Universal Spatial Structure of Nonequilibrium Steady States

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We describe a large family of nonequilibrium steady states (NESS) corresponding to forced flows over obstacles. The spatial structure at large distances from the obstacle is shown to be universal, and can be quantitatively characterized in terms of certain collective modes of the strongly coupled many body system, which we define in this work. In holography, these modes are spatial analogues of quasinormal modes, which are known to be responsible for universal aspects of relaxation of time dependent systems. These modes can be both hydrodynamical or nonhydrodynamical in origin. The decay lengths of the hydrodynamic modes are set by η/s, the shear viscosity over entropy density ratio, suggesting a new route to experimentally measuring this ratio. We also point out a new class of nonequilibrium phase transitions, across which the spatial structure of the NESS undergoes a dramatic change, characterized by the properties of the spectrum of these spatial collective modes.

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Equilibrium many-body systems are known to exhibit universal behavior, as famously exemplified by their critical phenomena near second-order phase transitions. These are characterized by a small number of universal modes that scale according to computable critical exponents and leave their imprint on macroscopic physical properties of the system.

This state of affairs contrasts with the situation when such systems are not in equilibrium [1], with universal results few and far between. Determining the physical characteristics of such a system is typically strongly situation dependent. A notable exception is the dynamical crossing of a second-order phase transition at a finite rate τ_Q. As proposed by Kibble [2] and Zurek [3], the number of topological defects that form in the broken symmetry phase is given in terms of a scaling law, depending on a small set of universal modes. The exact details of the quench through the transition are unimportant, only the rate of approach to the critical point enters into the scaling law [3].

Given the success of the KZ mechanism [4], and the recent experimental interest it has created, for example, Refs. [5,6], one may ask whether other scenarios exist that are able to strongly constrain out of equilibrium dynamics using a small set of universal collective modes, leaving an imprint on the macroscopic spatial structure of the system.

In this work we consider a large class of nonequilibrium steady states (NESS) are set up as follows: consider a many body system forced to flow over an obstacle. This gives rise to a strong nonlinear disturbance in the vicinity of the obstacle, while the flow far from it on either side is simple with a constant velocity $\mathbf{v}_L$ on the left and $\mathbf{v}_R$ on the right (see Fig. 1). One then wants to know what the steady state looks like at large distances, in other words how the strongly nonlinear behavior around the obstacle relaxes spatially toward its asymptotic values. This is a difficult problem, in general out of technical reach of current methods. The AdS/CFT correspondence gives rise to a powerful computational framework particularly in the nonequilibrium setting. Indeed this approach has been used to elucidate the temporal equilibration (Furthermore, previous studies of holographic NESS include current driven [7–10] as well as heat-driven [11,12] cases.) of strongly coupled plasmas [13,14] and superfluids [15]. In each case, the late-time behavior is very accurately predicted by the spectrum of low-lying quasinormal modes (QNM) [16], whose relevance to thermalization was first pointed out in Ref. [17].

In this Letter we use holography to explicitly find the full nonlinear solution for certain strongly coupled theories, whose dual solutions are black holes without Killing horizons. The spatial structure is indeed universally characterized by a stationary version of QNMs (Modes of this kind have been studied in holography in a variety of other contexts [18–22].), which we define and obtain in a few illustrative examples. For a given choice of asymptotic flow velocity, $v = v_L$ or v_R, these modes form a discrete set of

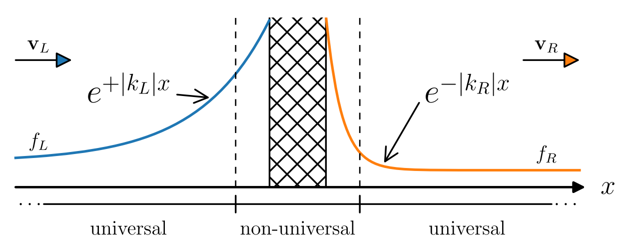

FIG. 1. Schematic representation of the NESS considered, showing the imprint of spatial collective modes which describe the return to equilibrium far from an obstacle. The vertical axis f refers to a quantity of interest, such as the expectation value of the stress tensor, with a value f_L on the left and f_R on the right.

