

## Two-Dimensional Pulse Propagation without Anomalous Dispersion

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Anomalous dispersion is a surprising phenomenon associated with wave propagation in an even number of space dimensions. In particular, wave pulses propagating in two-dimensional space change shape and develop a tail even in the absence of a dispersive medium. We show mathematically that this dispersion can be eliminated by considering a modified wave equation with two geometric spatial dimensions and, unconventionally, two timelike dimensions. Experimentally, such a wave equation describes pulse propagation in an optical or acoustic medium with hyperbolic dispersion, leading to a fundamental understanding and new approaches to ultrashort pulse shaping in nanostructured metamaterials.

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**Introduction.**—A light pulse traveling through a 3-D medium (3 is the number of space dimensions) changes its shape and develops a tail as it propagates if different frequency components of the wave travel at different speeds. This pulse broadening, called *dispersion*, occurs in a medium with a frequency-dependent refractive index. Dispersion of light does not occur in 3-D in a vacuum.

In contrast, in 2-D (three-dimensional *space-time*) a pulse changes its shape and forms a tail in a *vacuum* even though the wave speed  $c$  is a frequency-independent constant. This dispersion in vacuum in 2-D cannot be explained in terms of a variable wave speed, and so this phenomenon is called *anomalous dispersion* [1,2]. Anomalous dispersion in vacuum is a geometrical phenomenon associated with the dimension of space in which the waves propagate and it occurs for all kinds of waves, electromagnetic and acoustic. It occurs in two-dimensional but not in three-dimensional space.

The rumbling of thunder provides physical evidence of anomalous dispersion. If we model a lightning bolt as a long (approximately translationally invariant) vertical line source, the sound waves that are produced propagate outward as *two-dimensional* cylindrical waves. Although the lightning bolt is instantaneous, an observer does not hear a “bang,” but rather a rumbling that fades like  $1/t$ , as sound emanating from successively more distant sections of the lightning bolt reach the observer [3,4]. This constitutes an intuitive explanation of anomalous dispersion. For electromagnetic waves, this implies that for 2-D waveguides conventional ways to compensate ultrashort pulse broadening will not succeed even if all chromatic dispersion is carefully compensated.

Classic works on mathematical physics mention the fundamental role of the number of spatial dimensions in determining the existence or absence of anomalous dispersion [1,2]. The present work extends this consideration, showing that the number of time dimensions can also

play a role. Additional effective timelike dimensions, identified by a different sign in the corresponding term of the wave equation, can be introduced when waves propagate in metamaterials with hyperbolic dispersion [5]. In this Letter, we show that anomalous dispersion can be completely eliminated if we consider a modified wave equation that can be realized in hyperbolic metamaterials, both electromagnetic and acoustic, and is mathematically analogous to a 2-D wave equation with two timelike dimensions instead of one.

Electromagnetic fields in a nondispersive and nonmagnetic anisotropic material with a uniaxial dielectric permittivity tensor  $\bar{\epsilon} = \text{diag}(\epsilon^x, \epsilon^x, \epsilon^z)$  obey the wave equation [5]:

$$u_{tt} = \frac{c^2}{\epsilon^z} (u_{xx} + u_{yy}) + \frac{c^2}{\epsilon^x} u_{zz},$$

where subscripts denote partial derivatives, the scalar field  $u \equiv E^z$  represents the  $z$  component of the electric field, and  $c$  is the speed of light. This equation becomes the usual homogeneous 3-D wave equation when  $\epsilon^x = \epsilon^z$ . However, it can also represent the wave equation in two time and two space dimensions when  $\epsilon^x$  is negative and  $\epsilon^z$  is positive. Such materials are called *hyperbolic materials*. Anisotropic materials with diagonal components of the effective permittivity tensor having opposite signs occur naturally [6,7] or can be constructed as metal-dielectric multilayers [8]. Light propagates inside them as cones [9,10] and their two-time character has been studied [5,11,12], raising the intriguing possibility of observing dispersionless propagation in 2-D. Hyperbolic metamaterials exist for acoustic waves too [13]. Space-times with two time dimensions also occur in  $M$ -theory [14].

