Radiatively Generating the Higgs Potential and Electroweak Scale via the Seesaw Mechanism

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The minimal seesaw scenario can radiatively generate the Higgs potential to induce electroweak symmetry breaking while supplying an origin of the Higgs vacuum expectation value from an underlying Majorana scale. If the Higgs potential and (derived) electroweak scale have this origin, the heavy $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y$ singlet states are expected to reside at $m_N \sim 10$–500 PeV for couplings $|\omega| \sim 10^{-4.5} - 10^{-6}$ between the Majorana sector and the standard model. In this framework, the usual challenge of the electroweak scale hierarchy problem with a classically assumed potential is absent as the electroweak scale is not a fundamental scale. The new challenge is the need to generate or accommodate PeV Majorana mass scales while simultaneously suppressing tree-level contributions to the potential in ultraviolet models.

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Introduction.—The standard model (SM) provides a successful description of many particle physics measurements but does not explain the experimental evidence for dark matter and neutrino masses. These facts argue for extensions of the SM that experience some form of decoupling in an effective field theory (EFT) setting [1] so that the corrections to the SM in an effective field theory (SMEFT) framework are small perturbations.

Minimal (viable) extensions of the SM are usually beset with an inability to address the electroweak (EW) scale hierarchy problem. A SMEFT statement of which is that threshold corrections to $H^\dagger H$ can be generated by integrating out sectors extending the SM proportional to large scales $\Lambda \gg m_h$. Without parameter tuning the Higgs mass is expected to be proximate to the cutoff scale of the theory for this reason. Symmetries, such as supersymmetry, can suppress these threshold corrections. This is frequently done while assuming the EW scale is a fundamental parameter and the SM Higgs potential has a classical form that leads to $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_{\mathrm{em}}$. As a result, one generally expects new particles related to stabilizing symmetries, such as superpartners, to be around the TeV scale and found at the LHC. Unfortunately, this has not (as yet) occurred.

In this Letter we do not adopt the assumptions that the EW scale and Higgs potential are fixed classically without a dynamical origin. We develop an alternative approach using the minimal seesaw scenario [2–5]. The idea is that the Higgs mass and potential are generated by threshold corrections from the Majorana [6] sector and the anomalous breaking of scale invariance in the Coleman-Weinberg (CW) potential [7]. The result is an origin of the observed neutrino masses within a SM extension that avoids parameter tuning. An origin for the EW scale is introduced that suggests a new perspective on the EW scale hierarchy problem, modifying the usual concerns that lead to expectations of TeV scale new states into an alternate framework. The purpose of this Letter is to demonstrate this possibility —"the neutrino option." The scenario presented is falsifiable; it sensitively depends on an experimentally determined top, Higgs and neutrino masses. Higher order renormalization group equations (RGEs) and threshold matching calculations are also critical in this scenario. This approach motivates more theoretical and experimental progress in all of these areas in order to falsify or confirm this possible origin of the Higgs potential and EW scale.

Seesaw model and threshold corrections.—We use the seesaw formalism of Refs. [8,9]. The extension of the SM Lagrangian with $p, q = \{1,2,3\}$ singlet fields N_p is given by

$$2\mathcal{L}_{N_p} = \overline{N_p}(i\partial\!\!\!/ - m_p)N_p - \overline{\ell_L^\beta}\,\tilde{H}\,\omega_\beta^{p,\dagger}N_p, \\ - \overline{\ell_L^{c\beta}}\tilde{H}^*\omega_\beta^{p,T}N_p - \overline{N_p}\omega_\beta^{p,*}\tilde{H}^T\ell_L^{c\beta} - \overline{N_p}\omega_\beta^p\tilde{H}^\dagger\ell_L^\beta. \tag{1}$$

The $\omega_\beta^p = \{x_\beta, y_\beta, z_\beta\}$ are each complex vectors in flavor space. These vectors have absorbed the Majorana phases θ_p. The mass eigenstate Majorana fields are defined such that they satisfy the Majorana condition [6]: $N_p^c = N_p$. These fields are related to chiral right-handed fields N_R that are singlets under $\mathrm{SU}(3) \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ as [9] $N_p = e^{i\theta_p/2}N_{R,p} + e^{-i\theta_p/2}(N_{R,p})^c$. The superscript c stands for charge conjugation; defined on a Dirac spinor as $\psi^c = -i\gamma_2\gamma_0\bar{\psi}^T$. Integrating out the seesaw model at tree level, the results can be mapped to the SMEFT up to dimension seven [8]. The threshold corrections of interest lead to dimension two and four terms. They come about due to integrating out the heavy N_p states at one loop. Considering a Higgs potential parametrization

