Order and Symmetry Breaking in the Fluctuations of Driven Systems

N. Tizón-Escamilla,[1] C. Pérez-Espigares,[2,3] P. L. Garrido,[1] and P. I. Hurtado[1,*]

[1]*Departamento de Electromagnetismo y Física de la Materia, and Institute Carlos I for Theoretical and Computational Physics, Universidad de Granada, Granada 18071, Spain*

[2]*University of Modena and Reggio Emilia, via G. Campi 213/b, 41125 Modena, Italy*

[3]*School of Physics and Astronomy and Centre for the Mathematics and Theoretical Physics of Quantum Non-Equilibrium Systems, University of Nottingham, Nottingham, NG7 2RD, United Kingdom*

(Received 23 June 2016; revised manuscript received 7 October 2016; published 31 August 2017)

Dynamical phase transitions (DPTs) in the space of trajectories are one of the most intriguing phenomena of nonequilibrium physics, but their nature in realistic high-dimensional systems remains puzzling. Here we observe for the first time a DPT in the current vector statistics of an archetypal two-dimensional (2D) driven diffusive system and characterize its properties using the macroscopic fluctuation theory. The complex interplay among the external field, anisotropy, and vector currents in 2D leads to a rich phase diagram, with different symmetry-broken fluctuation phases separated by lines of first- and second-order DPTs. Remarkably, different types of 1D order in the form of jammed density waves emerge to hinder transport for low-current fluctuations, revealing a connection between rare events and self-organized structures which enhance their probability.

DOI: 10.1103/PhysRevLett.119.090602

Introduction.—The theory of critical phenomena is a cornerstone of modern theoretical physics [1,2]. Indeed, phase transitions of all sorts appear ubiquitously in most domains of physics, from cosmological scales to the quantum world of elementary particles. In a typical second-order phase transition, order emerges continuously at some critical point, as captured by an order parameter, signaling the spontaneous breaking of a symmetry and an associated nonanalyticity of the relevant thermodynamic potential. Conversely, first-order transitions are characterized by an abrupt jump in the order parameter and a coexistence of different phases [1,2]. In recent years, these ideas have been extended to the realm of fluctuations, where dynamical phase transitions (DPTs) (i.e., in the space of trajectories) have been identified in different systems, both classical [3–17] and quantum [18–21]. Important examples include glass formers [22–29], micromasers and superconducting transistors [30,31], or applications such as DPT-based quantum thermal switches [32–34].

DPTs appear when conditioning a system to have a fixed value of some time-integrated observable, such as, e.g., the current or the activity. The different dynamical phases correspond to different types of trajectories adopted by the system to sustain atypical values of this observable. Interestingly, some dynamical phases may display emergent order and collective rearrangements in their trajectories, including symmetry-breaking phenomena [5,9–11], while the large deviation functions (LDFs) [35] controlling the statistics of these fluctuations exhibit nonanalyticities and Lee-Yang singularities [36–43] at the DPT reminiscent of standard critical behavior. This is a finding of crucial importance in nonequilibrium physics, as these LDFs play a role akin to the equilibrium thermodynamic potentials for nonequilibrium systems, where no bottom-up approach exists yet connecting microscopic dynamics with macroscopic properties [3,4,44]. Moreover, the emergence of coherent structures associated with rare fluctuations implies in turn that these extreme events are far more probable than previously anticipated [11,45].

Despite their conceptual importance, observing DPTs is challenging, as the spontaneous emergence of large fluctuations in macroscopic systems is unlikely [3], so one may question their physical relevance. However, recent breakthroughs have shown that fluctuations admit a control-theory (or active) interpretation [3,46,47] where rare trajectories become typical under the action of an external control field. Among the fields that drive the system to the desired fluctuation, the one minimizing the dissipated energy is univocally related to the typical trajectory for the spontaneous emergence of such a fluctuation [3]. In this way, a DPT at the trajectory level corresponds to a singular change in the optimal control field, and this could be easily observed in actual experiments. In this sense, DPTs are not only of conceptual but also of practical importance, especially for realistic $d>1$ systems [28,29] amenable to control for technological applications. However, up to now most works on DPTs have focused on toy 1D models [9–21] or fluctuations of scalar (1D) observables in $d>1$ [22–32], and the challenge remains to understand DPTs in the fluctuations of fully vectorial observables in d dimensions and how they are affected by the (possible) system anisotropy.

In this Letter, we address this challenge and report compelling evidence of a rich DPT and new physics in the statistics of vectorial currents in an archetypal 2D driven diffusive system, the weakly asymmetric simple exclusion process (WASEP) [48]. To crack this problem, we use massive cloning Monte Carlo simulations for rare event statistics [49–51], together with the macroscopic fluctuation theory (MFT) to understand the fluctuation phase diagram [3]. We find a second-order DPT between a homogeneous fluctuation phase with structureless trajectories and Gaussian current statistics, and a non-Gaussian phase for small currents. This non-Gaussian phase is characterized by the emergence of coherent jammed states in the form of traveling-wave trajectories, thus breaking the spatiotemporal translation symmetry. Such jammed states, which are surprisingly extended and noncompact, hamper particle flow enhancing the probability of low-current fluctuations [10], and we introduce a novel order parameter for their detection. Interestingly, for mild or no anisotropy, different symmetry-broken phases appear (depending on the current vector) separated by lines of first-order DPTs, a degeneracy which disappears beyond a critical anisotropy. Dynamical coexistence of the different traveling-wave phases appears along these first-order lines.