The homogeneous linear wave equation

$$u_{tt} = c^2 \nabla^2 u \quad (1)$$

describes waves  $u(\mathbf{x}, t)$  that travel with constant (frequency-independent) wave speed  $c$  through a uniform medium. This wave equation describes how an initial pulse at  $t = 0$  given by the initial conditions

$$u(\mathbf{x}, 0) = q(\mathbf{x}), \quad u_t(\mathbf{x}, 0) = p(\mathbf{x}) \quad (2)$$

evolves into the wave  $u(\mathbf{x}, t)$  at time  $t$ . This initial-value problem for the wave equation has an explicit quadrature solution in any space dimension.

Wave propagation in odd-dimensional space is fundamentally different from wave propagation in even-dimensional space. When the space dimension  $D$  is odd and  $D > 1$ , waves obey *Huygens's principle* [1,2]; that is, waves created by an instantaneous point source at  $t = 0$  (e.g., a light pulse) take the form of an expanding bubble. An observer sees blackness until the wave arrives, sees an instantaneous flash as the wave passes by, and immediately afterward sees blackness again [see Fig. 1(a)]. Such a wave propagates on the surface of the light cone. In contrast, in even-dimensional space an instantaneous point source gives rise to a wave that develops a tail. An observer sees blackness until the wave arrives and then sees a flash. However, the medium does not immediately return to quiescence; rather, the wave amplitude decays to 0 like  $t^{-\alpha}$ , where  $\alpha > 0$  depends on  $D$ . When  $D = 2$ , the wave amplitude decays to 0 like  $1/t$ . The tail of the wave propagates less rapidly than  $c$  as a consequence of anomalous dispersion. Such a wave propagates on the surface *and in the interior* of the light cone [see Fig. 1(b)]. Therefore, in a 2-D vacuum, *Huygens's principle* does not apply as there is anomalous dispersion [1,2].

*2-D wave propagation without anomalous dispersion.*—One might wonder whether it is possible to produce a medium whose dispersive properties exactly cancel the anomalous dispersion that occurs in 2-D wave propagation. Such a medium actually exists. The effect of anomalous dispersion can be canceled if we modify the 2-D wave equation by adding lower derivatives in time:

$$u_{tt} - t^{-1}u_t + t^{-2}u = c^2(u_{xx} + u_{yy}). \quad (3)$$

The one-derivative term models gain, which speeds up the lagging tail of a 2-D wave. However, this term is too strong, so to reduce its effect we also introduce the time-dependent term  $t^{-2}u$ . For this artificial medium a flash bulb gives rise to a wave that does not disperse; the wave remains confined to the surface of the light cone and does not leak into the interior of the light cone [see Fig. 1(c)]. Below we demonstrate analytically this dispersionless propagation. Furthermore, we show that the wave equation (3) is equivalent to a *constant-coefficient* wave equation in *two* space and *two* time dimensions.

*1-D homogeneous wave equation.*—The 1-D wave equation is special because its solutions obey Huygens's

principle when  $p = 0$ ,  $q \neq 0$ , and they exhibit anomalous dispersion when  $q = 0$ ,  $p \neq 0$ . The general solution  $u(x, t) = f(x + ct) + g(x - ct)$  to the 1-D wave equation is a superposition of two waves of unchanging shape traveling at constant speed  $c$ , one moving to the left and the other moving to the right. The exact solution satisfying the initial conditions (2) is given by *D'Alembert's formula*:

$$u(x, t) = \frac{q(x + ct) + q(x - ct)}{2} + \frac{1}{2c} \int_{x-ct}^{x+ct} ds p(s). \quad (4)$$

