

Revealing Nonclassicality of Inaccessible Objects

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Some physical objects are hardly accessible to direct experimentation. It is then desirable to infer their properties based solely on the interactions they have with systems over which we have control. In this spirit, here we introduce schemes for assessing the nonclassicality of the inaccessible objects as characterized by quantum discord. We consider two probes individually interacting with the inaccessible object but not with each other. The schemes are based on monitoring entanglement dynamics between the probes. Our method is robust and experimentally friendly, as it allows the probes and the object to be open systems and makes no assumptions about the initial state, dimensionality of involved Hilbert spaces, and details of the probe-object Hamiltonian. We apply our scheme to a membrane-in-the-middle optomechanical system, to detect system-environment correlations in open system dynamics as well as nonclassicality of the environment, and we foresee potential benefits for the inference of the nonclassical nature of gravity.

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What should be known about an inaccessible object to conclude that it is “not classical”? In this Letter, inspired by quantum communication scenarios, we show that it is sufficient to verify whether such an object can be used to increase quantum entanglement between remote probe particles that individually interact with it but are not directly coupled to each other.

Specifically, we prove that such a gain in quantum entanglement is possible only if, during its evolution, the object shares with the probes quantum correlations in the form of quantum discord [1–5]. In turn, the presence of quantum discord between the probes and the object entails a nonclassical feature of the object itself. According to the definition of discord, two or more subsystems share quantum correlations if there is no von Neumann measurement on one of them that keeps the total state unchanged. This can happen only when nonorthogonal (indistinguishable) states are involved in the description of the physical configuration of the measured subsystem. This indistinguishability is the nonclassical feature that we aim to detect. We formulate analytical criteria revealing such nonclassicality based on operations performed only on the probes and without any detailed modeling of the inaccessible object in question.

We emphasize that the nonclassicality is revealed under a set of minimal assumptions. Namely, (i) the object may remain inaccessible at all times; i.e., it need not be directly measured. In particular, its quantum state and Hilbert space dimension can remain unknown throughout the whole assessment. Our method is thus valid when the object is an elementary system or an arbitrarily complex one.

(ii) The details of the interaction between the object and the probes may also remain unspecified. (iii) Every party can be open to its own local environment. These properties make our method applicable to a large number of experimentally relevant situations.

We demonstrate the revealing power of our criteria for nonclassicality through the study of an optomechanical system, which is a platform of enormous experimental interest. This is clearly not the only situation that can benefit from the results of our investigation. We conclude the Letter with a discussion of a set of physical problems, from the revelation of system-environment correlations in open system dynamics to the quest for the possible quantum nature of gravity, that would be fully suited to the framework presented here.

The formal criteria.—Consider the scenario depicted in Fig. 1. System C is assumed to be the inaccessible object and to mediate the interaction between two remote probes, labeled A and B . Therefore, from now on, we refer to system C as the *mediator*. It is essential for our method that the probes are not directly coupled and interact only via the mediator. Therefore, the Hamiltonian for the process under scrutiny can be written as $H_{AC} + H_{BC}$, with H_{JC} the interaction Hamiltonian between the mediator C and probe $J = A, B$. Our work is developed in the context of entanglement distribution with continuous interactions [6]. We first focus on the partition $A:BC$ and demonstrate a result which will be instrumental to design our criteria for the inference of nonclassicality of C based on entanglement dynamics in AB only. Previous studies on the resources allowing for entanglement distribution showed that any

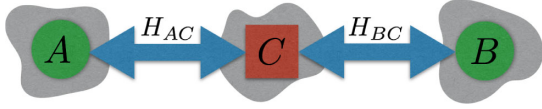


FIG. 1. Probes A and B individually interact with a mediator object C but not with each other. We allow C to be inaccessible; i.e., no measurement can be performed on it, and its state may remain unknown. We show conditions under which the gain of entanglement in AB implies nonzero quantum discord $D_{AB|C}$. Our protocols make no assumptions about the dimensions of each subsystem or explicit form of H_{AC} and H_{BC} and allow each subsystem to be open to its own environment (represented by gray-colored shadows).

three-body density matrix, i.e., the state of ABC at any time t in the present context, satisfies the inequality [7,8]

$$|E_{A:BC}(t) - E_{AC:B}(t)| \leq D_{AB|C}(t). \quad (1)$$

Here $E_{X:Y}$ is the relative entropy of entanglement in the partition $X:Y$ [9], and $D_{X|Y}$ is the relative entropy of discord [10], also known as the one-way quantum deficit [11]. Note that relative entropy of discord is, in general, not symmetric, i.e., $D_{X|Y} \neq D_{Y|X}$. Equation (1) shows that the change in entanglement due to the relocation of C is bounded by the quantum discord carried by it.