Model.—The 2D WASEP belongs to a broad family of driven diffusive systems of fundamental and technological interest [3,4,11]. Microscopically, this model is defined on a 2D square lattice of size $N = L \times L$ with periodic boundaries where $M \leq N$ particles evolve, so the global density is $\rho_0 = M/N$. Each lattice site may contain at most one particle, which performs stochastic jumps to neighboring empty sites along the $\pm\alpha$ direction ($\alpha = x, y$) at a rate $r_{\pm}^{\alpha} \equiv \exp[\pm E_\alpha/L]/2$, with $\mathbf{E} = (E_x, E_y)$ being an external field. For large $\mathbf{E}$ and moderate system sizes, the field per unit length $\mathbf{E}/L$ is strong enough to induce an effective anisotropy in the medium [52], enhancing diffusivity and mobility along the field direction, an effect that can be accounted for in our theory below by an effective anisotropy parameter ϵ.

Trajectory statistics.—We are interested in the statistical physics of an ensemble of trajectories conditioned to a given total vector current $\mathbf{Q}$ integrated over a long time t. In the spirit of equilibrium statistical mechanics, this trajectory ensemble is fully characterized by a dynamical partition function $Z_t(\boldsymbol{\lambda}) = \sum_{\mathbf{Q}} P_t(\mathbf{Q}) e^{\boldsymbol{\lambda}\cdot\mathbf{Q}}$, where $P_t(\mathbf{Q})$ is the probability of trajectories of duration t with total current $\mathbf{Q}$, or equivalently by the associated dynamical free energy (DFE) $\mu(\boldsymbol{\lambda}) = \lim_{t\to\infty} t^{-1} \ln Z_t(\boldsymbol{\lambda})$. The intensive vector $\boldsymbol{\lambda}$ is conjugated to the extensive current $\mathbf{Q}$, in a way similar to the relation between temperature and energy in equilibrium systems. However, and unlike temperature, the parameter $\boldsymbol{\lambda}$ is nonphysical and cannot be directly manipulated, a main difficulty when studying DPTs which can be, however, circumvented using the active interpretation of fluctuation formulas [3]. In any case, fixing $\boldsymbol{\lambda}$ is equivalent to conditioning the system to have an intensive current $\mathbf{q}_\lambda \equiv \mathbf{Q}_\lambda/t = \boldsymbol{\nabla}_\lambda \mu(\boldsymbol{\lambda})$, so by varying $\boldsymbol{\lambda}$ one can move from one dynamical phase to another.

Macroscopic fluctuation theory.—At the mesoscopic level, driven diffusive systems like WASEP are characterized by a density field $\rho(\mathbf{r}, t)$ obeying a continuity equation $\partial_t \rho + \boldsymbol{\nabla}\cdot\mathbf{j} = 0$, with a current field $\mathbf{j}(\mathbf{r}, t) \equiv -\hat{D}(\rho)\boldsymbol{\nabla}\rho + \hat{\sigma}(\rho)\mathbf{E} + \boldsymbol{\xi}$. The field $\boldsymbol{\xi}(\mathbf{r}, t)$ is a Gaussian white noise of weak amplitude $\propto L^{-1}$ (the inverse system size) which accounts for microscopic random fluctuations at the mesoscopic level, and $\mathbf{E}$ is the external field driving the system out of equilibrium. The deterministic part of $\mathbf{j}(\mathbf{r}, t)$ is given by Fick's law, with $\hat{D}(\rho) \equiv D(\rho)\hat{\mathcal{A}}$ and $\hat{\sigma}(\rho) = \sigma(\rho)\hat{\mathcal{A}}$ the diffusivity and mobility matrices, respectively. The constant diagonal matrix $\hat{\mathcal{A}}$ measures the system underlying anisotropy, i.e., the possible change of microscopic jump rates from one spatial direction to another. We are interested in the statistics of trajectories $\{\rho(\mathbf{r}, t), \mathbf{j}(\mathbf{r}, t)\}_0^\tau$ constrained to a fixed current $\mathbf{q} = \tau^{-1}\int_0^\tau dt \int d\mathbf{r}\,\mathbf{j}(\mathbf{r}, t)$ during a long time τ in a closed system with periodic boundaries. The associated nonequilibrium steady state is homogeneous, with constant (and conserved) density ρ_0 and average current $\langle\mathbf{q}\rangle = \sigma_0\hat{\mathcal{A}}\mathbf{E}$, with $\sigma_0 \equiv \sigma(\rho_0)$. The MFT offers precise variational formulas for the DFE $\mu(\boldsymbol{\lambda})$ starting from the above fluctuating hydrodynamics equations [3] and with the only input of two transport coefficients, which for 2D WASEP are $D(\rho) = 1/2$ and $\sigma(\rho) = \rho(1-\rho)$, and an anisotropy matrix that we parametrize here as $\hat{\mathcal{A}}_{xx} = 1 + \epsilon$ and $\hat{\mathcal{A}}_{yy} = 1 - \epsilon$. This MFT problem can be solved using standard techniques (see Supplemental Material [53]), and we now summarize its predictions.

