the effect of adiabatic STT can be effectively described by the following Lifshitz invariants in the spin-wave energy:

$$H_{DM} = \frac{v_s}{2} \int d^3 r [m_1 \cdot (\nabla_n \times m_1) + m_2 \cdot (\nabla_n \times m_2)]. \quad (13)$$

In the lattice-model language, this expression corresponds to the monoaxial Dzyaloshinskii-Moriya (DM) interaction between the next-nearest-neighboring sites with effective strength $D_{eff} = \hbar^2 \gamma v_s/(M_s a^4)$, which can be estimated as $\hbar v_s/a \approx 0.5$ K for $j_c = 10^{12}$ A/m^2, where a is the lattice constant. This situation partly resembles spin-current-induced DM interaction in ferromagnets with Rashba coupling [59]. Recently, next-nearest-neighboring DM interactions attracted attention in view of the magnonic Nernst effect in antiferromagnets [34,35].

Linear response.—The amount of spin-wave chirality induced by the spin current can be estimated using the linear response theory. For this purpose, we treat H_{DM} as a perturbation to the magnon Hamiltonian in Eq. (10). The spin-wave chirality density can be calculated using the Kubo formula [60]

$$\langle \rho_\chi \rangle = -i \int_{-\infty}^t dt' e^{-s(t-t')} \langle [\hat{\rho}_\chi(t), \hat{H}_{DM}(t')] \rangle, \quad (14)$$

where the average is taken with the equilibrium density matrix $\hat{\rho}_0 = \exp(-\hat{H}/k_B T)$, $s \to 0^+$, and the operators $\hat{\rho}_\chi$ and $\hat{H}_{DM}$ are obtained from Eqs. (9) and (13) by the Holstein-Primakoff transformation [47].

Straightforward calculations show that the total spin-current-induced chirality at the temperature T is obtained as follows [47]:

$$C_\chi = -2 v_s \sum_p \frac{\partial n_p}{\partial \omega_p} p_n^2 [\varepsilon_m(p) + \varepsilon_l(p)], \quad (15)$$

where $n_p = (\exp(\omega_p/k_B T) - 1)^{-1}$ is the equilibrium magnon distribution. For linearly dispersing magnons, this expression can be integrated explicitly, providing $C_\chi = 4\pi^2 v_s \hbar \Omega_{ex}(k_B T)^4/(45 c_s^5)$, where $\Omega_{ex} = \gamma M_s \delta$ is the exchange frequency.

To characterize the asymmetry created by the spin current, we propose to normalize the induced chirality in Eq. (15) on the total contribution to C_χ from the magnons with positive chirality at the thermal equilibrium $C_+ = \sum_{p_n > 0} \omega_p p_n \langle a_{Lp}^\dagger a_{Lp} + a_{R-p}^\dagger a_{R-p} \rangle$. Without spin current, this amount of chirality is compensated by exactly the same number of magnons with negative chirality providing $C_\chi = 0$. Therefore, we can introduce a dimensionless parameter $g = C_\chi/C_+$ that can be interpreted as an amount of degeneracy lifted by the spin current. For magnons with linear dispersion, we can estimate this quantity as [47]

$$g = k \frac{v_s}{c_s} \frac{\hbar \Omega_{ex}}{k_B T}, \quad (16)$$

where $k \approx 0.69$. In a typical antiferromagnetic insulator with $c_s = 10$ km/s and $\Omega_{ex}/2\pi = 10$ THz, we estimate $g \approx 0.3\%$ at room temperatures for $j_c = 10^{12}$ A/m^2.

Summary.—The symmetry analysis for spin-wave dynamics in antiferromagnets has been developed by drawing an analogy with Maxwell's equations. The conservation law for the spin-wave chirality has been established. This quantity, which has been determined by the difference in numbers of left- and right-polarized magnons, is directly related to Lipkin's zilch in electrodynamics [4]. In this respect, we would like to mention Refs. [6,7] relevance to recent discussions of magnon spin current [30,31,35,36].

We also note that our symmetry approach has a potential extension to coupled magneto-optical excitations in antiferromagnets [61].

Pure spin current in antiferromagnets can lift the degeneracy with respect to polarization. In this situation, spin-wave chirality determines the asymmetry in the spin-wave energy absorption rate, similar to its twin optical effect [1]. The efficiency of the spin current is determined by the ratio v_s/c_s, which can reach 10^{-3} for current densities $j_c = 10^{11}$–10^{12} A/m^2. Such current densities were previously used to observe the Doppler shift of spin waves in ferromagnets [55]. The experiments can be probed in thin-film interfaces of antiferromagnetic insulator and nonmagnetic metal (for example, spin current generated by the spin-Hall effect across Pt/NiO interface was used in Ref. [62]) or in metallic antiferromagnets with intrinsic spin-Hall effect [57,63].

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