purely imaginary wave numbers $k(v)$ and the leading mode, i.e., the one with the smallest $|\mathrm{Im}k|$ can be hydrodynamical or nonhydrodynamical, and will be denoted k_*. The relaxation towards the asymptotic flow happens at the exponential rate $\propto e^{-\mathrm{Im}k_* x}$, so that the relaxation towards the right boundary value corresponds to a mode with $\mathrm{Im}k_* > 0$, while the left mode has $\mathrm{Im}k_* < 0$. A drastic reorganization of the spatial structure of the NESS occurs whenever a dominant mode crosses the real axis for a certain critical velocity v_c. In this case, as $v \to v_c^-$ the downstream spatial relaxation rate will tend toward zero, only to be, for $v > v_c$, dominated by the previously subleading mode. The upstream spatial relaxation rate undergoes a similar transition as v is decreased through v_c. This constitutes a new nonequilibrium phase transition, and we conjecture that transitions of this form exist in systems outside of holography. Indeed, we provide examples of such transitions purely from the point of view of hydrodynamics.

The physical setup considered in this work should be regarded as a spacelike version of a quench [23]. Instead of switching on a source at some time t_0 and then asking about the temporal relaxation towards a new equilibrium, we consider an obstacle (modeled by a source) at some spatial location x_0 and asking about the spatial relaxation towards the asymptotic equilibrium. In both cases the asymptotic physics is fully universal and determined by a spectrum of discrete collective modes of the system. The importance of QNMs in holography cannot be overstated, and attempts are being made to define and explore them beyond AdS/CFT [24]. Here we point out that an equally rich and universal story is present when considering NESS, opening the exciting possibility to access these modes via measurements of the spatial structure of driven critical systems in the lab. In particular, for modes that are hydrodynamic in origin the spatial decay rate (in units of the temperature) depends directly on the shear viscosity in units of the entropy density, η/s. This applies for any system with an effective hydrodynamic description, greatly extending the scope beyond holography and raising the possibility of an experimental measurement of η/s using the spatial structure of NESS. To this end, we note that recent experiments have demonstrated the presence of hydrodynamic electron flow in $PdCoO_2$ [25], as well as graphene [26].

Relativistic hydrodynamics in d dimensions.— Hydrodynamics describes a wide class of systems in the form of a universal theory that arises in a long wavelength limit. In this section we construct the spatial collective modes that appear in this effective theory. We stress that while hydrodynamics does contain certain spatial collective modes, there can be additional "higher" modes in a more complete theory that do not exist in the hydrodynamic limit. This is the case for holography, discussed in the next section.

To first order the Landau frame stress tensor is

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu + p\Delta^{\mu\nu} - \eta\sigma^{\mu\nu} - \zeta\Delta^{\mu\nu}\partial\cdot u + O(\partial)^2, \quad (1)$$

subject to the conservation equations, $\partial_\mu T^{\mu\nu} = 0$. u^μ is a timelike unit-normalized d-velocity field, while $\Delta^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu$ projects orthogonal to u^μ. η and ζ are the shear and bulk viscosities. The shear tensor is given by $\sigma^{\mu\nu} \equiv 2\Delta^{\mu\rho}\Delta^{\nu\sigma}[\partial_{(\rho}u_{\sigma)} - (1/(d-1))\eta_{\rho\sigma}\partial\cdot u]$.

To find the collective modes, we solve the conservation equations for linear perturbations about a long-range equilibrium state characterized by ε, p, and a $(d-1)$ velocity, $\mathbf{v}$, such that $u^\mu = \gamma(1,\mathbf{v})$, where $\gamma = 1/\sqrt{1-\mathbf{v}\cdot\mathbf{v}}$. The perturbations we seek are of the form, $\varepsilon(x^\mu) = \varepsilon + \delta\varepsilon e^{ik_\sigma x^\sigma}$ with similar expressions for $p(x^\mu)$ and $u^\mu(x^\nu)$, all of which are time independent in the laboratory frame, i.e., $k_\mu = (0,\mathbf{k})$. Energy conservation immediately gives $\delta\varepsilon = -(\varepsilon+p)k\cdot\delta u/(k\cdot u)$, and for a speed of sound c_s we also write $\delta p = c_s^2\delta\varepsilon$. Thus, the remaining unsolved conservation equations determine δu^μ, which are either transverse or longitudinal with respect to the obstacle. Transverse perturbations, $k\cdot\delta u_T = 0$, obey the dispersion relation