Dynamical phase diagram.—Small current fluctuations ($|\mathbf{q} - \langle\mathbf{q}\rangle| \ll 1$ or $|\boldsymbol{\lambda}| \approx 0$) typically result from the random superposition of mostly independent local jumps which sum incoherently to yield the desired current, so the typical trajectories associated with these small fluctuations are still homogeneous, as the stationary ones [5,9]. According to the central limit theorem, this leads to Gaussian current statistics corresponding to a quadratic dynamical free energy $\mu_G(\mathbf{z}) \equiv (\mathbf{z}\cdot\hat{\sigma}_0\mathbf{z} - \mathbf{E}\cdot\hat{\sigma}_0\mathbf{E})/2$, with $\mathbf{z} \equiv \boldsymbol{\lambda} + \mathbf{E}$. This homogeneous phase is depicted in light gray in Fig. 1. A local stability analysis then shows that this Gaussian, homogeneous regime eventually becomes unstable against small but otherwise arbitrary spatiotemporal perturbations in trajectories. For WASEP, this happens for large enough external fields and currents $\mathbf{q}\cdot\hat{\mathcal{A}}^{-1}\mathbf{q} \leq \sigma_0^2 \Xi_c$, or equivalently $\mathbf{z}\cdot\hat{\mathcal{A}}\mathbf{z} \leq \Xi_c$, where Ξ_c is a critical threshold; see the black lines separating gray and colored regions in Figs. 1(a)–1(c). This transition can be shown to be of second-order type, as $\partial^2_{|\mathbf{z}|}\mu(\mathbf{z})$ is discontinuous at the critical line [53].

Interestingly, the dominant perturbation immediately after the instability kicks in takes the form of a traveling density wave with structure only along one-dimension (1D), either x or y [see Figs. 1(e) and 1(f)]. This collective rearrangement breaks the system spatiotemporal translation

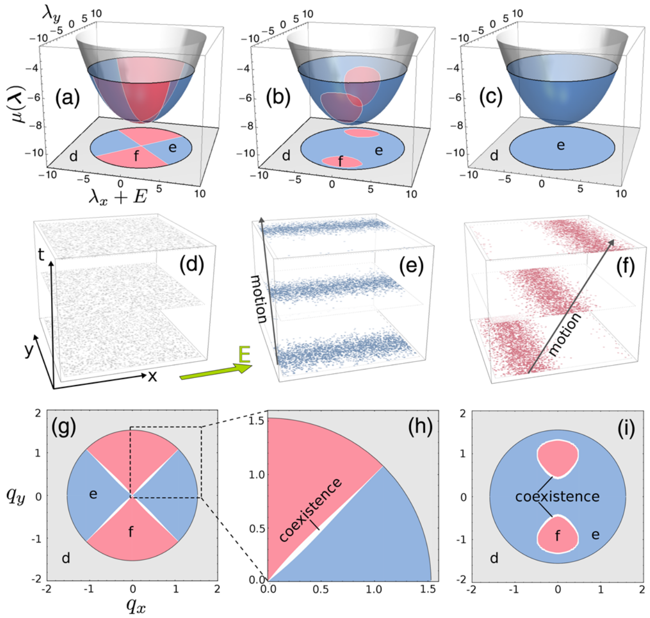

FIG. 1. Top row: $\mu(\boldsymbol{\lambda})$ for the 2D WASEP in an external field $\mathbf{E} = (10, 0)$, as derived from the MFT, in the case of (a) no anisotropy, $\epsilon = 0$, (b) mild anisotropy, $0 < \epsilon < \epsilon_c$, and (c) strong anisotropy, $\epsilon > \epsilon_c$. The projections show the phase diagram in $\boldsymbol{\lambda}$ space for each case, and letters indicate the typical spatiotemporal trajectories in each phase, displayed in the middle row (d)–(f). A DPT appears between a Gaussian phase (light gray) with homogeneous trajectories (d) and two different non-Gaussian symmetry-broken phases for low currents characterized by jammed density waves (e),(f). The first DPT is second-order, while the two symmetry-broken phases are separated by lines of first-order DPTs. Bottom row: Phase diagram in current space for anisotropy $\epsilon = 0$ (g),(h) and $0 < \epsilon < \epsilon_c$ (i). The coexistence pockets (white) are apparent.

