Emergent $1/f$ Noise in Ensembles of Random Telegraph Noise Oscillators

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The emergence of $1/f$ noise from an aggregate of $1/f^2$ noise signals in magnetic nanodots undergoing random telegraph oscillations in their magnetization is reported. This emergence is found to occur with as few as two random telegraph noise (RTN) oscillators producing $1/f$ noise across two decades of frequency bandwidth, and with fewer than ten such oscillators producing $1/f$ noise across over four decades. The RTN fluctuations observed are as small as one part in 10 000 compared to dc voltage signals but still generate easily observable $1/f$ noise at up to 10^5 Hz. These observations may explain the historic difficulty in identifying RTN oscillator sources of $1/f$ noise.

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Low-frequency $1/f$ noise is perhaps both the most interesting, and vexing of all observed noise; its ubiquity is well known, having been observed in systems ranging from voltage fluctuations in electronic systems [1–3], quasar intensities [4], neuron firing events [5], the flood levels of the Nile river [6], various magnetic systems [7,8], and even in music and speech [9]. Although discovered decades ago, it continues to attract significant interest [10–12] with most theoretical work to understand the physical mechanism(s) behind $1/f$ noise focusing on semiconductor systems, with little generality allowing extension to other systems [3]. The most widely accepted explanation to date is van der Ziel's picture of a $1/f$ noise spectrum arising from a distribution of activated processes producing Lorentzian spectra. He showed mathematically that a distribution of Lorentzian spectra, as might arise from a collection of nonidentical random telegraph noise (RTN) oscillators, could produce a $1/f$ signal in the aggregate by imposing rather reasonable restrictions on the distribution [13]. This model has significant appeal in that it addresses the major concern of the divergence of the power spectral density (PSD) at zero frequency for $1/f$ noise [1,14]. However, despite its mathematical appeal, an explicit, unambiguous experimental demonstration of this emergent behavior in a physical system with well characterized individual RTN oscillators collectively exhibiting $1/f$ noise has yet to be achieved in the over 65 years since the model's postulation.

In this Letter, we present data exhibiting an explicit emergence of $1/f$ noise from fewer than ten well-characterized two level oscillators exhibiting RTN [15]. The RTN Lorentzian spectra, which have $1/f^2$ noise signatures, arise from magnetization fluctuations in mesoscale square permalloy dots. The magnetization fluctuations are observed through resistance measurements, with the resistance fluctuations due to the anisotropic magnetoresistance (AMR) [16] of the dots. The sample geometry allows AMR measurements on both individual dots, and on multiple dots in series. Noise spectra with $1/f$ dependence are shown to emerge in the aggregate from a small number of RTN oscillators, demonstrating the robustness of the $1/f$ evolution from $1/f^2$ noise; the van der Ziel explanation is formulated for a continuum of Lorentzian signals, but our measurements show that $1/f$ signals can emerge over many decades of frequency space for fewer than ten constituent Lorentzians, and two decades of $1/f$ noise was observed for only two dots exhibiting RTN. We also note the size of the individual RTN voltage signal necessary to generate $1/f$ noise in our systems is quite small compared to the total sample dc voltage; RTN amplitudes on the order of 10^{-4} the dc voltage signal generate easily measurable $1/f$ noise over several decades of frequency space at frequencies up to 10^5 Hz. Both the few RTN oscillators needed and the low signal magnitude from each give a possible explanation on why it has been difficult determining the source of $1/f$ noise in the vast majority of systems.

We also examine the range of energy barriers the van der Ziel model would predict necessary to generate $1/f$ noise, and compare it to our experimentally determined barrier height distribution for the individual dots [17]. There have been numerous studies of RTN and $1/f$ noise in semiconductors and in magnetic tunnel junctions [1,18,19], as well as observation of both $1/f$ and RTN in other systems [20], but an unambiguous quantification of the individual, discrete RTN signals leading to the $1/f$ noise spectrum has not yet been explicitly demonstrated.

In what follows, we describe the fabrication process of the magnetic dot samples measured in this work, followed by our experimental methods used to measure the fluctuating magnetization of both single dots and collections of dots. Next we present data showing aggregate $1/f$ noise arising from collections of single dot RTN signals before applying a statistical analysis of multiple samples to compare to the van der Ziel model. Finally, we discuss the ramifications of our results in the broader context of $1/f$ noise in general.