$$k = -i\frac{\varepsilon+p}{\eta c^2}v\cos\theta + O(k^2), \quad (2)$$

where we denote $v = |\mathbf{v}|$, $k = \sqrt{\mathbf{k}\cdot\mathbf{k}}$, and $\mathbf{v}\cdot\mathbf{k} = vk\cos\theta$, obtained by solving for v order by order in small k, and then inverting. Despite being time independent, this mode is related to the usual shear diffusion pole. Specifically, if we perform a Lorentz transformation to the rest frame of the fluid where the wave vector picks up a frequency $k^\mu = (\omega,\mathbf{q})$, at this order these quantities obey a dispersion relation of the form $\omega = -iDq^2$ with diffusion constant $D = (\eta c^2/(\varepsilon+p))$. [The transformation between the fluid rest frame quantities ω, $\mathbf{q}$ and the laboratory frame quantities, $\mathbf{k}$, $\mathbf{v}$ is given in the Supplemental Material [27] (5).] Note, however, that q here is imaginary. Next, the longitudinal sector, $\delta u^\mu = \delta u_L \Delta^{\mu\nu}k_\nu$, has a dispersion relation,

$$k = -i\frac{\varepsilon+p}{\frac{d-2}{d-1}\eta + \frac{1}{2}\zeta}\frac{1}{c^2}\frac{\sqrt{1-(\frac{v_0}{c})^2}\cos\theta}{(1-(\frac{v_0}{c}\sin\theta)^2)^2}(v \mp v_0) + O(k)^2, \quad (3)$$

where $v_0 \equiv c_s\sec\theta/\sqrt{1+[(c_s/c)\tan\theta]^2}$ and again we have reintroduced the speed of light, c. Similarly this mode is related to sound; in the rest frame of the fluid it obeys the dispersion relation $\omega = \mp c_s q - (i/2)\{[(d-2)/(d-1)2\eta + \zeta]/[\varepsilon+p]\}c^2q^2$, but again note q is imaginary.

The appearance of η, ζ in $k(v)$ suggests a new route to their measurement (as well as other transport coefficients which appear at higher orders in k)—by measuring the long range spatial structure of NESS in the laboratory. Specifically, using $\varepsilon + p = Ts$ we see that k/T in Eq. (2) depends only on η/s and parameters of the flow (v, θ), while Eq. (3) depends additionally on ζ/s and c_s. The preceding analysis relies only on universal properties of hydrodynamics, and is thus independent of holographic duality.

Holography for CFT_3.—Moving to a complete theory allows us to construct a complete spectrum—hydrodynamic and otherwise—as well as demonstrate its role in explicitly constructed NESS.

As before, we construct the spectrum of spatial collective modes by linearly perturbing the equilibrium solution reached far from the obstacle. In this case the equilibrium configuration is given by the Schwarzschild black brane metric, boosted along a planar horizon direction by u^μ. (One could also consider different equilibrium states, for instance those with charge, superconductors, insulators, etc.) The computation is reminiscent of a QNM calculation where the boundary condition at the event horizon is ingoing. Here the spatial collective modes are time independent by construction, so an ingoing condition cannot apply. We define the modes to be those which are regular on the future event horizon. Further technical details can be found in the Supplemental Material [27].