symmetry by localizing particles in a jammed region to facilitate a low-current fluctuation. This solution can be extended to all currents below the critical line, and we find that different 1D density waves dominate different current vector regimes, depending on the anisotropy parameter ϵ; see Figs. 1(a)–1(c). Lines of first-order DPTs separate both density-wave phases where the DFE $\mu(\boldsymbol{\lambda})$ exhibits a jump in its first derivative [53], so the current $\mathbf{q}_{\boldsymbol{\lambda}} = \boldsymbol{\nabla}_{\boldsymbol{\lambda}}\mu(\boldsymbol{\lambda})$ corresponding to a given $\boldsymbol{\lambda}$ jumps discontinuously at these lines. In this way, the first-order DPT lines in $\boldsymbol{\lambda}$ space correspond to pockets in $\mathbf{q}$ space where dynamical coexistence emerges between the two traveling-wave phases; see Figs. 1(g)–1(i). This means that if we were to observe an atypical current $\mathbf{q}$ sitting in one of these pockets, either by an unlikely spontaneous fluctuation or by an active control of the current with an optimal field, we would observe the dynamical coexistence of the two different traveling density waves.

Strikingly, particular 2D traveling-wave solutions (such as, e.g., traveling compact packets) do not improve the variational problem for $\mu(\boldsymbol{\lambda})$ when compared to their 1D counterparts. This is surprising, as one would naively expect the system to minimize the interface between the high- and low-density regions while developing a macroscopic jam to sustain a low-current fluctuation. This phenomenological picture does not emerge in our theory and is not observed in the simulations below.

What are the key ingredients responsible of the new physics here described and not observed in previous works [9–32]? First, by considering vectorial currents, it becomes apparent that current rotations can trigger first-order transitions between different symmetry-broken jammed dynamical phases. This is certainly not present in simpler 1D models [9–21] and cannot show up when studying fluctuations of scalar observables in $d > 1$ [22–32]. Second, by including anisotropy in our analysis (a main feature of many realistic $d > 1$ systems not considered before), it becomes clear its strong effect on the relative shape and position of the different jammed phases; see Figs. 1(a)–1(c). In this way, it is the interplay between vectorial currents and anisotropy in $d > 1$ that gives rise to the rich and complex dynamical phase diagram here described. Mathematically, the novel competition between different symmetry-broken dynamical phases is due to the appearance of a structured vector field coupled to the current [58–60].

Numerical results.—The previous results call for independent numerical verification, as they derive from an effective mesoscopic theory which relies on a few hypotheses [3,53]. To search for this DPT, we explored the current statistics of the 2D WASEP using massive cloning Monte Carlo simulations [49–51]. In particular, we simulated systems with density $\rho_0 = 0.3$, several system sizes up to $N = 144$, and a strong external field $\mathbf{E} = (10, 0)$. The cloning Monte Carlo method relies on a controlled modification of the system stochastic dynamics such that the rare events responsible for a given fluctuation are no longer rare and involves the parallel simulation of multiple copies of the system [49–51]. The number of clones needed to observe a given rare event grows exponentially with the system size, all the more the rarer the event is [61,62]. In particular, to pick up and characterize reliably the DPT in the 2D WASEP, we needed the extraordinary number of $N_c = 5.12 \times 10^5$ clones evolving in parallel for a long time.

According to the MFT, Gaussian current statistics corresponding to a quadratic DFE $\mu_G(\mathbf{z})$ are expected for $\mathbf{z} \cdot \hat{A}\mathbf{z} \geq \Xi_c$; see Fig. 1 and the discussion above. This is fully confirmed in Fig. 2, which shows the measured $\mu(\mathbf{z})$ for $N = 144$ as a function of $z = |\mathbf{z}|$ for different current orientations $\phi = \tan^{-1}(z_y/z_x)$. This confirms that mild current fluctuations stem from the random superposition of weakly correlated, localized events which sum up incoherently to yield Gaussian statistics. Interestingly, we find a weak dependence of $\mu(\mathbf{z})$ on ϕ in this Gaussian regime, a clear hallmark of the effective anisotropy mentioned above. Indeed, this ϕ dependence can be used to

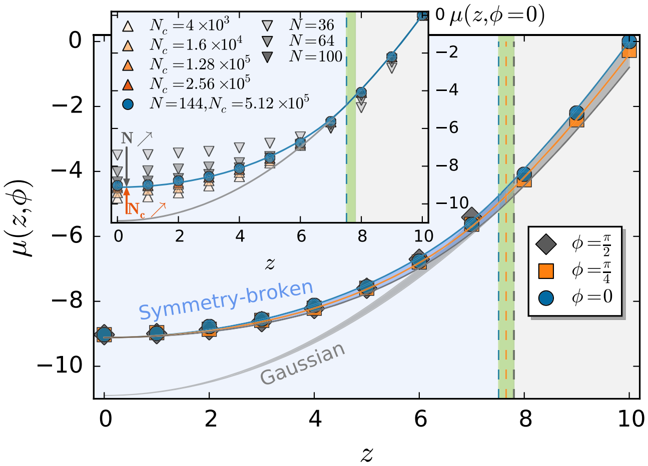

FIG. 2. Main: $\mu(\boldsymbol{\lambda})$ vs $z = |\boldsymbol{\lambda} + \mathbf{E}|$ as obtained in simulations for $N = 144$, $N_c = 5.12 \times 10^5$, and different $\phi = \tan^{-1}(z_y/z_x)$, together with MFT predictions for anisotropy $\epsilon = 0.038$. A DPT from a Gaussian regime (light-gray ribbon) to a symmetry-broken, non-Gaussian phase (blue ribbon) is apparent upon crossing $z_c(\phi)$, with $\mathbf{z}_c \cdot \hat{A}\mathbf{z}_c = \Xi_c$ (green vertical stripe). Different ϕ correspond to different MFT lines within the shaded ribbons. Inset: Convergence to the $\phi = 0$ MFT prediction (blue line) for $N = 144$ as N_c increases (upward triangle) and for optimal N_c as N increases (downward triangle).

