Non-Fermi Glasses: Localized Descendants of Fractionalized Metals

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Non-Fermi liquids are metals that cannot be adiabatically deformed into free fermion states. We argue for the existence of "non-Fermi glasses," phases of interacting disordered fermions that are fully many-body localized (MBL), yet cannot be deformed into an Anderson insulator without an eigenstate phase transition. We explore the properties of such non-Fermi glasses, focusing on a specific solvable example. At high temperature, non-Fermi glasses have qualitatively similar spectral features to Anderson insulators. We identify a diagnostic based on ratios of correlators that sharply distinguishes between the two phases even at infinite temperature. Our results and diagnostic should generically apply to the high-temperature behavior of MBL descendants of fractionalized phases.

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Introduction.—Adiabatic continuity, exemplified by the Fermi liquid, is a central theme in many-body physics [1]. The interactions in Fermi liquids only "dress" the elementary excitations, which retain the character of the microscopic fermions; the quasiparticle residue Z, which measures the overlap between the quasiparticle and bare fermion creation operators, remains finite, and the (bare) electron spectral function—measured by tunneling experiments—exhibits dispersing quasiparticle modes. In strongly correlated systems, however, the Fermi liquid picture can break down as $Z \to 0$ at a quantum phase transition to a "non-Fermi liquid" phase. Non-Fermi liquid phases often exhibit fractionalization: their elementary excitations have distinct quantum numbers from the bare fermions. For instance, an electron can fractionalize into elementary collective excitations that carry spin and charge, and have *different* velocities. The single-electron spectral function will then not exhibit sharply dispersing modes but rather an incoherent, multiparticle continuum; we take the consequent absence of a quasiparticle residue to be the key property of a "fractionalized" phase, and our results pertain generally to such phases. Despite this sharp distinction in single-particle properties, however, thermodynamics and transport might be insensitive to the distinction between Fermi liquids and non-Fermi liquids, as in the "orthogonal metal" [2].

In clean systems such as Fermi liquids, adiabatic continuity only applies to elementary excitations above the ground state (states with zero energy density). However, strongly disordered systems in the many-body localized (MBL) phase can exhibit adiabatic continuity in the entire spectrum [3,4]. In particular, a typical many-body eigenstate of an MBL system can be regarded (almost everywhere [5]) as a product state (or Slater determinant) of localized orbitals, perturbatively dressed by interactions [6,7], much as the Fermi liquid is a dressed version of a Fermi gas—an apt name might be a "Fermi glass" [8,9]. This leads us to ask: do there exist fractionalized, "non-Fermi glass" MBL phases, which cannot be regarded as perturbatively dressed Slater determinants, and if so, what are their properties?

In this work, we embark on the study of non-Fermi glasses by exploring the fate of the orthogonal metal [2] in the presence of strong quenched disorder. We term the resulting phase the orthogonal many-body localized (OMBL) insulator. Although the OMBL insulator has the same transport and thermodynamic properties as the Fermi glass, or conventional MBL (CMBL) insulator, the two phases are separated by an eigenstate phase transition [10,11]. The usual spectral signatures of fractionalization are absent in this strongly disordered limit. Since both the OMBL and CMBL phases are localized, the local single-particle spectral function in both phases is dominated by sharp peaks. These are related to localized integrals of motion [6], which can be accessed by electron tunneling in both phases. The *spatially averaged* spectral functions retain a sharp distinction at zero temperature, featuring a soft gap in the OMBL phase but not in the CMBL phase; however, at nonzero temperatures, this soft gap is filled in. Thus, the eigenstate phase transition between OMBL and CMBL phases is spectrally "hidden." Nonetheless, the two phases are distinct even at infinite temperature; this distinction is hidden in the structure of spatial correlations. We identify a specific ratio of correlation functions (namely, that of the single-particle propagator to the pair propagator) that sharply distinguishes them and argue that such ratios generically diagnose fractionalization in the MBL setting.