Let us start from the simple case where the overall probe-mediator system is closed (which allows us to ignore for now the gray-colored shadows in Fig. 1). If the interaction Hamiltonians H_{JC} satisfy $[H_{AC}, H_{BC}] = 0$, the evolution operator from the initial time $t = 0$ to some finite time τ is just $U = U_{BC}U_{AC}$, where $U_{JC} = \exp(-iH_{JC}\tau)$ and we set $\hbar = 1$. This situation is equivalent to first interacting C with A and then C with B (or in reversed order). However, note that the density matrix $\rho' = U_{AC}\rho_0 U_{AC}^\dagger$ obtained by “evolving” the initial state through U_{AC} only does not describe the state of the system at τ . Nevertheless, we now show the relevance of the properties of state ρ' for entanglement gain.

Consider the following forms of Eq. (1) written for the initial state ρ_0 and the instrumental state ρ' , respectively:

$$\begin{aligned} E_{AC:B}(0) - E_{A:BC}(0) &\leq D_{AB|C}(0), \\ E'_{A:BC} - E'_{AC:B} &\leq D'_{AB|C}. \end{aligned} \quad (2)$$

Note that $E_{AC:B}(0) = E'_{AC:B}$, because U_{AC} is local in this partition. The state at time τ is given by $\rho_\tau = U_{BC}\rho' U_{BC}^\dagger$, and thus $E_{A:BC}(\tau) = E'_{A:BC}$, this time owing to U_{BC} being local. Summing the above inequalities, we obtain a bound on the entanglement gain:

$$E_{A:BC}(\tau) - E_{A:BC}(0) \leq D_{AB|C}(0) + D'_{AB|C}. \quad (3)$$

This opens up the possibility to create entanglement at time τ without producing discord at both $t = 0$ and τ but rather by utilizing nonclassicality in the instrumental state. In

other words, the gain of entanglement in $A:BC$ could be mediated by object C , which gets nonclassically correlated by U_{AC} and then decorrelated by U_{BC} . Therefore, C is classically correlated at times $t = 0$ and τ . We now give a concrete example of this type of entanglement creation.

Consider the interaction Hamiltonian

$$H = \sigma_A^x \otimes \mathbb{1} \otimes \sigma_C^x + \mathbb{1} \otimes \sigma_B^x \otimes \sigma_C^x, \quad (4)$$

where $\sigma^j (j = x, y, z)$ is the Pauli- j matrix. As the initial state, we choose the classically correlated state

$$\rho_0 = \frac{1}{2} |011\rangle\langle 011| + \frac{1}{2} |100\rangle\langle 100|, \quad (5)$$

where, e.g., $\sigma^z|0\rangle = |0\rangle$. One can now readily check that the relative entropy of entanglement $E_{A:BC}$ grows from 0 to 1 in the time span from $t = 0$ to $\tau = \pi/4$, whereas discord $D_{AB|C}$ remains zero at these two times. The gain is indeed due to nonclassical correlations of the instrumental state: Applying only U_{AC} for a time $\tau = \pi/4$ produces discord $D'_{AB|C} = 1$.

For general noncommuting interaction Hamiltonians, one can pursue a similar analysis with the help of the Suzuki-Trotter expansion. The evolution operator U is now discretized into short-time interactions of C with A and then B (or the reversed order) as

$$U = \lim_{n \rightarrow \infty} (e^{-iH_{BC}\Delta t} e^{-iH_{AC}\Delta t})^n, \quad (6)$$

where $\Delta t = \tau/n \rightarrow 0$. Accordingly, Eq. (3) holds with τ replaced by Δt . It is now natural to ask if a scenario exists where entanglement could be increased via interactions with a classical C at all times by exploiting the discord in the instrumental state. The example given above is not of this sort, because, although we have $D_{AB|C} = 0$ at $t = 0$ and τ , it is nonzero for $t \in (0, \tau)$. It turns out that, for short evolution times, the discord of the instrumental state cannot be exploited as the following theorem demonstrates.