*3-D homogeneous wave equation.*—The exact solution  $u(\mathbf{x}, t)$  to the initial-value problem for the 3-D wave equation  $u_{tt} = c^2(u_{xx} + u_{yy} + u_{zz})$  is obtained by using a construction invented by Kirchhoff. The quadrature solution is given compactly by *Poisson's formula* [1,2]:

$$u(x, y, z, t) = \frac{\partial}{\partial t} (t\omega_{ct}[q]) + t\omega_{ct}[p]. \quad (5)$$

The *spherical mean*  $\omega_{ct}[\phi]$  of the function  $\phi(x, y, z)$  is an integral over the surface of a three-dimensional sphere of radius  $ct$  centered at  $(x, y, z)$ :

$$\omega_{ct}[\phi] \equiv \int_{\alpha^2 + \beta^2 + \gamma^2 = 1} \frac{d\Omega}{4\pi} \phi(x + ct\alpha, y + ct\beta, z + ct\gamma), \quad (6)$$

where  $d\Omega$  is an infinitesimal solid angle. Equations (5) and (6) are the complete solution to (1) and (2).

*2-D homogeneous wave equation.*—The solution to the initial-value problem for the 2-D wave equation  $u_{tt} = c^2(u_{xx} + u_{yy})$  can be expressed in Poisson form (5), but now the 3-D spherical mean is a weighted average over the surface of a 2-D disk centered at  $(x, y)$ :

$$\omega_{ct}[\phi] \equiv \frac{1}{2\pi} \iint_{\alpha^2 + \beta^2 \leq 1} d\alpha d\beta \frac{\phi(x + ct\alpha, y + ct\beta)}{\sqrt{1 - \alpha^2 - \beta^2}}. \quad (7)$$

We derive the integral in (7) from (6) by applying Hadamard's *method of descent* in which we project from  $D = 3$  down to  $D = 2$  by assuming that  $\phi(x, y, z)$  is independent of  $z$ .

The method of descent may be used to project the 2-D solution (5) and (7) down to  $D = 1$ , allowing us to recover D'Alembert's solution (4) to the 1-D wave equation. To do so we choose  $\phi(x, y)$  in (7) to be independent of  $y$ .

*Verification of Huygens's principle in 3-D.*—The 3-D solution  $u(x, y, z, t)$  in (5)–(6) depends on values of  $q$  and  $p$  only at points that are *exactly* a distance  $ct$  from  $(x, y, z)$ . Points that are further from  $(x, y, z)$  than  $ct$  do not affect the solution  $u(x, y, z, t)$  because wave disturbances from such points cannot travel faster than  $c$ . However, the fact that points that are *closer* to  $(x, y, z)$  than  $ct$  also do not affect the solution  $u(x, y, z, t)$  is a surprise because there is ample time for waves emanating from these nearby points to reach

the point  $(x, y, z)$ . It is this feature of 3-D wave propagation that leads to Huygens's principle. As stated earlier, 1-D and 2-D wave propagation do not obey Huygens's principle.

*Solution to the initial-value problem for a 3-D point disturbance.*—Consider a 3-D medium that is initially quiescent and suppose that at  $t = 0$  there is a momentary light pulse at the origin. We represent such a disturbance by

$$u(x, y, z, 0) = 0, \quad u_t(x, y, z, 0) = \delta(x)\delta(y)\delta(z). \quad (8)$$

To see how this disturbance propagates in time, we substitute (8) into Poisson's formula (5) and evaluate the integral in (6). We obtain the spherical wave form  $u(x, y, z, t) = (1/4\pi cr)\delta(r - ct)$  where  $r = \sqrt{x^2 + y^2 + z^2}$ . This 3-D wave resulting from the point disturbance (8) is precisely the expected expanding bubble. An observer at a distance  $r$  from the 3-D point disturbance waits a time  $t = r/c$  and then detects a momentary flash followed by total quiescence. There is no remnant of this disturbance when  $t > r/c$ .