Disordered orthogonal metals.—We first schematically review the construction of the orthogonal metal [2] of spinless electrons. The electron operator at site i, c_i is

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written as the product of a fermion operator f_i and a Pauli matrix σ_i^x acting on an auxiliary subspace. This doubling of degrees of freedom on each site is associated with a Z_2 gauge redundancy [12]. Thus, the original theory (with only "physical" c fermions) can be recast as a theory of f fermions and σ spins [here, subject to a transverse-field Ising-model (TFIM) Hamiltonian, $H_{\mathrm{TFIM}} = h\sum_i \sigma_i^z + J\sum_{\langle ij\rangle}\sigma_i^x\sigma_j^x$] coupled to a fluctuating Z_2 gauge field. This has two possible $T \to 0$ phases. When the Z_2 gauge theory is confining or the TFIM is ordered, the propagating degrees of freedom are c fermions, and we have a conventional metal. However, when the Z_2 gauge theory is deconfined and the TFIM is in its paramagnetic phase, the c fermion is fractionalized into separately propagating f and σ degrees of freedom: the orthogonal metal. In spatial dimension $d > 1$, where Z_2 gauge theory has both confined and deconfined phases, both orthogonal and conventional metals are possible.

This phase structure readily generalizes to excited eigenstate properties in the presence of strong randomness. In two dimensions, pure Z_2 gauge theory with random couplings is related by duality to a random transverse-field Ising model (RTFIM). For strong randomness, the gauge theory has confined and deconfined "eigenstate phases" that survive the inclusion of localized dynamical matter [10]. The OMBL phase then exists in the regime where (1) the gauge theory is in its deconfined, localized eigenstate phase, (2) the σ spins are in the localized paramagnetic phase, and (3) the f fermions are localized. In this regime, the "matter" degrees of freedom (f's and σ's) are weakly coupled to the gauge sector, which can, therefore, be neglected. At high temperatures, any of the three sectors could also be in a *thermal* phase; if so, it will infect the others [13]. To avoid this, we focus on the strong randomness limit where all sectors are localized. We develop our argument by first constructing and studying a $d = 1$ exactly solvable toy model of the orthogonal *Anderson* (noninteracting) insulator. We then argue that our results generalize to interacting systems and to $d > 1$. When the gauge field is deconfined, we can regard f and σ as separately propagating degrees of freedom, and the c fermion as a composite of them, with the proviso that only gauge-neutral quantities (i.e., c fermions) are measurable. Since 1D gauge theories always confine, we must *impose* this on our toy model since, strictly speaking, it cannot emerge via fractionalization. However, our analysis of spectral and correlation properties is symmetry based and, thus, quite general.

Exactly solvable model.—Our $d = 1$ exactly solvable model (introduced absent disorder in Ref. [2]) consists of fermions (c_i) and Ising spins (σ_i), with Hamiltonian

$$H_{\mathrm{ES}} = \sum_i J_i\sigma_i^x\sigma_{i+1}^x + h_i\sigma_i^z(-1)^{c_i^\dagger c_i} - t_i(c_i^\dagger\sigma_i^x\sigma_{i+1}^x c_{i+1} + \mathrm{H.c.}) + \mu_i c_i^\dagger c_i, \quad (1)$$

where J_i, h_i, t_i, and μ_i are random variables. We now rewrite Eq. (1) in terms of $\tau_i^z \equiv \sigma_i^z(-1)^{f_i^\dagger f_i}$, $\tau_i^x = \sigma_i^x$, $f_i \equiv \tau_i^x c_i$: the two sectors decouple, $H_{\mathrm{ES}} = H_f + H_\tau$, with

$$H_f = \sum_i t_i(f_i^\dagger f_{i+1} + \mathrm{H.c.}) + \mu_i f_i^\dagger f_i,$$
$$H_\tau = \sum_i h_i\tau_i^z + J_i\tau_i^x\tau_{i+1}^x, \quad (2)$$

