Next-to-Leading Order Computation of Exclusive Diffractive Light Vector Meson Production in a Saturation Framework

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We perform the first next-to-leading order computation of the $\gamma^{(*)} \to V\ (\rho, \phi, \omega)$ exclusive impact factor in the QCD shock-wave approach and in the most general kinematics. This paves the way to the very first quantitative study of high-energy nucleon and nucleus saturation beyond the leading order for a whole range of small-x exclusive processes, to be measured in ep, eA, pp, and pA collisions at existing and future colliders.

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Introduction.—Among the various achievements of the HERA experiments, two landmark results emerged from $e^{\pm}p$ deep inelastic scattering (DIS). First, diffractive events represent a fraction of up to 10% of the total $e^{\pm}p$ cross section for DIS [1,2]. Second, the study of the kinematical domain where the photon virtuality Q^2 is moderate and the Bjorken x variable is asymptotically small may be interpreted as an indication that the proton saturates, both in inclusive and diffractive deep inelastic scattering, as first exhibited within the Golec-Biernat and Wüsthoff model [3]. It has been further realized that exclusive diffractive processes could give an excellent lever arm to scrutinize the proton's internal structure at asymptotic energies. In particular, the exclusive diffractive production of a light vector meson $V\ (\rho, \phi, \omega)$ [4–6]

$$\gamma^{(*)} p \to V p \tag{1}$$

was studied at HERA both for forward [7] and large t [7,8] kinematics. On top of the photon virtuality, the transverse momenta exchanged in the t channel give access to the impact parameter distribution of partons inside the proton via a Fourier transformation.

Understanding the highly energetic proton state is theoretically particularly appealing and phenomenologically valuable. First, at large center of mass energy $\sqrt{s}$, the proton is a dense system with high field strengths, still in the weak-coupling regime, and perturbative effective resummation methods must be applied. Second, in the context of relativistic heavy ion collisions, in view of producing and studying the quark-gluon plasma, the colliding nuclei in initial stages are saturated. Thus, saturation is one of the most important and longstanding problems of QCD.

In different frameworks, either based on a QCD shock-wave formalism [9], a large-N_c dipole model [10,11], or an effective perturbative weak-coupling field theory approach [12], a color glass condensate (CGC) picture has emerged, describing the small-x dynamics of QCD towards the saturation regime.

Still, it has been realized that in order to get a detailed understanding of the properties of the high-energy proton, precise quantitative predictions are absolute prerequisites. This means that one should go beyond leading order computations, a task which is particularly difficult to achieve in the above mentioned frameworks. A first step towards such an improvement was performed at the level of the evolution kernel for a CGC, including first the running coupling effects [13] and finally the whole next-to-leading order (NLO) correction to the kernel [14–16]. First steps have been made concerning the corrections to the coupling to a probe in inclusive and semi-inclusive

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processes. The NLO impact factor has been obtained for the inclusive coupling to a γ^* [17] and for semi-inclusive hadron production, involving the coupling to a parton, in view of studying $p_\perp$-broadening effects [18–20]. Finally, the first computation of an exclusive NLO impact factor in the CGC framework was performed in Refs. [21,22].

Although the application in phenomenology of the inclusive results of Ref. [18] met conceptual problems due to the appearance in some cases of negative cross sections at high momenta, several suggestions for improvements of divergence subtraction and CGC resummation methods [20,23–28] were proposed to overcome this issue. The new range of exclusive processes proposed in this Letter will surely provide a new contribution to uncovering a proper solution to this problem, complementarily to the inclusive and semi-inclusive observables studied until now.

In this Letter, we study the exclusive production of a neutral longitudinally polarized vector meson with NLO accuracy. As first noticed in Ref. [29], one can describe in DIS the exclusive production of a meson from a $q\bar{q}$ pair based on the collinear QCD factorization scheme. At moderate energies, the amplitude is given as a convolution of quark or gluon generalized parton distributions in the nucleon, the distribution amplitude (DA) for the light meson, and a perturbatively calculable hard scattering amplitude [30,31]. The DAs and generalized parton distributions are subject to specific QCD evolution equations [32]. Still, such a factorization is proven only for the twist-2 dominated transition between a longitudinally polarized photon and a longitudinally polarized vector meson [30]. Explicit breaking of collinear factorization occurs at twist 3, through end-point singularities, in the exclusive electroproduction of transversely polarized vector mesons [33]. As a remedy, an improved collinear approximation scheme [34] has been proposed and applied to ρ electroproduction [35]. At high energies, where the exchange of t-channel gluons dominates, k_T factorization applies. The end-point singularities are naturally regularized by the transverse momenta of these t-channel gluons [36,37], providing models [38] to describe HERA data.