estimate that $\epsilon \approx 0.038$ properly describes the observed weak anisotropy; see the inset in Fig. 3. This effective anisotropy is slightly larger than the critical anisotropy $\epsilon_c \approx 0.035$, beyond which a single symmetry-broken phase dominates the non-Gaussian regime [see Fig. 1(c)], an observation consistent with additional results below. The Gaussian, incoherent fluctuation regime ends up for $\mathbf{z} \cdot \hat{A}\mathbf{z} < \Xi_c$, where clear deviations from the quadratic form $\mu_G(\mathbf{z})$ become apparent; see Fig. 2. This change of behavior, in excellent agreement with MFT predictions, signals the onset of the DPT to a symmetry-broken phase characterized by non-Gaussian current fluctuations and traveling density-wave trajectories. A clear convergence to the MFT prediction is observed in the Gaussian and non-Gaussian regimes as both N and the number of clones N_c increase; see the inset in Fig. 2.

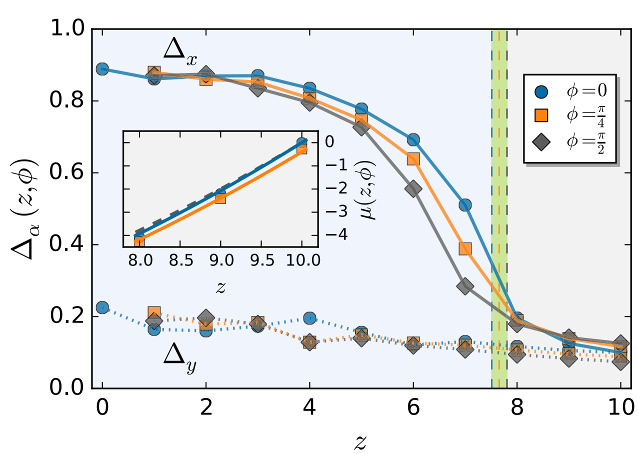

FIG. 3. Tomographic α coherences, with $\alpha = x, y$, as a function of z for different current angles ϕ measured for $N = 100$ and $\mathbf{E} = (10, 0)$. Inset: DFE $\mu(\mathbf{z})$ vs z in the Gaussian regime for $\phi = 0, \pi/4$; see Fig. 2. Solid (dashed) lines are MFT predictions with anisotropy $\epsilon = 0.038$ ($\epsilon = 0$).

The smoking gun of any continuous phase transition, such as the DPT here reported, is a smooth but apparent change in an order parameter [1]. To distinguish between the different jammed density-wave phases which are expected to appear for low current fluctuations [see Figs. 1(e) and 1(f)], we introduce now a structural order parameter capable of discerning the jam direction, if any (see [53] for a detailed description). In particular, we take 1D slices of our 2D system along a given direction, $\alpha = x$ or y, and compute the center-of-mass position for each slice. Clearly, a small average dispersion $\langle \sigma_\alpha^2 \rangle_\lambda$ of the centers of mass across the different slices signals the formation of a jam along the α direction [Figs. 1(e) and 1(f)], while random homogeneous configurations typical of the Gaussian phase [Fig. 1(d)] are characterized by a large dispersion. We hence define the tomographic α coherence (i.e., the center-of-mass coherence across the different slices along the α axis) as $\Delta_\alpha(\boldsymbol{\lambda}) \equiv 1 - \langle \sigma_\alpha^2 \rangle_\lambda$, and Fig. 3 shows this order parameter measured in simulations across the DPT for $\alpha = x, y$. Remarkably, $\Delta_x(z)$ increases steeply for $\mathbf{z} \cdot \hat{A}\mathbf{z} \leq \Xi_c$ and all angles ϕ of the current vector, while $\Delta_y(z)$ remains small and does not change appreciably across the DPT, clearly indicating that only one of the two possible symmetry-broken phases appear in our simulations, as expected from the MFT in the supercritical anisotropy regime $\epsilon > \epsilon_c$ and consistent with the measured effective anisotropy $\epsilon \approx 0.038 > \epsilon_c$; see the inset in Fig. 3. Note also that the behavior of both $\Delta_\alpha(z)$ across the DPT is consistent with the emergence of a traveling wave with structure in 1D and not in 2D, as in the latter case both $\Delta_\alpha(z)$ should increase upon crossing $z_c(\phi)$. Moreover, the steep but continuous change of $\Delta_x(\mathbf{z})$ across the DPT is consistent with a second-order transition, in agreement with the MFT.