Theorem.—For three open systems A , B , and C with Hamiltonian $H = H_{AC} + H_{BC}$ and each coupled to its own local environment, the entanglement satisfies the condition $E_{A:BC}(\tau) \leq E_{A:BC}(0)$ if $D_{AB|C}(t) = 0$ at any time $t \in [0, \tau]$.

Proof.—The proof is presented in Supplemental Material [12].

We emphasize the generality of this theorem, where both the mediator and probes are open to their own local environments. This matches a large number of experimentally relevant situations, some of them being addressed in the last part of this Letter. The setup where A , B , and C are closed systems is then a special case of the theorem above in which we have $E_{A:BC}(\tau) = E_{A:BC}(0)$ if $D_{AB|C}(t) = 0$ [12]. Such a theorem extends the monotonicity of entanglement under local operations and classical communication (LOCC) [21] to the case of continuous

interactions. In general, zero-discord states are good models for classical communication, as they allow for continuous projective measurements on C that do not disturb the whole multipartite state.

We are now in a position to study the presence of discord $D_{AB|C}$ from observing AB only. In light of the theorem above, a promising candidate for this goal is the entanglement gain. However, we now show that some features of the initial tripartite state need to be ensured, but they can be guaranteed by only operating on AB .

Let us consider Eq. (4) and choose the initial state

$$\rho_0 = \frac{1}{2}|\psi_+\rangle\langle\psi_+| \otimes |+\rangle\langle+| + \frac{1}{2}|\phi_+\rangle\langle\phi_+| \otimes |-\rangle\langle-|, \quad (7)$$

where $\sigma^x|\pm\rangle = \pm|\pm\rangle$, and $|\psi_+\rangle = (1/\sqrt{2})(|01\rangle + |10\rangle)$ and $|\phi_+\rangle = (1/\sqrt{2})(|00\rangle + |11\rangle)$ are two Bell states between subsystems AB . As the initial state in Eq. (7) contains the eigenstates of H_C , the system remains classical, as measured on C , at all times. Furthermore, the classical basis is the same at all times. Yet, one can verify that the relative entropy of entanglement between the probes is given by $E_{A:B}(t) = 1 - S_{AB}(t)$, where $S_{AB}(t)$ is the von Neumann entropy of the AB state at time t , and oscillates between 0 and 1. Hence, in general, entanglement gain in the partition $A:B$ does *not* signify the nonclassicality of C (nonzero $D_{AB|C}$).

Similar considerations have been presented in Ref. [22] to provide a counterexample of the impossibility of entanglement gain via LOCC. However, the partition $A:BC$ is entangled already from the beginning (in our example, we have $E_{A:BC} = 1$). The subsequent evolution only localizes such entanglement to the $A:B$ partition. This example emphasizes that the ancillary particles within the framework of LOCC (here C) are not allowed to be initially correlated with the principal system (here AB), even if the correlations are classical.

Furthermore, the only way of gaining entanglement in subsystem AB via classical C is to localize it from the already present entanglement in $A:BC$. This is a consequence of our theorem and reinforces its role as a proper generalization of the monotonicity of entanglement to continuous interactions. Namely,

$$E_{A:B}(\tau) \leq E_{A:BC}(\tau) \leq E_{A:BC}(0). \quad (8)$$

Now, if we ensure by operating on the probes only that the initial entanglements coincide, i.e., $E_{A:BC}(0) = E_{A:B}(0)$, entanglement gain in system AB is possible only due to nonzero discord $D_{AB|C}$. As we are interested in observing entanglement gain, it is natural to start with as small entanglement as possible. This leads us to propose the application of an entanglement-breaking channel to one of the available systems, at time $t = 0$. Indeed, after application of the channel, we have $E_{A:B}(0) = E_{A:BC}(0) = 0$. In a more concrete example, the channel is a von Neumann