In this Letter, we will carry out, for the first time in the shock-wave context, a complete NLO calculation for exclusive diffractive meson production in $\gamma^{(*)}p$ or $\gamma^{(*)}A$ collisions, with completely general kinematics, by combining the collinear factorization and high energy small-x factorization techniques. We will show the full infrared safe results for the $\gamma_L^* \to V_L$ and $\gamma_T^{(*)} \to V_L$ transitions. The details of the calculation will be provided in a separate article [39].

The present result provides the first calculation of higher order corrections, in a complete NLO framework, of a vast class of exclusive processes. Indeed, the complete generality of the kinematics allows it to be applied to a wide range of experimental conditions. It can describe either the electroproduction of vector mesons with general kinematics, or their photoproduction at large transferred momentum. As a result, it can be used both at ep and eA colliders, like the future EIC [40] or LHeC [41] and in ultraperipheral collisions at RHIC or at the LHC [42,43].

The shock-wave framework.—Let us define two lightlike vectors n_1 and n_2 such that the partons in the upper (lower) impact factor have large momentum components along n_1 (n_2). We write the Sudakov expansion of any vector p as

$$p^\mu \equiv p^+ n_1^\mu + p^- n_2^\mu + p_\perp^\mu. \tag{2}$$

Normalizing the light-cone basis so that $n_1 \cdot n_2 = 1$, we write the scalar product of two vectors as

$$\begin{aligned} p\cdot q &\equiv p^+q^- + p^-q^+ + p_\perp \cdot q_\perp \\ &= p^+q^- + p^-q^+ - \vec{p}\cdot\vec{q}. \end{aligned} \tag{3}$$

Within the shock-wave formalism, the computation is performed in a frame where the target is highly boosted. We separate the gluonic field into an external (internal) field containing the gluons with momentum components along n_1 below (above) the cutoff $e^\eta p_\gamma^+$, where p_γ is the momentum of the photon and η is the rapidity divide, which eventually separates the gluons belonging to the projectile impact factor from the ones attributed to the shock wave.

We work in the QCD light-cone gauge $n_2 \cdot A = 0$. In the high energy limit and in this gauge, the external field b_η^μ is located at zero light-cone time z^+ and has the eikonal Lorentz structure

$$b_\eta^\mu(z) = b_\eta^-(\vec{z})\delta(z^+)n_2^\mu. \tag{4}$$

We define the high-energy Wilson line operator as

$$U_{\vec{z}}^\eta \equiv \mathcal{P}\exp\left(ig\int_{-\infty}^{+\infty} dz^+ b_\eta^-(z)\right). \tag{5}$$

The scattering amplitude is obtained by convoluting the impact factor with the matrix elements of operators built from Wilson line operators acting on the target states.

In the context of such a NLO diffractive process, we introduce the dipole and double dipole operators in momentum space from Wilson line operators in the fundamental representation of $SU(N_c)$ as

$$\begin{aligned} &[\mathrm{Tr}(U_1^\eta U_2^{\eta\dagger}) - N_c](\vec{p}_1,\vec{p}_2) \\ &\quad\equiv \int d^d\vec{z}_1 d^d\vec{z}_2 e^{-i(\vec{p}_1\cdot\vec{z}_1)-i(\vec{p}_2\cdot\vec{z}_2)}[\mathrm{Tr}(U_{\vec{z}_1}^\eta U_{\vec{z}_2}^{\eta\dagger}) - N_c], \end{aligned} \tag{6}$$

and

$$[\mathrm{Tr}(U_1^\eta U_3^{\eta\dagger})\mathrm{Tr}(U_3^\eta U_2^{\eta\dagger}) - N_c \mathrm{Tr}(U_1^\eta U_2^{\eta\dagger})](\vec{p}_1, \vec{p}_2, \vec{p}_3)$$
$$\equiv \int d^d\vec{z}_1 d^d\vec{z}_2 d^d\vec{z}_3 e^{-i(\vec{p}_1\cdot\vec{z}_1)-i(\vec{p}_2\cdot\vec{z}_2)-i(\vec{p}_3\cdot\vec{z}_3)}$$
$$\times [\mathrm{Tr}(U^\eta_{\vec{z}_1} U^{\eta\dagger}_{\vec{z}_3})\mathrm{Tr}(U^\eta_{\vec{z}_3} U^{\eta\dagger}_{\vec{z}_2}) - N_c \mathrm{Tr}(U^\eta_{\vec{z}_1} U^{\eta\dagger}_{\vec{z}_2})], \quad (7)$$

where $d = 2 + 2\epsilon$ is the transverse dimension. In these equations, $\vec{z}_1, \vec{z}_2, \vec{z}_3$ are, respectively, the transverse coordinates of the interaction points of the quark, the antiquark, and the gluon with the external shock-wave field. Their conjugate transverse momenta $\vec{p}_1, \vec{p}_2, \vec{p}_3$ are the incoming effective momenta acquired via interaction with the t-channel shock-wave field.