Data from this work were acquired from 10 samples, with a given sample consisting of between 2 to 81 dots. The samples were fabricated on Si_3N_4 coated Si substrates, patterned through a two-step liftoff process. Arrays of 250 nm square dots with 1 μm center-to-center spacing were patterned into single-layer resist using electron beam lithography at 100 keV using a Vistec EBPG 5000+ system. Following development of the resist, a 3 nm seed layer of Ta, a 10 nm layer of permalloy (Py) ($Ni_{80}Fe_{20}$), and a 3 nm capping layer of Ru were deposited by dc sputtering under zero applied magnetic field. After liftoff, substrates were placed on a rotating stage, and a short ion mill step at a steep angle (75°) to the substrate normal was performed to remove fencing from the dots that could cause unwanted shape anisotropy energy. Finally, nonmagnetic contacts consisting of a 5 nm W/15 nm Au bilayer were patterned by a similar process. The final sample geometry is shown in the inset in Fig. 1(a) with larger collections of dots made by daisy chaining up to 9 of the 9 dot samples.

The magnetization fluctuations of the dots were observed through resistance measurements, with resistance values corresponding to the net magnetization direction through the AMR. The resistance is related to the magnetization direction by

$$R(\theta) = R_\perp + (R_\parallel - R_\perp)\cos^2\theta, \quad (1)$$

where θ is the angle between the magnetization and the current, and $R_\perp (R_\parallel)$ is the resistance when the current and magnetization are perpendicular (collinear) [16]. We define the current to be in the positive x direction, so θ may be interpreted simply as the direction of the net magnetization. As seen in the Fig. 1 inset, for current contacts at the ends of a chain, a judicious choice of voltage contacts allows a four-terminal resistance measurement [21] on any individual dot, or any collection of adjacent dots. To measure the fluctuations, a dc current of ~300 μA was supplied by a battery powered source. The resulting voltages were amplified by a Stanford Instruments SR560 preamp before being measured by both a Tektronix DPO4012b oscilloscope to record time records, and by a Hewlett-Packard 35670A spectrum analyzer to record noise PSD. The preamp was ac coupled using a discrete passive filter at a roll-off frequency of 0.5 Hz to block the large dc voltage component, while still remaining sensitive to jumps in the magnetization, which would be manifest as jumps in resistance, as per Eq. (1).

For the dots discussed here, their configurational anisotropy energies [22,23] are similar to those previously reported by Endean et al. [17], with the easy axes of the magnetization perpendicular to the sides of the square dots.