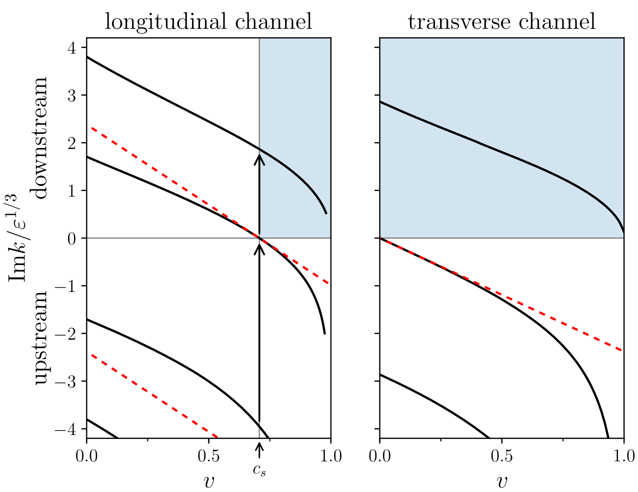

FIG. 2. The discrete spectrum of spatial collective modes as a function of asymptotic flow velocity $k(v)$ for a CFT_3, computed holographically using stationary perturbations of boosted Schwarzschild-AdS_4. Here we show the case of flow incident angle $\theta = 0$ (black). There is a $(v, k) \to (-v, -k)$ symmetry that connects some of the modes shown through $v = 0$. Also shown is the conformal relativistic hydrodynamic spectrum (red dashed) valid to first order in small k. All modes found have $\mathrm{Re}k = 0$. On the downstream side, for some flow velocities v there are no modes of hydrodynamic origin (blue shaded region). In the longitudinal channel there is a phase transition as the velocity is increased through c_s (arrows) giving rise to discontinuities in k_*.

The leading (i.e., longest range) parts of the resulting spectrum are displayed in Fig. 2. We also show the modes obtained in the first-order hydrodynamic approximation, with appropriate transport coefficients $\eta = s/(4\pi)$, $\zeta = 0$ and $c_s = 1/\sqrt{2}$. All modes found have $\mathrm{Re}k = 0$. As previously advertised the holographic theory contains additional modes that are not present in hydrodynamics and for some v these nonhydrodynamic modes give the dominant long distance contribution.

A new nonequilibrium phase transition is also visible in Fig. 2. In the longitudinal channel, as v is increased through c_s, there is a sudden change in the dominant mode, k_*, on either the upstream or downstream side. For instance, on the downstream side the hydrodynamic mode decay length becomes ever longer as v is increased, and becomes suddenly dominated by a short nonhydrodynamic mode once $v > c_s$.

Holography for CFT_2 *and* CFT_∞.—In low and high spacetime dimension analytic treatment of the spatial collective modes becomes possible. For $d = 2$ equilibrium is given by the BTZ black brane. For a scalar field perturbation about the zero velocity background there is a discrete set of modes labeled by $n \in \mathbb{Z}$, with dispersion relations $\omega = \pm q - i4\pi T[(\Delta/2) + n]$ [32,33], where T is the Hawking temperature of the black hole and Δ is the dimension of the dual operator. Exploiting Lorentz invariance to reach the modes of interest, we pick $\omega = -\gamma k v$, $q = \gamma k$, obtaining

$$k = i\frac{4\pi T}{\gamma(v \pm 1)}\left(\frac{\Delta}{2} + n\right), \tag{4}$$

where $\mathrm{Re}k = 0$ and—comparing to Eqs. (2) and (3)—a suggestive factor of $4\pi T$, despite η not being defined in $d = 2$. In the limit $d \to \infty$ there is a decoupled sector of perturbations that are supported in a near horizon region, corresponding to modes with ω, $q \sim d^0$ [34–36]. These can be constructed analytically [37]. Once more using Lorentz invariance an appropriate choice of ω, q gives $\mathrm{Re}k = 0$. For small k these modes match the large d limit of the hydrodynamic modes computed earlier.

Nonlinear holographic NESS construction.—In the previous sections we constructed individual spatial collective modes. Here we show that these modes govern the behavior the NESS far from the obstacle by explicit construction and checking the asymptotics. These are given holographically by families of black branes with non-Killing horizons, in which the obstacle is provided by x-dependent deformations of the CFT metric, $\gamma_{\mu\nu}$, i.e.,

$$\gamma_{\mu\nu} = \eta_{\mu\nu} + s_{\mu\nu}(x). \tag{5}$$

We consider sources whose components are Gaussian centred, on $x = 0$. The details of the obstacle are not important, as the spectrum of collective modes is a property of the theory itself. We only have to ensure that the obstacle excites the part of the spectrum we are interested in. The source terms in Eq. (5) can act as a source for shear, and we allow for velocity components transverse to the obstacle.