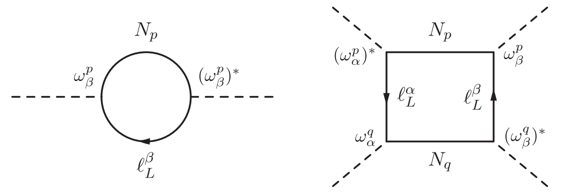

FIG. 1. One loop corrections matching onto and generating the Higgs potential at the scale(s) m_p in the seesaw model.

$$V_c(H^\dagger H) = -\frac{m^2}{2}(H^\dagger H) + \lambda(H^\dagger H)^2, \tag{2}$$

and neglecting the effect of running down from the scale(s) $\mu = m_p$, the diagrams of Fig. 1 give a threshold matching to the Higgs potential terms

$$\Delta m^2 = m_p^2 \frac{|\omega_p|^2}{8\pi^2} F_1, \quad \Delta\lambda = -5\frac{(\omega_q \cdot \omega^{p,\star})(\omega_p \cdot \omega^{q,\star})}{64\pi^2} F_2, \tag{3}$$

where, in the assumption of approximately degenerate N_p states, $F_1, F_2 \simeq 1$. Here, the repeated indices are summed over and the results can be compared to past results in Refs. [10,11]. We have used dimensional regularization in $d = 4 - 2\epsilon$ dimensions and $\overline{\mathrm{MS}}$. The counterterms of the SM and the full theory cancel the ϵ divergences in each case. The mismatch of the SMEFT and full theory Lagrangian when $p^2/m_p^2 \to 0$ defines the threshold matching. Note that the λ threshold corrections can be subdominant to other quantum corrections in the full CW potential in the parameter space of interest, where $|\omega_p| \ll 1$ and $m_p \gg 246$ GeV.

Induced CW potential.—The threshold corrections to $H^\dagger H$ can be naturally dominant in defining the Higgs potential below the scales $\mu \simeq m_p$ as the SM is classically scale invariant in the limit that the vacuum expectation value (VEV) of the Higgs field $v \to 0$ [7,12–14] ($m \to 0$) in Eq. (2). This point of enhanced symmetry is anomalous, even before its soft breaking by the threshold matching. However, the additional SM breakings of scale invariance through quantum corrections are associated with dimensionful parameters that are smaller than m_p^2 in a consistent version of this scenario at the threshold matching scale.

A breaking of the scaleless limit of the SM is due to QCD, which generates the scale Λ_{QCD} by dimensional transmutation [7] at low scales as $(\Lambda_{\mathrm{QCD}}/\mu)^{b_0} = \exp[-8\pi/\hbar g_3^2(\mu)]$, where $b_0 = 11 - (2/3)n_f$ [15,16]. The quark masses that result lead to V_c contributions such as $\Delta m^2 = (N_c y_t^2 \Lambda_{\mathrm{QCD}}^2/32\pi^2)(1 + 3\log[\mu^2/\Lambda_{\mathrm{QCD}}^2]) + \cdots$, which subsequently induces a VEV for the Higgs, leading to gauge boson masses $\propto \Lambda_{\mathrm{QCD}}$. As we are assuming $m_p^2|\omega_p|^2 \gg \Lambda_{\mathrm{QCD}}^2$ (for each p), these contributions are naturally subdominant for $H^\dagger H$, and anyway, at the threshold matching scale we consider, the QCD coupling has run to scales such that $g_3(m_p) < 1$. Renormalization of the CW potential also introduces an anomalous breaking of scale invariance. Consider defining $V_{\mathrm{CW}}(\langle H^\dagger H\rangle)$ as the one loop CW potential expanding around the scaleless limit of the SM while neglecting the threshold corrections. The standard result [7,12,13] can be minimized via $\partial V_{\mathrm{CW}}/\partial\langle H^\dagger H\rangle = 0$. The VEV scale obtained is exponentially separated from the renormalization scale. This scale is associated with the asymptotic nature of the perturbative expansions used in constructing the CW potential, that also predicts S-matrix elements that are used to fix SMEFT Lagrangian parameters. This scale can be either suppressed or enhanced depending on the net sign of the quantum correction in the CW potential, and a suppression is consistent with an EFT analysis.