Summary.—We have presented compelling evidence of a complex dynamical phase transition in the current vector statistics of a paradigmatic model of transport in 2D, characterizing its properties with the tools of the macroscopic fluctuation theory. Our analysis of MFT equations predicts a rich phase diagram, with nonanalyticities of first- and second-order type in the current dynamical free energy, accompanied by emergent order in different symmetry-broken phases characterized by traveling density waves. This richness is aided by the complex interplay among anisotropy, external field, and vector currents in $d > 1$, key features missing in the simpler models studied in the past. Interestingly, our results show that order and coherence may emerge out of an unlikely fluctuation, proving the deep connection between rare events and self-organized structures which enhance their probability. This is expected to be a general feature of many complex dynamical systems [45]. The mapping between exclusion processes and dual quantum spin systems [63–66] suggests a connection between the DPT here uncovered and a rich quantum phase transition yet to be explored. It would be also interesting to determine the universality class of this DPT and the dynamical exponents of the different fluctuation phases [7,17].

Financial support from Spanish Projects No. FIS2013-43201-P (Ministerio de Economía y Competitividad - MINECO) and No. FPU13/05633, Italian Research Funding Agency (Ministero dell'Istruzione, dell'Università e della Ricerca - MIUR) through Fondo per gli Investimenti della Ricerca di Base (FIRB) project Grant No. RBFR10N90W, Italian Istituto Nazionale di Alta Matematica (INdAM) *Francesco Severi*, University of Granada, Junta de Andalucía Project No. P09-FQM4682, H2020-FETPROACT-2014 Grant No. 640378 (Rydberg Quantum Simulations - RySQ) and Granada Excellence Network of Innovation Laboratories (GENIL) PYR-2014-13 project is acknowledged.

*phurtado@onsager.ugr.es

[1] J. J. Binney, N. J. Dowrick, A. J. Fisher, and M. Newman, *The Theory of Critical Phenomena: An Introduction to the Renormalization Group* (Oxford University Press, New York, 1992).

[2] J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena*, 4th ed., International Series of Monographs on Physics (Clarendon Press, Oxford, 2002).

[3] L. Bertini, A. De Sole, D. Gabrielli, G. Jona-Lasinio, and C. Landim, Macroscopic fluctuation theory, Rev. Mod. Phys. **87**, 593 (2015).

[4] B. Derrida, Non-equilibrium steady states: Fluctuations and large deviations of the density and of the current, J. Stat. Mech. (2007) P07023.

[5] T. Bodineau and B. Derrida, Distribution of current in nonequilibrium diffusive systems and phase transitions, Phys. Rev. E **72**, 066110 (2005).

[6] L. Bertini, A. De Sole, D. Gabrielli, G. Jona-Lasinio, and C. Landim, Nonequilibrium current fluctuations in stochastic lattice gases, J. Stat. Phys. **123**, 237 (2006).

[7] V. Lecomte, U. C. Täuber, and F. van Wijland, Current distribution in systems with anomalous diffusion: Renormalization group approach, J. Phys. A **40**, 1447 (2007).

[8] T. Bodineau, B. Derrida, and J. L. Lebowitz, Vortices in the two-dimensional simple exclusion process, J. Stat. Phys. **131**, 821 (2008).

[9] P. I. Hurtado and P. L. Garrido, Spontaneous Symmetry Breaking at the Fluctuating Level, Phys. Rev. Lett. **107**, 180601 (2011).

[10] C. Pérez-Espigares, P. L. Garrido, and P. I. Hurtado, Dynamical phase transition for current statistics in a simple driven diffusive system, Phys. Rev. E **87**, 032115 (2013).

[11] P. I. Hurtado, C. P. Espigares, J. J. del Pozo, and P. L. Garrido, Thermodynamics of currents in nonequilibrium diffusive systems: Theory and simulation, J. Stat. Phys. **154**, 214 (2014).

[12] S. Vaikuntanathan, T. R. Gingrich, and P. L. Geissler, Dynamic phase transitions in simple driven kinetic networks, Phys. Rev. E **89**, 062108 (2014).

[13] R. L. Jack, I. R. Thompson, and P. Sollich, Hyperuniformity and Phase Separation in Biased Ensembles of Trajectories for Diffusive Systems, Phys. Rev. Lett. **114**, 060601 (2015).

[14] O. Shpielberg and E. Akkermans, Le Chatelier Principle for Out-of-Equilibrium and Boundary-Driven Systems: Application to Dynamical Phase Transitions, Phys. Rev. Lett. **116**, 240603 (2016).

[15] L. Zarfaty and B. Meerson, Statistics of large currents in the Kipnis-Marchioro-Presutti model in a ring geometry, J. Stat. Mech. (2016) P033304.

[16] Y. Baek, Y. Kafri, and V. Lecomte, Dynamical Symmetry Breaking and Phase Transitions in Driven Diffusive Systems, Phys. Rev. Lett. **118**, 030604 (2017).