Our construction proceeds numerically based on the method of Ref. [23], which formulates the stationary gravitational problem such that the bulk coordinates penetrate the future event horizon. (See the Supplemental Material [27] for details, which includes Refs. [28–31].) As emphasized in Ref. [23], one must supply enough data in the form of boundary conditions to fix all the moduli of the corresponding flow. In addition to ε, v of Ref. [23], we fix a third modulus, θ, the asymptotic incident angle of the flow. In general there is refraction and $\theta_L \neq \theta_R$.

We have constructed solutions which are asymptotically subsonic-to-subsonic, as well as supersonic-to-supersonic, with and without transverse flow. For these solutions we seek local fluid variables by using the field theory stress tensor, $\langle T_{\mu\nu}\rangle$ [38]. We solve the following eigenvalue problem at each point on the boundary,

$$\langle T_{\mu\nu}\rangle U^\mu = -\varepsilon U_\nu, \qquad \gamma_{\mu\nu}U^\mu U^\nu = -1, \tag{6}$$

for the three undetermined pieces of ε, U^μ. Asymptotically on the left or right these are the moduli of the solution, i.e., asymptotically $U^\mu = \gamma(v)(1, v\cos\theta, v\sin\theta)$.

To check for the presence of the collective modes we note some quantity f in the channel of interest will take the form $f = C + A_k e^{-\mathrm{Im}kx}$. To numerically extract the value of k we then compute

$$\kappa_f(x) = -\frac{1}{\varepsilon^{1/3}}\frac{\partial_x^2 f}{\partial_x f}, \tag{7}$$

and then $\mathrm{Im}k/\varepsilon^{1/3} = \lim_{x\to\pm\infty}\kappa_f(x)$. To illustrate we use an example where a mode of nonhydrodynamic origin is dominant. One place this occurs is in the transverse channel, downstream in a subsonic flow (as we may predict from the spectrum of Fig. 2). We give an example of this flow in Fig. 3 where we show κ_ε and κ_{v^y}, where $v^y = U^y/U^t$. These quantities display excellent agreement with the longest range spatial collective mode obtained by direct construction, confirming the expectation that the spatial collective modes determine the long distance behavior of the nonlinear NESS.

Finally, we turn to a demonstration of the proposed nonequilibrium phase transitions in the longitudinal channel at $v = c_s$. In Fig. 4 we consider the downstream, right-hand side of a NESS in two cases, $v_R < c_s$ and $v_R > c_s$. In each case we show the spatial decay of ε and the longitudinal collective mode spectrum on the complex-k plane. Beginning with $v_R < c_s$, the long range behavior is governed by the smaller $\mathrm{Im}k > 0$ mode, as the plot of ε indicates. As v_R is increased, this mode descends down the

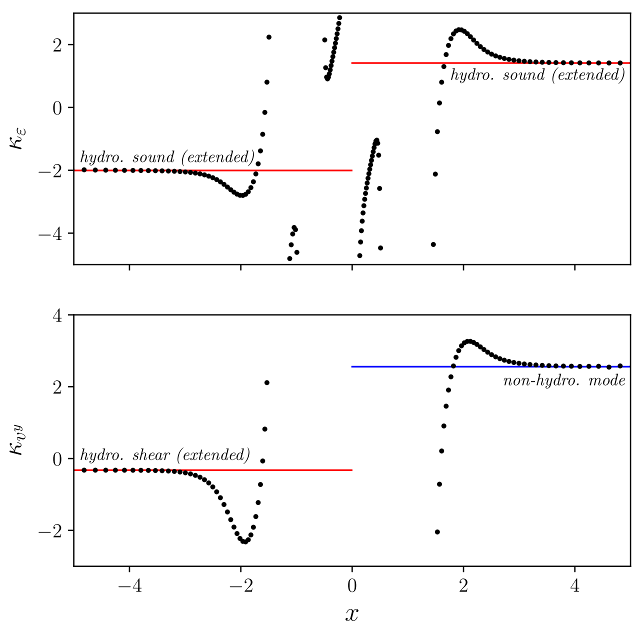

FIG. 3. Asymptotically subsonic-to-subsonic NESS, with finite transverse velocity. We show (with black circles) κ_ε for the longitudinal channel (upper panel) and κ_{v^y} for the transverse channel (lower panel), as defined in Eq. (7). Also shown are the values of $\mathrm{Im}k/\varepsilon^{1/3}$ for the spatial collective modes, computed directly given the left or right moduli of the asymptotic equilibrium. The red solid lines are continuously connected to the hydrodynamic modes labeled, while the blue solid line is a nonhydrodynamic mode.