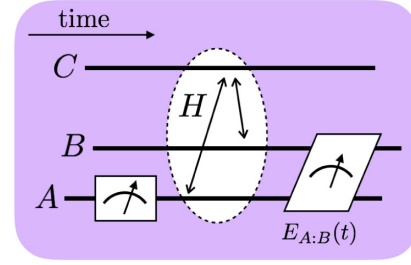


FIG. 2. Proposed protocol detecting nonclassicality of inaccessible object C . Each system A , B , and C can be an open system, with its own local environment. The protocol makes no assumptions about the initial tripartite state and the explicit expression for the Hamiltonian $H_{AC} + H_{BC}$ and has the following steps: (i) von Neumann measurement in subsystem A (or any entanglement-breaking channel); (ii) evolution of the whole ABC ; (iii) entanglement estimation of AB . We show in the main text that nonzero entanglement reveals positive discord $D_{AB|C}$.

measurement. An arbitrary measurement is allowed, and the experimentalist should choose the one having the potential for the biggest entanglement gain. Note that the measurement results need not be known. Our main detection method is illustrated and summarized in Fig. 2. Note that entanglement estimation in step (iii) can be realized with entanglement witnesses [23,24], rendering state tomography unnecessary. (See Supplemental Material [12] for a criterion based on a comparison between entanglement and initial purities of the probes).

Optomechanics.—We address now the practical implications of our criteria for scenarios of current technological relevance. In particular, we consider experiments of cavity optomechanics [25] as the paradigm of an open mesoscopic quantum system for which the criteria identified above hold the potential to be practically significant. In fact, one of the goals of optomechanics is to infer the nonclassicality of the state of a massive mechanical system, in a similar spirit as “certification” in Refs. [26,27], without affecting its (in general, fragile) state. A possible setting for such a task is given by a so-called membrane-in-the-middle configuration, where a mechanical oscillator (*a membrane*) is suspended at the center of a two-sided optical cavity [28]. By driving the cavity with laser fields from both its input mirrors, respectively, we realize a situation completely analogous to that in Fig. 1 (cf. Fig. 3). We now show that our scheme detects nonclassicality of the membrane without measuring it.

The interaction Hamiltonian for the setup in Fig. 3 reads [28]

$$H_{\text{int}} = -\hbar G_{0a} a^\dagger a q + \hbar G_{0b} b^\dagger b q. \quad (9)$$

This is complemented by local terms affecting each subsystem individually (cf. Supplemental Material for the details [12]). Here a and b are the annihilation operators for the respective fields, q is the dimensionless positionlike

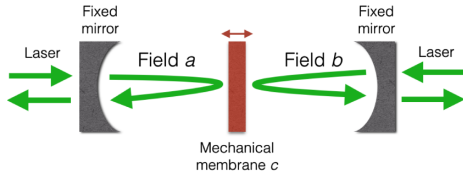


FIG. 3. Optomechanics setup. The mechanical membrane c is mediating interaction between driven cavity fields a and b . The membrane is interacting with its local environment at temperature T resulting in the Brownian motion, and the fields are independently interacting with their respective driving lasers through the fixed mirrors.

quadrature of the membrane, and $G_{0a(b)}$ represents the strength of the coupling between field a (b) and the membrane. All the other interactions are local; i.e., a (b) is coupled to its own environment, and c is coupled to its thermal phonon reservoir, responsible for the Brownian motion of the membrane. Thus, our theorem directly applies here, and we can implement the detection method in Fig. 2.

In order to independently confirm the nonclassicality of the membrane and demonstrate that there is considerable entanglement to be detected, we now calculate the ensuing entanglement dynamics. We choose the logarithmic negativity to quantify entanglement. Starting from the experimentally natural state where c is in a thermal state and a and b are coherent states, we calculate the dynamics of $E_{a:b}$ and $E_{ab:c}$. As initially there is no entanglement, the first step in Fig. 2 can be omitted. The results of our analysis are presented in Fig. 4 for varying power of the right laser. The parameters used in our simulations all adhere to present-day technology [29]. We see that nonzero $E_{a:b}(\tau)$ is always accompanied by nonzero $E_{ab:c}$ at some time $(0, \tau)$. Note