[17] D. Karevski and G. M. Schütz, Conformal Invariance in Driven Diffusive Systems at High Currents, Phys. Rev. Lett. **118**, 030601 (2017).

[18] J. P. Garrahan and I. Lesanovsky, Thermodynamics of Quantum Jump Trajectories, Phys. Rev. Lett. **104**, 160601 (2010).

[19] C. Ates, B. Olmos, J. P. Garrahan, and I. Lesanovsky, Dynamical phases and intermittency of the dissipative quantum Ising model, Phys. Rev. A **85**, 043620 (2012).

[20] I. Lesanovsky, M. van Horssen, M. Gutua, and J. P. Garrahan, Characterization of Dynamical Phase Transitions in Quantum Jump Trajectories beyond the Properties of the Stationary State, Phys. Rev. Lett. **110**, 150401 (2013).

[21] F. Carollo, J. P. Garrahan, I. Lesanovsky, and C. Pérez-Espigares, Fluctuating hydrodynamics, current fluctuations and hyperuniformity in boundary-driven open quantum chains, arXiv:1703.00355.

[22] J. P. Garrahan, R. L. Jack, V. Lecomte, E. Pitard, K. van Duijvendijk, and F. van Wijland, Dynamical First-Order Phase Transition in Kinetically Constrained Models of Glasses, Phys. Rev. Lett. **98**, 195702 (2007).

[23] J. P. Garrahan, R. L. Jack, V. Lecomte, E. Pitard, K. van Duijvendijk, and F. van Wijland, First-order dynamical phase transition in models of glasses: An approach based on ensembles of histories, J. Phys. A **42**, 075007 (2009).

[24] L. O. Hedges, R. L. Jack, J. P. Garrahan, and D. Chandler, Dynamic order-disorder in atomistic models of structural glass formers, Science **323**, 1309 (2009).

[25] D. Chandler and J. P. Garrahan, Dynamics on the way to forming glass: Bubbles in space-time, Annu. Rev. Phys. Chem. **61**, 191 (2010).

[26] E. Pitard, V. Lecomte, and F. Van Wijland, Dynamic transition in an atomic glass former: A molecular-dynamics evidence, Europhys. Lett. **96**, 56002 (2011).

[27] T. Speck, A. Malins, and C. P. Royall, First-Order Phase Transition in a Model Glass Former: Coupling of Local Structure and Dynamics, Phys. Rev. Lett. **109**, 195703 (2012).

[28] R. Pinchaipat, M. Campo, F. Turci, J. E. Hallett, T. Speck, and C. P. Royall, Experimental Evidence for a Structural-Dynamical Transition in Trajectory Space, Phys. Rev. Lett. **119**, 028004 (2017).