In summary, the soft breaking of the scaleless limit of the SM [17] is such that the threshold corrections to $H^\dagger H$ due to integrating out the N_p states can be a dominant contribution to V_{CW} fixing a high scale boundary condition for the Higgs potential. This occurs for interesting parameter space when tuning of the threshold corrections against bare parameters is avoided expanding around the classically scaleless limit of the SM Lagrangian.

Running down to the scale $\mu = \hat{m}_t$.—We assume that the Higgs potential is (dominantly) given by Eq. (3) when integrating out the Majorana sector. This condition can be obtained requiring (i) smaller breaking of scale invariance in the Higgs sector and (ii) the bare tree-level Higgs potential at the scale m_p is negligible compared to the threshold contributions. The realization of these conditions in a full UV model represents a challenge alternative to the usual hierarchy problem. In such a scenario, a nontrivial consistency condition is the successful generation of the SM potential at lower energy scales and field values. The measured masses of the SM states and their couplings ensures this can occur in a very nontrivial fashion. Note also that despite the fact that the seesaw boundary condition fixes $\lambda < 0$, this coupling can still run to positive values at lower scales as it is not multiplicatively renormalized. To demonstrate that a successful lower scale phenomenology can result from these seesaw boundary conditions, we take $V_c(H^\dagger H)$ to be fixed to $m^2(m_p) \equiv \Delta m^2$ and $\lambda(m_p) \equiv \Delta\lambda$. The parameters m^2 and λ are then run down according to the coupled SM RGEs. The β functions for running above the top mass are introduced as $\beta(x) = (4\pi)^2 dx/d\ln\mu^2$ and are taken (at leading order) from the summary in Ref. [27] as

$$\beta(g_Y^2) = g_Y^4\frac{41}{6}, \quad \beta(g_2^2) = g_2^4\left(-\frac{19}{6}\right), \quad \beta(g_3^2) = g_3^4(-7),$$
$$\beta(\lambda) = \left[\lambda\left(12\lambda + 6Y_t^2 - \frac{9}{10}\left(5g_2^2 + \frac{5g_Y^2}{3}\right)\right) - 3Y_t^4 + \frac{9}{16}g_2^4 + \frac{3}{16}g_Y^4 + \frac{3}{8}g_Y^2 g_2^2\right], \tag{4}$$

$$\beta(m^2) = m^2\left[6\lambda + 3Y_t^2 - \frac{9}{20}\left(5g_2^2 + \frac{5g_Y^2}{3}\right)\right], \quad (5)$$

$$\beta(Y_t^2) = Y_t^2\left[\frac{9}{2}Y_t^2 - 8g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{12}g_Y^2\right]. \quad (6)$$

Here g_2, g_3 are the coupling constants of the $SU(2)_L$ and $SU(3)_c$ gauge groups, while g_Y is the coupling of the $U(1)_Y$ group. $Y_i = \sqrt{2}m_i/v$ is the Yukawa coupling of a fermion to the Higgs field with $v^2 = 1/\sqrt{2}\hat{G}_F$. Contributions proportional to Y_b and Y_τ have been neglected.

The differential system is solved by fixing the boundary conditions to be

$$\lambda(m_p) = -n^2\frac{5}{64\pi^2}|\omega|^4, \qquad m^2(m_p) = \frac{n|\omega|^2}{8\pi^2}m_p^2, \quad (7)$$

$$\hat{Y}_t(m_t) = Y_t^{(0)} + Y_t^{(1)}(m_t) = 0.9460, \quad (8)$$

$$\hat{g}_Y(m_t) = g_Y^{(0)} + g_Y^{(1)}(m_t) = 0.3668, \quad (9)$$

$$\hat{g}_2(m_t) = g_2^{(0)} + g_2^{(1)}(m_t) = 0.6390, \quad (10)$$

$$\hat{g}_3(m_t) = 1.1671, \quad (11)$$

for different choices of m_p and $|\omega|$, which approximate to one common universal scale and coupling in what follows.