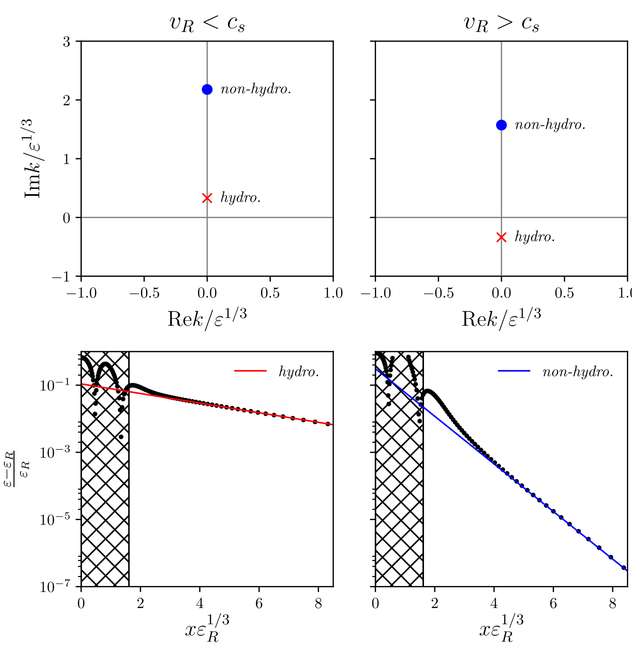

FIG. 4. Demonstration of the new nonequilibrium phase transition on the downstream, right-hand side of a NESS, from $v_R < c_s$ (left column) to $v_R > c_s$ (right column). Top row: locations of the spatial collective modes at these v_R in the complex k plane, displaying one mode of hydrodynamic origin (red x) and one nonhydrodynamic mode (blue circle). Bottom row: Spatial profile of ε on the right-hand side of a NESS (black circles) together with an amplitude-fit collective mode from the spectrum above with the longest decay length (solid lines).

imaginary-k axis and crosses the real axis at $v_R = c_s$. For $v_R > c_s$ this mode is in the lower half plane, no longer decays as $x \to +\infty$, and so it can no longer appear on the right-hand side of a regular NESS. The behavior of ε is thus suddenly dominated by the second, nonhydrodynamic mode which is now the longest range contribution.

Discussion.—We have defined and constructed "spatial collective modes" which, as we have argued, describe the universal spatial relaxation to equilibrium at large distances in a wide class of NESS. In the hydrodynamic limit the decay length of the modes, $L \equiv |\mathrm{Im}k|^{-1}$, depend directly on η/s, suggesting a new route to its experimental measurement. For example, for flows at standard temperature with $\theta = 0$ and $v = \beta\,\mathrm{m\,s}^{-1}$, the decay lengths in the transverse sector are $L\beta \simeq 7.46$ mm for graphene (Taking $c = v_F$ from Ref. [39] and η/s from Ref. [40].) ($L \simeq 7.46$ nm at $v = v_F$) and $L\beta \simeq 200$ m for $\mathcal{N} = 4$ SYM plasma.

The often delicate issue of heating in NESS (e.g., Refs. [41,42]) here is sidestepped, since the spatial pattern of the heat flow itself is universal and predicted by our mechanism. We have constructed explicit examples of non-Killing black holes in holography which confirm the role played by these modes, and demonstrated novel nonequilibrium phase transitions resulting from a reorganization of their spectrum. It is our hope that these modes, which may be viewed as the spatial analogues of QNMs, provide fruitful targets for further theoretical and experimental work on NESS.

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