[29] B. Abou, R. Colin, V. Lecomte, E. Pitard, and F. van Wijland, Activity statistics in a colloidal glass former: Experimental evidence for a dynamical transition, arXiv:1705.00855.
[30] J. P. Garrahan, A. D. Armour, and I. Lesanovsky, Quantum trajectory phase transitions in the micromaser, Phys. Rev. E **84**, 021115 (2011).
[31] S. Genway, J. P. Garrahan, I. Lesanovsky, and A. D. Armour, Phase transitions in trajectories of a superconducting single-electron transistor coupled to a resonator, Phys. Rev. E **85**, 051122 (2012).
[32] D. Manzano and P. I. Hurtado, Symmetry and the thermodynamics of currents in open quantum systems, Phys. Rev. B **90**, 125138 (2014).
[33] D. Manzano and E. Kyoseva, An atomic symmetry-controlled thermal switch, Sci. Rep. **6**, 31161 (2016).
[34] D. Manzano and P. I. Hurtado, Harnessing symmetry to control quantum transport, arXiv:1707.07895.
[35] H. Touchette, The large deviation approach to statistical mechanics, Phys. Rep. **478**, 1 (2009).
[36] C.-N. Yang and T.-D. Lee, Statistical Theory of Equations of State and Phase Transitions. I. Theory of Condensation, Phys. Rev. **87**, 404 (1952).
[37] P. F. Arndt, Yang-Lee Theory for a Nonequilibrium Phase Transition, Phys. Rev. Lett. **84**, 814 (2000).
[38] R. A. Blythe and M. R. Evans, Lee-Yang Zeros and Phase Transitions in Nonequilibrium Steady States, Phys. Rev. Lett. **89**, 080601 (2002).
[39] S. M. Dammer, S. R. Dahmen, and H. Hinrichsen, Yang-Lee zeros for a nonequilibrium phase transition, J. Phys. A **35**, 4527 (2002).
[40] R. A. Blythe and M. R. Evans, The Lee-Yang theory of equilibrium and nonequilibrium phase transitions, Braz. J. Phys. **33**, 464 (2003).
[41] C. Flindt and J. P. Garrahan, Trajectory Phase Transitions, Lee-Yang Zeros, and High-Order Cumulants in Full Counting Statistics, Phys. Rev. Lett. **110**, 050601 (2013).
[42] J. M. Hickey, C. Flindt, and J. P. Garrahan, Intermittency and dynamical Lee-Yang zeros of open quantum systems, Phys. Rev. E **90**, 062128 (2014).
[43] K. Brandner, V. F. Maisi, J. P. Pekola, J. P. Garrahan, and C. Flindt, Experimental Determination of Dynamical Lee-Yang Zeros, Phys. Rev. Lett. **118**, 180601 (2017).
[44] J. Barré, C. Bernardin, and R. Chetrite, Density large deviations for multidimensional stochastic hyperbolic conservation laws, arXiv:1702.03769.
[45] K. D. N. T. Lam, J. Kurchan, and D. Levine, Order in extremal trajectories, J. Stat. Phys. **137**, 1079 (2009).
[46] R. Chetrite and H. Touchette, Variational and optimal control representations of conditioned and driven processes, J. Stat. Mech. (2015) P12001.
[47] R. Chetrite and H. Touchette, Nonequilibrium Markov processes conditioned on large deviations, Ann. Inst. Henri Poincaré **16**, 2005 (2015).
[48] B. Derrida, An exactly soluble non-equilibrium system: The asymmetric simple exclusion process, Phys. Rep. **301**, 65 (1998).
[49] C. Giardinà, J. Kurchan, and L. Peliti, Direct Evaluation of Large-Deviation Functions, Phys. Rev. Lett. **96**, 120603 (2006).
[50] V. Lecomte and J. Tailleur, A numerical approach to large deviations in continuous time, J. Stat. Mech. (2007) P03004.
[51] C. Giardinà, J. Kurchan, V. Lecomte, and J. Tailleur, Simulating rare events in dynamical processes, J. Stat. Phys. **145**, 787 (2011).
[52] The rates $r_\pm^\alpha \equiv \exp[\pm E_\alpha/L]/2$ converge for large L to the standard ones found in the literature [4,5], namely, $\frac{1}{2}(1 \pm E_\alpha/L)$, but avoid problems with negative rates for small L. Indeed, the hydrodynamic description of both variants of the model is identical in the thermodynamic limit. However, for finite, moderate values of L the field per unit length $(\mathbf{E}/L)$ is too strong, leading to an effective anisotropy in the system. In fact, by expanding the microscopic transition rate $r_\pm^\alpha$ to second order in the field per unit length, i.e., $r_\pm^\alpha \approx \frac{1}{2}[1 \pm E_\alpha/L + \frac{1}{2}(E_\alpha/L)^2] + \mathcal{O}[(E_\alpha/L)^3]$, it is easy to show using a simple random walk argument that the second-order perturbation results in an effective increase of diffusivity and mobility along the field direction and an associated decrease in the orthogonal direction.
[53] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.119.090602 for the details, which includes Refs. [54–57].
[54] P. I. Hurtado, C. Pérez-Espigares, J. J. del Pozo, and P. L. Garrido, Symmetries in fluctuations far from equilibrium, Proc. Natl. Acad. Sci. U.S.A. **108**, 7704 (2011).
[55] R. Villavicencio-Sánchez, R. J. Harris, and H. Touchette, Fluctuation relations for anisotropic systems, Europhys. Lett. **105**, 30009 (2014).
[56] C. Pérez-Espigares, F. Redig, and C. Giardinà, Spatial fluctuation theorem, J. Phys. A **48**, 35FT01 (2015).
[57] C. Kipnis, C. Marchioro, and E. Presutti, Heat flow in an exactly solvable model, J. Stat. Phys. **27**, 65 (1982).
[58] C. Pérez-Espigares, P. L. Garrido, and P. I. Hurtado, Weak additivity principle for current statistics in d-dimensions, Phys. Rev. E **93**, 040103(R) (2016).
[59] R. Villavicencio-Sanchez and R. J. Harris, Local structure of current fluctuations in diffusive systems beyond one dimension, Phys. Rev. E **93**, 032134 (2016).
[60] N. Tizón-Escamilla, P. I. Hurtado, and P. L. Garrido, Structure of the optimal path to a fluctuation, Phys. Rev. E **95**, 032119 (2017).
[61] P. I. Hurtado and P. L. Garrido, Current fluctuations and statistics during a large deviation event in an exactly solvable transport model, J. Stat. Mech. (2009) P02032.
[62] T. Nemoto, F. Bouchet, R. L. Jack, and V. Lecomte, Population dynamics method with a multi-canonical feedback control, Phys. Rev. E **93**, 062123 (2016).
[63] B. U. Felderhof, Spin relaxation of the Ising chain, Rep. Math. Phys. **1**, 215 (1971).
[64] F. C. Alcaraz, M. Droz, M. Henkel, and V. Rittenberg, Reaction-diffusion processes, critical dynamics and quantum chains, Ann. Phys. (N.Y.) **230**, 250 (1994).
[65] R. B. Stinchcombe and G. M. Schütz, Application of Operator Algebras to Stochastic Dynamics and the Heisenberg Chain, Phys. Rev. Lett. **75**, 140 (1995).
[66] R. B. Stinchcombe, Stochastic non-equilibrium systems, Adv. Phys. **50**, 431 (2001).