The number of heavy neutrino species has been denoted with n and fixed to $n = 3$. The $x^{(0)}$ and $x^{(1)}(\mu)$ stand for the tree and one-loop level contribution in the SM and we have denoted them as hatted quantities inferred from measured input parameters. The numerical inputs used are

$$\{\hat{m}_Z, \hat{m}_W, \hat{m}_t, \hat{m}_h\} = \{91.1875, 80.387, 173.2, 125.09\}$$

in GeV units, $\hat{G}_F = 1.166378710^{-5}\ \mathrm{GeV}^{-2}$, and $\hat{\alpha}_s = 0.1185$. The expressions for the $x^{(1)}(\mu)$ used are summarized in Ref. [27]. The RG running in QCD is at four loops for g_3 and two loops in the EW interactions using Ref. [27]. The quantity $\lambda(\mu = m_t)$ does not show significant dependence on the parameter $|\omega|$: for any value $|\omega| < 0.1$ we have $|\Delta\lambda| \lesssim 10^{-6}$, which is numerically insignificant in the running. Conversely, this quantity is quite sensitive to the scale m_p and the RGE order used. There is significant numerical sensitivity to the input parameters. In particular, the precise experimental determination of $\{\hat{m}_h, \hat{m}_t\}$ is critical for the consistency of the scenario. We show the dependence on these inputs on the inferred scale m_p in Fig. 2. This plot shows the value for $\lambda(\hat{m}_t)$ consistent with experimental measurements is obtained for $m_p \simeq 10^{1.3}$ PeV, assuming $\hat{m}_t = 173.2$ GeV. The quantity $m^2(\mu = \hat{m}_t)$ is sensitive to both m_p and $|\omega|$. Figure 2 shows its dependence on $|\omega|$ for the fixed value $m_p = 10^{1.3}$ PeV and the corresponding viable band associated to the uncertainty on the top mass determination. (This interesting region of parameter space in the seesaw model has been previously discussed in Ref. [28].)

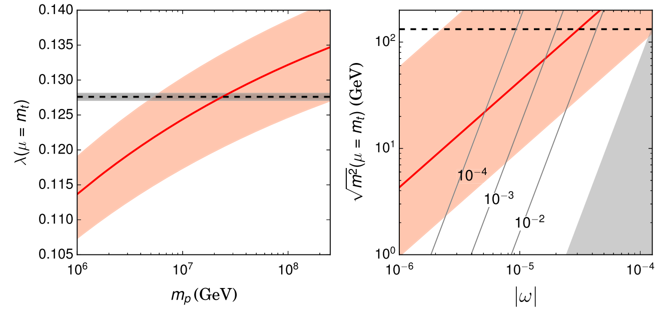

FIG. 2. Values of the parameters λ (left) and $\sqrt{m^2}$ (right) extrapolated at the scale $\mu = \hat{m}_t$ as a function of the heavy neutrino mass scale m_p and of $|\omega|$, respectively. The dashed lines and surrounding bands marks indicate the values consistent with the measured Higgs mass and its percentage error [29]. The red line in the left panel assumes $\hat{m}_t = 173.2$ GeV and the surrounding band corresponds to varying $\hat{m}_t$ between 171 and 175 GeV (a 2σ error variation [30]). In the right panel, the solid red line assumes $m_p = 10^{1.3}$ PeV, and the gray region is disfavored due to ΛCDM cosmology limits on the sum of neutrino masses [Eq. (12)]. The neutrino mass scales predicted (in eV) are the three solid lines.

This scenario can lead to a SM-like Higgs potential emerging from the combined effect of the threshold corrections and the SM RGEs as shown in Fig. 3.

Cosmological and low energy constraints.—The sum of the observed neutrino masses is $\sum_i m_\nu^i \simeq 3|\omega|^2/2\sqrt{2}\hat{G}_F m_p$ in the tree level approximation used here, while neglecting running effects. Assuming a ΛCDM cosmology, combined CMB, supernovae, and baryon acoustic oscillation data limit this sum [31]. This translates into a constraint of

$$\frac{3\sqrt{3}}{8\pi}\frac{|\omega|^2}{\hat{G}_F\sqrt{\Delta m^2}} \lesssim 0.23\ \mathrm{eV}, \qquad 95\ \mathrm{C.L.} \quad (12)$$