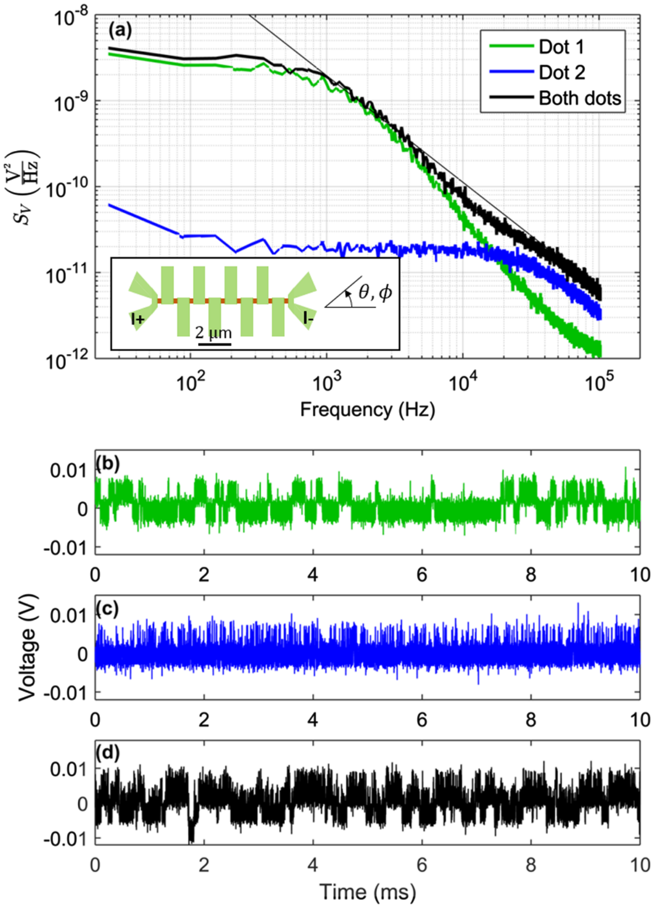

FIG. 1. PSD and voltage vs time data for two dots undergoing RTN for an applied field of 3.5 mT along the dot diagonal. The individual dots show clear RTN in (b) and (c), with the expected Lorentzian PSDs in (a). The aggregate time record (d) has the black PSD shown in (a). The fitted line has a slope of −1.24, within the accepted definition of $1/f$ noise [1,3]. The inset shows a schematic of the sample geometry; square dots (dark red) are electrically connected together by gold contacts (light green) entering the frame from the edges. By attaching the current leads shown, any other pair of contacts can be chosen as voltage leads to perform a 4-terminal resistance measurement across any combination of adjacent dots. The magnetization angle θ of a given dot and the applied field angle ϕ are both defined as shown.

The center-to-center spacing of 1 μm is sufficient to ensure that adjacent dots do not couple either magnetostatically, or through spin-torque when a current is applied. Using the configurational anisotropy model developed by Endean et al. [17], and relying on the Stoner-Wolfharth model [24], the energy landscape $E(\theta)$ is a function of both the configurational anisotropy and the applied field Zeeman energy, and in the simplest case is given by

$$E(\theta) = -\frac{E_A}{2}\cos(4\theta) - mH\cos(\theta - \phi), \quad (2)$$

where E_A is the height of the configurational anisotropy barrier, m is the magnetization of the dot, H is the applied field magnitude, ϕ is the field direction, and θ is the magnetization direction, with both angles measured relative to the x axis. For our dots with no applied field the height of the barrier along the dot diagonal, $\Delta E = E(\pi/4) - E(0) = E_A$, is large enough (several eV) to make any transition probability between two adjacent states (e.g., between $\theta = 0$ and $\theta = (\pi/2)$) vanishingly small at room temperature.

The application of a magnetic field along the dot diagonal ($\theta = (\pi/4)$) lowers the energy barrier along the diagonal allowing the barrier to be made comparable to the thermal energy. In this case, the dwell time for a given state, τ_d, will be on experimental time scales [15]. This activated switching can be described by an Arrhenius law [13],

$$\tau_d = \tau_0 e^{\Delta E/k_B T}, \tag{3}$$

where τ_0 is an attempt time, k_B is the Boltzmann constant, T is the temperature, and ΔE is the height of the magnetic field controlled energy barrier [25]. These fluctuations or random switchings between these two states are well known as RTN [26,27]. A single RTN signal with a symmetric energy barrier exhibits a PSD $S_V(f)$ with a Lorentzian frequency dependence

$$S_V(f) = \frac{\Delta V^2}{2} \frac{\tau_d}{1 + (\pi \tau_d f)^2}, \tag{4}$$

where ΔV is the measured voltage difference between the two states, and τ_d is the average dwell time for either energy well [27], as in Eq. (3). The Lorentzian line shape is easily identified by its frequency independent region below τ_d^{-1}, and a $1/f^2$ dependence at frequencies far above τ_d^{-1}.

Because of slight variations of identically fabricated dots, their energy landscapes will vary, leading to a distribution of anisotropy energy values E_A in Eq. (2). Since the average dwell time τ_d depends exponentially on the height of the barrier ΔE as in Eq. (3), even a small range of E_A's will give a very large spread of τ_d's across dots for the same applied field.

An example of RTN signals in both the time and frequency domains for two different dots both individually and collectively are shown in Fig. 1. The dots have different energy landscapes at the same applied field, leading to two separate Lorentzians with different τ_d's. We note that, despite the two dots each exhibiting Lorentzian PSDs, at this particular applied field the aggregate PSD shows an approximate $1/f$ dependence over 2 decades of frequency.

A signal extending across the full range of our frequency measurement, about 4 decades, can be generated by chaining more dots together as shown in Fig. 2. This figure shows data from four different chains of nine dots, measured as both individual chains, and together in series. From the time records shown in Figs. 2(c)–2(f), we note that only six RTN signals compose the aggregate $1/f$ signal covering four decades of frequency, with the single-chain data shown in Figs. 2(d) and 2(f) each showing single RTN signals, and that from Figs. 2(c) and 2(e) showing two RTN signals each.