*Anomalous dispersion in 1-D.*—Huygens's principle does not hold in 1-D because D'Alembert's solution (4) for  $u(x, t)$  depends on  $p(s)$  for  $x - ct \leq s \leq x + ct$  and not just on  $p(x + ct)$  and  $p(x - ct)$ . To see this, consider the evolution of a 1-D point disturbance given by the initial conditions  $q(x) = u(x, 0) = 0$ ,  $p(x) = u_t(x, 0) = \delta(x)$ . D'Alembert's formula (4) shows that these initial conditions spawn a wave in the form of a two-sided step function  $u(x, t) = (1/2c)\theta(ct - |x|)$ . An observer at  $x$  must wait a time  $t = |x|/c$  before the pulse arrives. After the wavefront passes, the medium does not return to its initially quiescent state; an upward displacement of  $1/(2c)$  persists for all time. Thus, 1-D wave propagation violates Huygens's principle [1,2].

There is a special class of initial conditions,  $p(x) = u_t(x, 0) = 0$ , that creates waves that do obey Huygens's principle. This is because waves arising solely from an initial displacement leave the medium quiescent after they have passed. For example, consider the initial conditions

$$u_t(x, 0) = 0, \quad u(x, 0) = \begin{cases} 1 - |x| & (|x| < 1), \\ 0 & (|x| \geq 1). \end{cases}$$

These initial conditions correspond to an initially triangular transverse displacement. D'Alembert's formula shows that the initial pulse splits into a right-going and a left-going triangular pulse, each having half the initial amplitude. The two pulses travel to the right and to the left with speed  $c$  and do not change their shape.

*2-D waves and anomalous dispersion.*—To show that 2-D wave propagation does not obey Huygens's principle, we examine the effect of a 2-D point disturbance:

$$u(x, y, 0) = 0, \quad u_t(x, y, 0) = \delta(x)\delta(y). \quad (9)$$

This pulse disturbance is an idealization of the initial conditions created, for example, by a fish surfacing on a quiet pond. Substituting (9) into (7) and (5), we obtain

$$u(x, y, t) = \frac{\theta(c^2 t^2 - x^2 - y^2)}{2\pi c \sqrt{c^2 t^2 - x^2 - y^2}}. \quad (10)$$

This formula describes a circular wave that propagates outward at speed  $c$ . An observer at  $x^2 + y^2 = r^2$  must wait until time  $t = r/c$  before the leading edge of the wave front arrives. The value of  $u$  at the leading edge is infinite, but after the wavefront passes, the trailing wave decays like  $1/t$  for large  $t$ .

This explains why thunder rumbles, as mentioned previously. The surface of the earth is locally flat and two dimensional. Assuming the lightning is a vertical line source, the sound of thunder propagates across the surface in the same way that the wave in (10) propagates from a pointlike disturbance, with a trailing rumbling. An alternative way to understand the rumbling is to view it as a 3-D wave in which the observer on the ground hears waves of the form (8) that emanate from successively higher and more distant points on the lightning bolt as  $t$  increases [3,4]. This explains the continued and tapering rumbling, and encapsulates the physical intuition behind the mathematics of Hadamard's method of descent which results in anomalous dispersion. The analogy breaks when considering the finite height of the lightning bolt, so that after some time the rumbling abruptly stops. One would not expect to notice this for the light wave, because the light speed is about a million times greater than the sound speed.

*Large-time asymptotic behavior of 2-D waves.*—The  $1/t$  decay in 2-D space of a trailing wave, as in (10), is a general property of any wave created by a localized disturbance. To show this, substitute  $a = ct\alpha$  and  $b = ct\beta$  into (7). Assuming that  $\phi$  has compact support, then

$$t\omega_{ct}[\phi] \sim \frac{1}{2\pi c^2 t} \iint_R dadb\phi(a, b) \quad (t \rightarrow \infty). \quad (11)$$

If the integral in (11) exists and is nonzero, then  $t\omega_{ct}[\phi] \propto 1/t$  as  $t \rightarrow \infty$ .