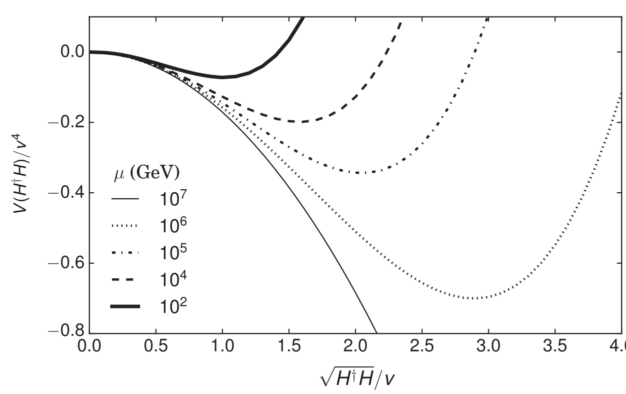

FIG. 3. The emergence of the Higgs potential due to running the seesaw boundary conditions down to $\mu \sim \hat{m}_t$.

The overall neutrino mass scale predicted is very sensitive to the uncertainty on $\hat{m}_t$, the chosen order of RGEs, and threshold loop corrections included in the numerical simulation. In Fig. 2 we show the absolute neutrino mass scales (gray lines) predicted at leading order as $|m_\nu| = 3|\omega|^2/2\sqrt{2}\hat{G}_F m_p$. One expects $|m_\nu|^2 \gtrsim \Delta m_{21}^2, \Delta m^2$ to avoid fine tuning, and a requirement of further model building in the Majorana sector.

In addition, a negative sign for λ and m^2 indicates a theory with a Hamiltonian unbounded from below. However, the corresponding decay time for the EW vacuum is exponentially small [32–35]. We have checked that the EW vacuum decays in this scenario are well approximated by the (negligible) result in the SM in Ref. [27]. The ratio of the scales at which $\beta(\lambda)$ vanishes in the SM, compared to the SM extension considered in this Letter (which fixes the size of the action of the bounce) is $\sim 1.000\,11$. The extrapolation of the theory far above the scale m_p is associated with a large theory uncertainty as the N_p could be embedded in an extended Majorana sector, with other states that can also modify the running of the couplings above the scale $\mu \simeq m_p$.

VI. Numerical stability of the results.—The results shown in Sec. IV are produced with one loop matching conditions and one loop RGEs. Increasing the RGE and threshold matching order used shows significant numerical sensitivity as the coupling λ is running to small and negative values asymptotically which introduces a sensitivity to the scale m_p where the seesaw boundary conditions are matched. This feeds into the required $|\omega|$ to produce the Higgs potential and EW scale, and, subsequently, the neutrino mass scale. For this reason, the minimal scenario is falsifiable. On the other hand, the uncertainty in $\hat{m}_t$ is significant. To illustrate this in Fig. 4 the best fit points for the cases where the boundary conditions of the scenario are evolved with one loop SM RGEs, two loop SM RGEs, and one loop RGEs for Δm and λ and two loop RGEs for the remaining SM parameters. (Formally the running should be described using the SMEFT RGEs, which include the effect of higher dimensional operators feeding into the running of the SM couplings [36]. We have checked that this effect is numerically subdominant in this model and neglected it.) This last case is shown as these parameters do not have a tree level matching coefficient in this scenario. Despite this, we have confirmed that using one loop or two loop RGEs the measured neutrino mass differences can be reproduced; see Sec. VIII.

Tree level decays and IceCube.—The tree level decays of the N_p states are well known; see Refs. [37,38]. Remarkably, the mass range selected for when the Higgs potential is radiatively generated in the minimal seesaw scenario is consistent with the measured energies of an excess of neutrinos reported by IceCube [39–41]. The $d\Gamma/dE_\nu$ spectrum that results from these decays is a sharp monochromatic peak at the scale $m_p/2$.