and each can be separately solved. The f fermions form an Anderson insulator regardless of the disorder strength (since we are in one dimension). Meanwhile, the τ spins are described by a RTFIM whose eigenstates can be either in a magnetically ordered ("paired" or "spin-glass") phase [10,14,15] when $h \ll J$, or a magnetically disordered ("paramagnetic") phase, when $h \gg J$. (Here, h, J characterize the strength of randomness in h_i, J_i.) We argue that these limits correspond to the conventional and orthogonal Anderson-localized phases, respectively, and on perturbing away from exact solvability, lead to CMBL and OMBL phases. We note that as outlined in Ref. [2], the complex f fermions carry *all* the electric charge; thus, the transport properties are independent of the phase of the τ spins. In the $T \to \infty$ limit that is our main focus, all eigenstates are equiprobable, and, hence, the thermodynamics in the two phases is identical (and trivial).

Spectral functions.—In the clean case, a sharp diagnostic of the orthogonal metal is the behavior of its single-particle spectral function. As discussed in the "Introduction" section, the conventional metal has sharp peaks corresponding to quasiparticle excitations, whereas the orthogonal metal has an incoherent two-particle continuum reflecting the fractionalization of the c fermions. This diagnostic evidently fails in the localized phase because *any* local spectral function is dominated by a finite number (set by the localization length ξ) of sharp peaks.

A second possibility is that the *spatially averaged* spectral function retains a distinction between the two phases; as we shall see, this is true in the disordered system at $T = 0$ but not at $T > 0$. Recall that the spectral function of an operator $\mathcal{O}$ is given by $A_{\mathcal{O}}(\omega) = -(1/\pi)\mathrm{Im}G^{\mathrm{ret}}_{\mathcal{O}}(\omega)$, where $G^{\mathrm{ret}}_{\mathcal{O}}(t) \equiv -i\Theta(t)\mathrm{Tr}\rho(\beta)[\mathcal{O}(t),\mathcal{O}^\dagger(0)]_\pm$ is the retarded Green's function of $\mathcal{O}$, and we choose a commutator (anticommutator) if $\mathcal{O}$ is bosonic (fermionic). The use of a density matrix ρ in this expression deserves comment in the context of MBL, where systems are usually considered in isolation. We imagine first coupling the system to a bath at temperature T with strength g and taking $g \to 0$ slowly so that the system remains in equilibrium with the bath [16]; then we may write $\rho = \mathcal{Z}^{-1}e^{-H/T}$, where $\mathcal{Z}$ is the partition function. In general, we have [17]

$$A_{\mathcal{O}}(\omega) = \frac{1 \pm e^{-\beta\omega}}{\mathcal{Z}}\sum_{m,n}|\langle m|\mathcal{O}|n\rangle|^2 e^{-\beta E_m}\delta(\omega - E_{mn}), \quad (3)$$

where $\beta = 1/T$ is the inverse temperature, m, n label many-body eigenstates $E_{mn} \equiv E_n - E_m$, and we take the negative (positive) sign for bosonic (fermionic) $\mathcal{O}$. At the solvable point, we may express eigenstates of H_{ES} as tensor products of eigenstates of H_τ, H_f, and decompose the energies, viz. $|m\rangle = |m_\tau\rangle \otimes |m_f\rangle$, $E_m = E^\tau_{m_\tau} + E^f_{m_f}$. Using this, the c-fermion spectral function is

$$A_{c_j}(\omega) = \frac{1+e^{-\beta\omega}}{\mathcal{Z}} \sum_{\substack{m_\tau, m_f, \\ n_\tau, n_f}} |\langle m_f|f_j|n_f\rangle|^2 e^{-\beta E^f_{m_f}} \times |\langle m_\tau|\tau^x_j|n_\tau\rangle|^2 e^{-\beta E^\tau_{m_\tau}} \delta(\omega - E^f_{m_f n_f} - E^\tau_{m_\tau n_\tau}). \quad (4)$$

This can also be derived by expressing A_c in terms of the f and τ spectral functions [17].