*Solution to the modified wave equation (3).*—This wave equation is singular at  $t = 0$ , so we do not try to solve (3) for the general initial conditions (2). However, we impose the special initial conditions

$$u(x, y, 0) = q(x, y) = 0, \quad u_t(x, y, 0) = p(x, y)$$

by following the Kirchhoff construction procedure. We rewrite (3) in spherical coordinates and seek radially symmetric solutions  $u(x, t)$  to

$$u_{tt} - t^{-1}u_t + t^{-2}u = c^2(u_{rr} + r^{-1}u_r). \quad (12)$$

We verify by direct differentiation that

$$u(r, t) = \frac{1}{2\pi c} \delta(r - ct) \quad (13)$$

exactly solves (12). This verification requires the use of the identity  $\delta(x) + x\delta'(x) = 0$ . By superposing solutions of this form, we construct a large class of solutions to (3) containing one arbitrary function of two arguments:

$$u(x, y, t) = t\omega_{ct}[p], \quad (14)$$

where

$$\omega_{ct}[\phi] \equiv \frac{1}{2\pi} \int_0^{2\pi} d\theta \phi(x + ct \cos \theta, y + ct \sin \theta). \quad (15)$$

Equation (14) is the exact solution to (3) for the initial conditions (2) with  $q(x, y) = u(x, y, 0) = 0$ . Of course, (14) is not the general solution to (3) because the general solution contains *two* arbitrary functions of two arguments each. We cannot express the general solution to (3) in Poisson form because (12) is time dependent, and thus the time derivative of a solution is not a solution.

Huygens's principle holds for this special initial condition because  $\omega_{ct}[\phi]$  in (15) is determined by the values of  $\phi(x_0, y_0)$  on the *surface* but not in the *interior* of the 2-D light cone  $(x - x_0)^2 + (y - y_0)^2 = c^2 t^2$ . Indeed, (13) is the expanding-bubble wave created by a flash bulb. There is no ringing effect if  $q = 0$ . This is the reverse of what happens for the 1-D wave equation where Huygens's principle holds if  $p = 0$ .

*Computational simulation of anomalous dispersion.*—The analytical arguments above can be verified by solving the equations numerically. We have used the differential equation solver COMSOL MULTIPHYSICS to implement (1) for three spatial dimensions [Fig. 1(a)], two spatial dimensions [Fig. 1(b)], and for modified wave equation (3)

[Fig. 1(c)]. A delta-function localized initial condition used in the derivations above in (8) and (9) cannot be modeled numerically. Instead, we use a Gaussian distribution with a spatial width, details of which are given in the caption. Such a field distribution represents an ultrashort broadband pulse or a unipolar wave packet [15] (a narrowband pulse would have field oscillations and the effect of this would be to nearly cancel the  $1/t$  tail that is characteristic of anomalous dispersion in 2-D space). Our simulations confirm the analytical results and show that solutions to the wave equation (1) in 3-D obey Huygens's principle, while in 2-D they exhibit anomalous dispersion. However, the modified 2-D wave equation (3) also obeys Huygens's principle with no anomalous dispersion and no trailing tail behind the pulses.

*Wave equation with two space and two time dimensions.*—The change of variable  $u(x, y, t) = tv(x, y, t)$  converts (3) to the wave equation  $v_{tt} + t^{-1}v_t = c^2(v_{xx} + v_{yy})$ . The left side of this equation is the two-dimensional *radial time derivative* of the wave equation

$$v_{\alpha\alpha} + v_{\beta\beta} = c^2(v_{xx} + v_{yy}), \quad (16)$$

where  $\mathbf{t} = (\alpha, \beta)$  is a *two-dimensional* vector time variable. This shows that anomalous dispersion does not occur for solutions to a linear homogeneous wave equation in a space of two time dimensions and two space dimensions as long as the initial disturbance has a vanishing derivative. Of course, the wave equation (16) contains four independent variables, while (3) only contains three. Therefore, the solutions that we proved to obey Huygens's principle and are free of anomalous dispersion obey the constraint of radial symmetry in the  $\alpha$  and  $\beta$  variables, which reduces the number of independent variables back to three. Although we do not have a general proof for dispersion-free solutions when this symmetry is broken, the Green function approach allowing evaluation of fields from point sources in any media [16,17] suggests that the effect is robust with respect to breaking this symmetry.