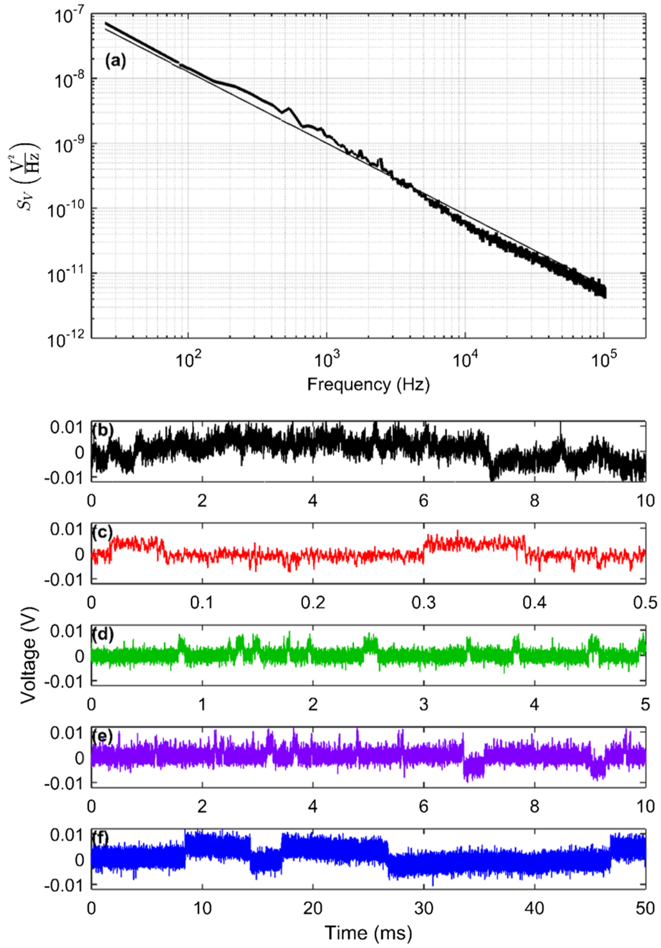

FIG. 2. PSD and voltage vs time data for four chains of nine dots, and the four chains in series, with an applied field of 3.5 mT along the dot diagonal. The composite PSD is shown in (a) with a fit line showing a slope of −1.1, and the corresponding time record is shown in (b). Time records (c)–(f) all show clear RTN signals, with (d) and (f) showing one RTN signal in the chain, and (c) and (e) showing two. We note that the slow voltage drifts in (f) such as the one seen between $t = 18$ and $t = 26$ ms are artifacts of the hi-pass filter on the measurement, which are visible over these long time scales. The roll-off frequency is over an order of magnitude lower than the lower frequency limit in (b).

The emergence of $1/f$ noise from an aggregate of Lorentzian signals was postulated by van der Ziel [13] by assuming a constant or flat distribution of activated processes with characteristic times τ_d [28]. He showed the PSD of those systems in aggregate will assume a $1/f$ dependence. Further, the range of ΔE's over which the distribution condition must be met can be relatively small and still produce a $1/f$ signal over many decades of frequency space, given Eq. (3)'s exponential dependence on ΔE. We note the requirement that the distribution of

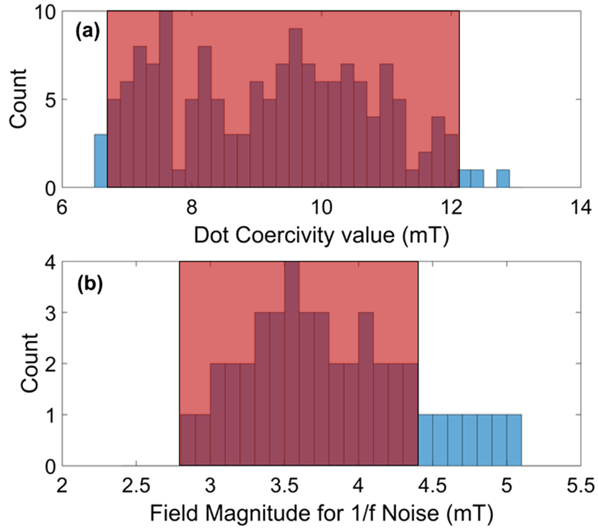

FIG. 3. For an approximately flat distribution of dot coercivities like the shaded region shown in (a), a range of applied fields where $1/f$ noise is expected can be predicted using Eq. (6). This prediction is depicted by the shaded region in (b), a histogram of the experimentally observed noise fields across samples.

energy barriers be flat can be relaxed to any slowly varying distribution [3,29] but for simplicity we proceed with the more restrictive flat distribution requirement.