Ground-state spectral functions.—The $T = 0$ spectral function only involves excitations about the ground state in each sector; as these must have positive energy, for $\omega \to 0$ only a limited set of transitions from each sector contribute to the spectral response [Fig. 1(a)]. In this limit, it is convenient to rewrite Eq. (4) as a convolution of $T = 0$ spectral functions of f and τ [17],

$$A^0_{c_j}(\omega) = \int_0^\omega d\Omega A^0_{f_j}(\Omega) A^0_{\tau^x_j}(\omega - \Omega), \quad (5)$$

where $A^0_{f_j}(\omega) = \sum_{n_f} |\langle 0_f|f_j|n_f\rangle|^2 \delta(\omega - E^f_{n_f})$, $A^0_{\tau^x_j}(\omega) = \sum_{n_\tau} |\langle 0_\tau|\tau^x_j|n_\tau\rangle|^2 \delta(\omega - E^\tau_{n_\tau})$.

Let us consider the f spectral function first, as its behavior is the same in both phases. Since the initial and final states $|0_f\rangle$, $|n_f\rangle$ lie in distinct fermion-parity sectors, they do not experience mutual level repulsion, and so the energy $E^f_{n_f}$ can be arbitrarily small (i.e., the tunneling density of states of an Anderson insulator is smooth and nonzero at the Fermi energy). Therefore, $\overline{A^0_{f_j}(\omega)} \overset{\omega\to 0}{\approx} \nu_f > 0$, where the bar denotes both spatial and disorder averaging.

We next consider the τ sector. In the spin-glass phase, τ^x_j has a nonvanishing ground-state expectation value, $\langle 0_\tau|\tau^x_j|0_\tau\rangle = m^x_j \neq 0$ leading to an $\omega = 0$ contribution to the τ spectral function. This vanishes in the paramagnetic phase. In addition, there are off-diagonal (i.e., finite-frequency) contributions to the τ spectral function. These go as continuously varying power laws [18,19], $\overline{A_{\tau^x_j}(\omega)} \overset{\omega\to 0}{\approx} m_x^2 \delta(\omega) + K(\omega/\omega_0)^{\gamma-1}$, where $m_x^2 = \overline{(m^x_j)^2}$, K is a constant, and $\gamma \geq 0$. In the exactly solvable model, $\gamma \to 0$ at the RTFIM transition. Using Eq. (5),

$$\overline{A^0_{c_j}(\omega)} \overset{\omega\to 0}{\approx} \nu_f \{m_x^2 + \nu_\tau(\omega/\omega_0)^\gamma\}. \quad (6)$$

For $h \ll J$, the τ's order ferromagnetically, $m_x \neq 0$, and there is a nonzero $\omega \to 0$ spectral response $\overline{A^0_{c_j}(\omega)} \sim$ const, corresponding to the conventional Anderson insulator. In the paramagnetic phase of the τ's for $h \gg J$, $m_x = 0$, and there is a "soft gap" to single-c-fermion tunneling: $\overline{A^0_{c_j}(\omega)} \sim \omega^\gamma$. This corresponds to the orthogonal Anderson insulator. The low-frequency $T = 0$ spectral response is, thus, sharply distinct in the two phases.

Spectral functions for $T > 0$.—Crucial to the $T = 0$ spectral distinction between the CMBL and OMBL phases is that in order to tunnel a c fermion, we must simultaneously tunnel an f fermion and create a spin excitation, both of which require a positive energy in the ground state. This, coupled with the fact that the off-diagonal spin response vanishes as $\omega \to 0$, generates a soft tunneling gap in the OMBL phase. For $T > 0$, however, $A_c(\omega)$ receives contributions from *all* initial states rather than just the ground state; therefore, a low-energy process for the c fermions can be built by offsetting a large positive energy difference in the f sector by a large negative energy difference in the τ sector or vice versa [Fig. 1(b)] [20]. In other words, at finite energy density, the low-frequency response of the c's is generically built from high-frequency responses of f's and τ's, and, therefore, the phase-space restrictions that lead to the distinct tunneling behavior in the OMBL and CMBL phases are absent. Formally, this can be seen directly from the $T \to \infty$ limit of Eq. (4): all Boltzmann factors equal the identity, and sums range over all states in each sector.