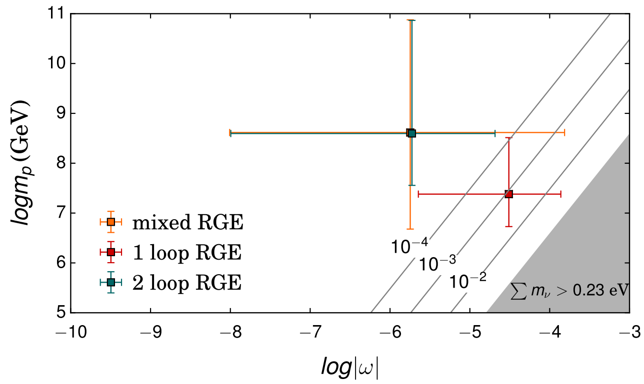

FIG. 4. Numerical sensitivity of the results, with all cases showing one loop matching to Δm, λ and including one loop corrections for the remaining SM parameters with one loop, two loop, or mixed RGEs for all parameters. The mixed case shows one loop running for Δm, λ and two loop running for all remaining SM parameters. The best fit points are indicated with a box in each case with error bars showing the experimental uncertainty in the top quark mass, which has been chosen to be its 2σ uncertainty [30]. Nevertheless, we have determined that the measured mass differences of the neutrinos can be accommodated in all three RGE cases.

The possibility that the N_p states can be viable dark matter candidates to induce the IceCube events has been examined in the literature. We agree with Refs. [42–44] that the required coupling for the event rate scales as $\Gamma_{\text{events}} \sim (|\omega|/10^{-29})^2 (m_p/1.2\ \text{PeV})/\text{yr}$, which is inconsistent with the preferred $|\omega|$ in the minimal neutrino option. Extended model building can possibly accommodate these observations.

Neutrino mass differences and mixing.—A common universal scale and coupling for the N_p states integrated out does not predict neutrino mass differences or mixing angles. Treating N_p, ω, and the charged lepton Yukawa Y_e as nondegenerate enlarges the number of free parameters (15 moduli+6 phases [9]) to a set larger than that of the experimental constraints (2 Higgs parameters+ $2\,\nu$ mass differences + 3 PMNS angles). As an existence proof, consider $\omega \equiv \omega_0 \mathbb{1} + \delta\omega$ with $(\delta\omega)_{ij} \ll \omega_0$. Correct values for $m^2(\hat{m}_t)$, $\lambda(\hat{m}_t)$, Δm_{12}^2 and Δm^2 (see Ref. [30]) in a normal hierarchy can be obtained, e.g., for $\{m_1, m_2, m_3\} = \{23.96, 24.77, 25.27\}$ PeV, $\omega_0 = 10^{-4.4}$, $\delta\omega_{11} = 4.08 \times 10^{-10}$, $\delta\omega_{22} = -1.88 \times 10^{-8}$, and $\delta\omega_{33} = -7.67 \times 10^{-9}$. Only the diagonal terms in ω were used in this example leading to $U_\nu = \text{diag}\{1, 1, 1\}$. The results can be perturbed by the free off diagonal entries that can combine with the unfixed charged lepton mass matrix rotation matrix U_ℓ so that $U_\ell^\dagger U_\nu = U_{\text{PMNS}}$.

Conclusions.—Because of a nontrivial interplay of the couplings of the SM and the mass scales of the SM states expanded around the classically scaleless limit, the minimal seesaw scenario can form a UV boundary condition that induces the Higgs potential at lower energies. This can occur as a simple mechanism to generate neutrino masses is introduced extending the SM.

In this scenario the EW scale is not fundamental but is due to the quantum threshold corrections matching the heavy singlet states onto the SMEFT, which is assumed to be expanded in its near scaleless limit. Instead of an expectation of new states at the TeV scale, the multi-PeV scale is the locus of a requirement of a mass generating mechanism for Majorana states, and possibly accompanying stabilizing symmetries. This change in perspective on the hierarchy problem is possibly valuable. The key point underlying this approach is to abandon attempts to stabilize the Higgs mass against threshold corrections at the TeV scale, due to the lack of experimental indications of new states associated with stabilizing symmetries. Instead, we advocate embracing these corrections as the origin of the Higgs potential. This approach can also be developed in other models.

Future experimental results supporting this scenario are a continued lack of discovery of states motivated by a traditional interpretation of the hierarchy problem at the LHC, and the eventual discovery of the Majorana nature of neutrinos in $0\nu\beta\beta$ decay. The scenario can be tested through consistency tests of the neutrino mass spectrum and the PMNS matrix due to a minimal seesaw scenario and more precise measurements of $\hat{m}_t, \hat{m}_h$. In this manner the neutrino option in generating the Higgs potential is falsifiable. Further phenomenological investigations, and the advance of higher order SM threshold and RGE calculations are also strongly motivated.

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