Average dwell times in the range $10^{-6}\ \mathrm{s} < \tau_d < 1\ \mathrm{s}$ produce the $1/f$ signal shown in Fig. 2(a) over the frequency range $10\ \mathrm{Hz} < f < 10^5\ \mathrm{Hz}$ [13]. Assuming an attempt time of $\tau_0 = 10^{-11}$ s [30] and taking $T = 293$ K, inverting Eq. (3) for both dwell time extremes gives a range of $0.25\ \mathrm{eV} \leq \Delta E \leq 0.63\ \mathrm{eV}$, where the energy barrier distribution must be constant. We suppose that we observe $1/f$ in a chain of dots at an applied field of magnitude H_N and angle $\phi = (\pi/4)$. Assuming that the magnetization in a given dot is jumping between approximately 0 and $\pi/2$ and that all dots have the same magnitude of magnetization, the second term in Eq. (2) becomes a constant, and so we require only a constant distribution of E_A's between dots.

The configurational anisotropy energy, $|E_A|$ of one of our dots, with a magnetization m, can be determined as in Ref. [17] by straightforward measurement of easy-axis coercivities H_c through the relationship

$$|E_A| = \frac{mH_c}{8}. \tag{5}$$

A histogram of coercivity values measured in ∼150 individual dots is shown in Fig. 3, displaying a relatively flat distribution of coercivities between 6.8 and 12.0 mT. To determine whether this region would predict the $1/f$ noise we see in our samples, we first combine Eqs. (2), (3), and (5) to find the noise field value that corresponds to RTN switching at a given dwell time τ_d for a dot with easy axis coercivity, H_C, for a field applied at an angle of $\pi/4$:

$$H_N = \frac{2k_B T}{(2-\sqrt{2})m}\ln\frac{\tau_0}{\tau_d} + \frac{H_C}{8-4\sqrt{2}}. \tag{6}$$

Using the established value of 8×10^5 A/m for the magnetization of Py [31] and the dwell time limits mentioned above, our model predicts $1/f$ noise for applied fields between 2.8 and 4.5 mT. Figure 3(b) shows a histogram of field magnitudes at which $1/f$ noise has been observed at an applied field angle of $\pi/4$, which shows agreement with our predicted field regime.

Our results have several consequences for our understanding of $1/f$ noise in general. For a system exhibiting $1/f$ noise where specific RTN oscillators are not known, identifying individual RTN oscillators in time record measurements may be the exception rather than the rule. This explains why identifying the sources of $1/f$ noise has been so difficult. As we have shown, in a macroscopic sample displaying a $1/f$ spectrum there can be as few as one RTN oscillator per decade of frequency explored and thus the specific RTN oscillators can be extremely rare. To make this point more strongly, the largest $1/f$ PSD frequency range explored to date is slightly more than six decades [32,33]. For this range as few as six or seven RTN oscillators could in principle explain the observations [34]. This is compounded by the small signal size associated with the individual RTN oscillators. In our case the individual voltage differences between the RTN states (after gain removal) is approximately 50 μV, compared to the total voltage drop across a chain which is ≈1.5 V. Thus, for a typical voltage measurement like ours, very small (on the order of one part in 10 000) voltage fluctuations of a few RTN oscillators can result in a $1/f$ spectrum which is observable at high frequencies (up to 10^5 Hz in our experiments), contingent on the noise floor of the measurement, of course; in general, this greatly complicates identifying the RTN oscillators in other systems. Last, the evolution of $1/f$ noise from RTN oscillators removes the zero frequency divergence in the power spectrum as the RTN spectrum becomes flat at low frequencies [1].

In summary, we have demonstrated the emergence of $1/f$ noise from discrete RTN signals in collections of magnetic dots. The overall statistics of the dot energies agree with the van der Ziel picture of $1/f$ noise, but for any given collection of dots exhibiting $1/f$ noise, the number of RTN signals comprising the signal is fewer than 10, showing that this effect is more robust than the original picture would indicate. We also find both the signal size of an individual RTN oscillator and the number of RTN oscillators necessary to produce $1/f$ noise make it very difficult to identify the relevant RTN oscillators that might constitute any given $1/f$ signal.

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