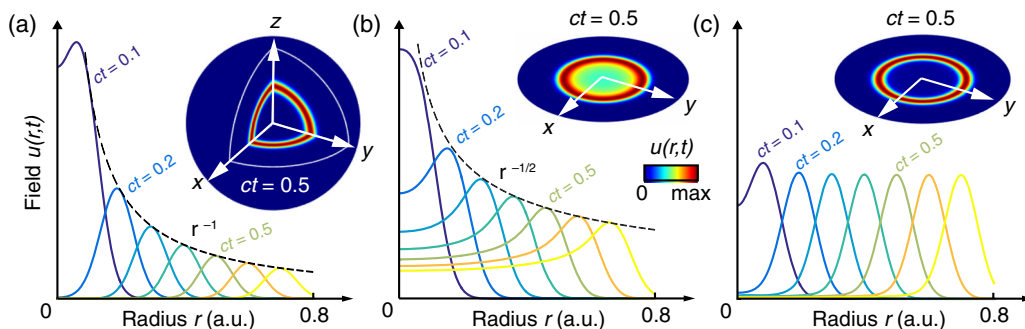


FIG. 1. Numerical solution of the wave equation in three cases: (a) conventional 3-D wave equation  $u_{tt} = c^2(u_{xx} + u_{yy} + u_{zz})$ ; (b) conventional 2-D wave equation  $u_{tt} = c^2(u_{xx} + u_{yy})$  showing anomalous dispersion; (c) modified two-dimensional wave equation  $u_{tt} - (1/t)u_t + (1/t^2)u = c^2(u_{xx} + u_{yy})$  with no anomalous dispersion. The insets show the solution at a fixed time. The initial conditions are zero initial value  $u(r, t = 0) = 0$  and a radially symmetric initial time derivative Gaussian distribution  $u_t(r, t = 0) = \exp[-r^2/(2\sigma_r^2)]$  with  $\sigma_r = 0.01$ . The radial variable is  $r = \sqrt{x^2 + y^2 + z^2}$  for (a) and  $r = \sqrt{x^2 + y^2}$  for (b) and (c).

Since the 3-D wave equation takes the form in (16) inside hyperbolic metamaterials, these anisotropic media behave like a two-dimensional space with no anomalous dispersion. Certain factors in a realistic physical system could hinder the practical elimination of dispersion: material dispersion, losses present in typical plasmonic-based realizations, finite size of the unit cells in the metamaterial, and existence of other field polarizations which do not obey the same wave equation inside the metamaterial. Some of these limitations may be relaxed in natural hyperbolic materials.

In conclusion, we have explained how the number of spatial dimensions affects ultrashort pulse propagation and showed that pulse spreading is different in the 1-D, 2-D, and 3-D cases. In the 2-D case, pulse spreading takes place even if material dispersion is absent. Using a mathematical mapping between a wave equation (with time-dependent coefficients) in 2-D space and a wave equation in 2-D space with two timelike coordinates, we have shown that the latter has no anomalous dispersion. Since the wave equation in 2-D space with two timelike coordinates is mathematically identical to 3-D pulse propagation inside hyperbolic materials, we have shown a possible realization for pulse broadening to be eliminated. Recent developments in ultrashort broadband pulses present challenges in dispersion compensation to avoid pulse distortion during propagation. The theory developed here explains for the first time how metamaterials can evade the distortion of electromagnetic or acoustic pulses in 2-D space. This development could be important for ultrafast optical applications such as in telecommunications, nonlinear optics, and basic physics research, as well as acoustic and optomechanical applications involving ultrashort pulses.

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