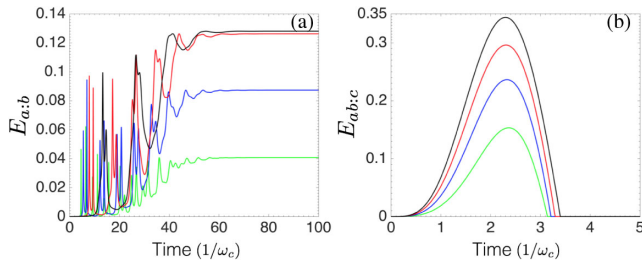


FIG. 4. Exemplary dynamics of entanglement (logarithmic negativity) $E_{a:b}$ and $E_{ab:c}$ for experimentally viable parameters. Mass of membrane 145 ng with a damping rate $2\pi \times 140$ Hz, temperature 0.3 K, and length of each cavity 25 mm with finesse 1.4×10^4 , and the wavelength of both lasers is 1064 nm. Here we fixed $P_a = 100$ mW, $\Delta_a = \omega_c$, and $\Delta_b = -\omega_c$, where $\omega_c = 2\pi \times 947$ kHz is the natural frequency of the membrane. We vary $P_b = 20$ (green lines), 40 (blue lines), 60 (red lines), and 80 mW (black lines). $P_{a(b)}$ stands for the power of the left (right) laser, and $\Delta_{a(b)}$ is its effective detuning (see Supplemental Material [12] for detailed calculations). Note that nonzero entanglement between the fields implies that the membrane is entangled with them in the process.

that entanglement is a stronger type of quantum correlation than discord. We have also performed similar calculations by varying the power of the left laser as well as the frequencies of the lasers within experimentally accessible ranges and observed consistent results (see Supplemental Material [12]).

System-environment correlations.—As a second relevant application of our study, let us consider again a closed-system dynamics and, in line with the assumed inaccessibility of the mediator, focus the attention to the probes only. We could thus think of C as an environment in contact with the open system AB . A vast body of literature exists on the study of the influence of initial system-environment correlations (SECs) on the evolution of the open system [30]. Proposals for the detection of SECs based on monitoring the dynamics of distinguishability [31–35] or purity [36,37] of the open system have been put forward. Such proposals have been implemented experimentally by means of quantum tomography [38,39]. Moreover, the possible nonclassical nature of SECs was linked to the impossibility of describing the evolution of an open system through completely positive maps [40]. Hence, detection schemes of quantum discord in the initial system-environment state have been proposed [41,42] and recently assessed experimentally [43–45].

Our scheme in Fig. 2 can also be used to reveal SECs, with the advantage that state tomography is not necessary. This is achieved by dividing the open system into A and B parts and monitoring the presence of entanglement between them. If one is interested only in the detection of correlations between AB and C , regardless of whether they are classical or not, the entanglement-breaking channel in Fig. 2 can be omitted. Indeed, for the initially uncorrelated state $\rho_0 = \rho_{AB} \otimes \rho_C$, we have $E_{A:BC}(\rho_{AB} \otimes \rho_C) = E_{A:B}(\rho_{AB})$, and no entanglement gain in AB is possible via classical C . Therefore, if one observes a gain, it would either be $\rho_0 \neq \rho_{AB} \otimes \rho_C$ or $D_{AB|C} > 0$ at some time. Both cases show correlations between AB and C . Finally, we note that previous schemes detect the nonclassicality of the system [41,42], i.e., presence of $D_{C|AB}$, whereas our schemes ascertain the nonclassicality of the environment, $D_{AB|C}$, which is perhaps a prime example of an inaccessible object.

Other applications.—A similar analysis can be done for remote quantum dots in a solid-state substrate [46] or spin-chain systems like in Ref. [47], as their physics also naturally distinguishes a mediating object that is inaccessible, e.g., locations of unpaired spins are unknown in a sample [47]. In a visionary perspective, system C could even be a gravitational field coupling massive systems A and B , which are mutually noninteracting. By determining experimentally the entanglement gain between A and B , one would conclude, according to our scheme, the nonclassical nature of the gravitational field between them. That is, if we were to embed into the quantum formalism description of the masses and the field, there would have to be nonorthogonal states in

the Hilbert space of the field, as this is required for the quantum discord $D_{AB|C}$ to be nonzero.

Conclusions.—We have proposed an entanglement-based criteria for the inference of nonclassicality of an inaccessible object. Our protocols are fully nondisruptive of the state of the system to probe and rely on only weak assumptions on the nature of the interactions involved. They are also robust against decoherence. These features make our proposal suitable to address nonclassicality at many levels, from experimentally relevant technological platforms such as quantum optomechanics to fundamental problems on the nature of gravity.

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