A sharp spectral distinction is absent not just at $T = \infty$ but generically at all $T > 0$. However, for temperatures much lower than the characteristic τ sector energy scales, a crossover persists in $A_c(\omega)$. When the τ's are paramagnetic, the c spectral function remains depleted at low frequencies, since only excitations with energies less than T are appreciably populated in equilibrium and can, thus, undergo "downhill" transitions. We, therefore, expect that in the OMBL phase, $\overline{A_c(T, \omega \to 0)} \sim T^\gamma$.

Diagnostic at $T = \infty$.—How can we distinguish between the orthogonal and conventional MBL phases for $T \to \infty$? Recall that we are ultimately interested in systems where the τ spins are not physically observable

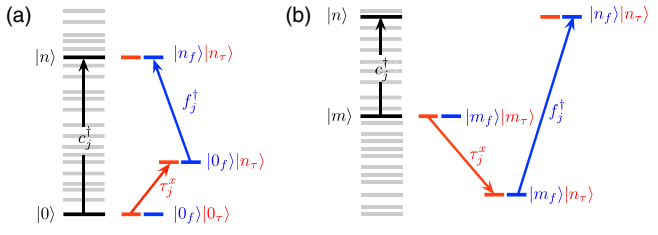

FIG. 1. Processes contributing to $A_c(\omega)$. (a) At $T = 0$, only excitations about the ground state of the f and τ sectors contribute, restricting the phase space as each has $E > 0$. (b) As $T \to \infty$, arbitrary low-energy c-tunneling processes can be put "on shell" by trading energy between sectors.

microscopic degrees of freedom to which one can couple directly; we must, therefore, build our desired diagnostic entirely out of c fermions. Although the spatial correlations of τ^x exhibit spin-glass order in one phase and decay exponentially in the other, one can only measure *products* of τ and f correlators. The f correlators decay exponentially in both phases, since the f fermions are always localized; therefore, the c correlators also decay exponentially in both phases. This overall exponential suppression masks the change from long-range behavior to exponential decay in the τ sector.

We consider instead a *ratio* of correlators, each computed at $T = \infty$ (i.e., via equal-weight eigenstate sums):

$$\mathcal{Q}(r) = \frac{\overline{(\sum_{i,m} |\langle m|c_i^\dagger c_{i+r}|m\rangle|^2)^2}}{\sum_{i,m} |\langle m|c_i^\dagger c_{i+1}^\dagger c_{i+r} c_{i+r+1}|m\rangle|^2}. \tag{7}$$

The numerator of Eq. (7) is sensitive to localization in both the f and τ sectors; in the spin-glass phase, it decays with the f-sector localization length (since the τ spins have spin-glass order), whereas in the paramagnet it decays with a localization length $\xi_c \simeq (1/\xi_f + 1/\xi_\tau)^{-1}$. The denominator, on the other hand, decays with the f-sector localization length in both phases: in the paramagnet, τ spins (related via a Jordan-Wigner transformation to real fermions) can be paired locally, but owing to charge conservation, the complex f fermions can only be paired up as in Fig. 2(b). Thus, the diagnostic (7) asymptotes to a constant value as $r \to \infty$ in the spin-glass phase of the τ's but decays exponentially in the paramagnet.

Because squaring *precedes* thermal averaging in Eq. (7), neither the numerator nor the denominator of $\mathcal{Q}(r)$ is an equal-time correlator. Rather, they are $\omega \to 0$ limits of spectral functions; unlike equal-time correlators, $\omega \to 0$ spectral functions remain sensitive to dynamics. $\mathcal{Q}$ is experimentally accessible: its numerator is the square of the $\omega \to 0$ limit of the spectral function of the operator $c_i^\dagger c_j$ that captures two-point correlations in tunneling; its denominator is the $\omega \to 0$ limit of the analogous Cooper pair correlator $\mathcal{O}_{ij} = c_i^\dagger c_{i+1}^\dagger c_{j+1} c_j$, which is related to the Josephson response. In the OMBL phase at $T = \infty$, fermionic pairs are more weakly localized than single fermions; this is a high-temperature manifestation of the relation between pairing and the orthogonal metal noted in Ref. [2]. Such "factoring out" of pieces of correlators to tease out hidden asymptotics is reminiscent of the diagnostics in Ref. [21], although the physics here is distinct.

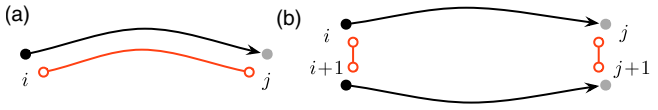

FIG. 2. Operator pairings in Eq. (7). Here, $f^\dagger$, f are denoted by black (grey) filled circles, and τ^x by a red unfilled circle. The numerator involves pairing (a), whereas the denominator is dominated (in the τ paramagnet) by pairings of the form (b) and contains long-distance f contractions. In the τ spin glass, τ^x has a classical expectation value (the spin-glass order parameter), so at leading order $\langle \tau_i^x \tau_j^x \rangle$ factorizes.

Extensions and discussion.—Having explored a simple solvable example of an orthogonal Anderson insulator, we now generalize our results. First, we argue that the orthogonal Anderson insulator extends into a bona fide OMBL phase. This is so, since perturbations of the solvable point that lead to short-range interactions in the decoupled theories do not destroy the localized phase of the f fermions or disrupt the phase structure of the RTFIM. Hence, the two insulators have sharply distinct eigenstates even with such perturbations and must be separated either by an eigenstate phase transition or an intermediate thermal phase. Though we disorder averaged independently for f and τ and this is no longer exact away from the solvable point, local interactions within either sector are innocuous [15,22], as they merely turn either noninteracting insulator into an MBL insulator. However, interactions will also introduce correlations between the disorder in the two sectors. The two distinct phases of the solvable model remain distinct in the presence of these disorder correlations, which preserve the Ising symmetry of the τ sector as well as the $U(1)$ symmetry of the charge sector. The presence (absence) of a soft gap in the zero-temperature c-fermion spectral function in the OMBL (CMBL) phase is due to the presence (absence) of Z_2 eigenstate order and, thus, persists in the interacting case. Further, the diagnostic (7) is robust to interactions. Both Figs. 2(a) and 2(b) will then involve terms with many c and $c^\dagger$ operators. However, the number of long-distance pairings is determined by the *imbalance* between $c^\dagger$'s and c's. Interactions do not change this, as long as $U(1)$ symmetry is preserved.

A second issue is that the toy model is 1D, yet the orthogonal phase is more natural in $d > 1$ where the gauge field can be deconfined. Exactly solvable, fine-tuned models of orthogonal *Anderson insulators* can be constructed in $d > 1$, but whether a robust MBL phase exists in $d > 1$ remains an open question due to the role of rare regions [23,24]. Even if strict MBL is absent, the CMBL and OMBL phases will persist as long-lived prethermal regimes [25,26]. Autocorrelators at intermediate times will be governed by MBL dynamics but will cross over to thermal behavior in the dc limit. As defined, our diagnostic (7) will behave thermally. However, instead of taking the dc limit, we could consider the same ratio of spectral functions at low nonzero frequency; it will be small (large) in the OMBL (CMBL) phase, with a sharp crossover between the two behaviors. This crossover should also occur in imperfectly isolated systems [16].

Our conclusions generalize from the OMBL phase to other fractionalized MBL phases, whose integrals of motion are orthogonal to the physical fermion or spin

operators. For spectral functions, this generalization is direct. Diagnostics analogous to Eq. (7) can be constructed for any fractionalized MBL phase whose physical particles have a conserved $U(1)$ charge carried by some (but not all) of the "partons." The physical fermion carries charge as well as other quantum numbers; however, composite local operators carry only charge and generalize the denominator in Eq. (7). It is tempting to speculate on the role of many-body mobility edges in this setting; all sectors presumably share a common mobility edge, since otherwise one will thermalize the rest. We defer this question to future work.

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