Factorization scheme.—At leading order accuracy, the factorized amplitude is the action of an operator $\mathcal{A}^\eta_{\mathrm{LO.}}$ on target states. This operator is the convolution of the dipole operator, a hard part Φ_0 to which we will refer as the impact factor, and a DA. The twist 2 DA φ for a longitudinally polarized vector meson V_L is defined via the matrix element of a nonlocal light-cone operator renormalized at scale μ_F

$$\langle V_L(p_V)|\bar{\Psi}(y)\gamma^\mu\Psi(0)|0\rangle_{y^2\to 0}$$
$$= f_V p_V^\mu \int_0^1 dx e^{ix(p_V\cdot y)}\varphi(x, \mu_F), \quad (8)$$

where the gauge link between fields was omitted since it does not contribute in the chosen light-cone gauge. We write the operator as follows:

$$\mathcal{A}^\eta_{\mathrm{LO}} \equiv -\frac{e_V f_V \varepsilon_\beta}{N_c}\int_0^1 dx\varphi(x,\mu_F)\int \frac{d^d\vec{p}_1}{(2\pi)^d}\frac{d^d\vec{p}_2}{(2\pi)^d}$$
$$\times (2\pi)^{d+1}\delta(p_V^+ - p_\gamma^+)\delta(\vec{p}_V - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2)$$
$$\times \Phi_0^\beta(x, \vec{p}_1, \vec{p}_2)[\mathrm{Tr}(U_1^\eta U_2^{\eta\dagger}) - N_c](\vec{p}_1, \vec{p}_2). \quad (9)$$

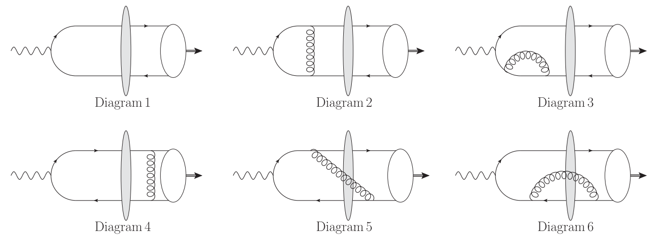

FIG. 1. Contributions to the impact factor for the $\gamma^* \to V$ transition. The gray blobs stand for the external (shock-wave) field while the white blobs denote the distribution amplitudes of the produced vector meson.

Here, ε_β is the polarization vector of the photon, f_V is the meson coupling, which is related to the vector meson decay into leptons, and e_V is an effective electric quark charge that takes into account the flavor content of the meson [44]. Φ_0 is obtained by computing diagram 1 in Fig. 1 using the effective shock-wave Feynman rules [22].

In practice to obtain a full physical amplitude one should first solve the Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov-Kovner (JIMWLK) [12] evolution equation for the Wilson line operators and then calculate their matrix elements using the final and the initial target states. Note that in our case the JIMWLK equation is reduced to the BK equation. For example, in the case of a scattering off a proton, the leading order amplitude $A^\eta_{\mathrm{LO}} \equiv \langle P'|\mathcal{A}^\eta_{\mathrm{LO}}|P\rangle$ will be given in terms of the nonforward dipole-proton scattering amplitude

$$\langle P'|[\mathrm{Tr}(U_1^\eta U_2^{\eta\dagger}) - N_c](\vec{p}_1, \vec{p}_2)|P\rangle. \quad (10)$$

At NLO accuracy the double dipole operator starts to contribute and we define similarly to the LO case the NLO operator

$$\mathcal{A}^\eta_{\mathrm{NLO}} \equiv -\frac{e_V f_V \varepsilon_\beta}{N_c}\int_0^1 dx\varphi(x,\mu_F)\int \frac{d^d\vec{p}_1}{(2\pi)^d}\frac{d^d\vec{p}_2}{(2\pi)^d}\frac{d^d\vec{p}_3}{(2\pi)^d}(2\pi)^{d+1}\delta(p_V^+ - p_\gamma^+)\delta(\vec{p}_V - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$
$$\times \frac{\alpha_s\Gamma(1-\epsilon)}{(4\pi)^{1+\epsilon}}\left\{\left(\frac{N_c^2-1}{N_c}\right)\Phi_1^\beta(x,\vec{p}_1,\vec{p}_2)[\mathrm{Tr}(U_1^\eta U_2^{\eta\dagger}) - N_c](\vec{p}_1,\vec{p}_2)(2\pi)^d\delta(\vec{p}_3)\right.$$
$$\left. + \Phi_2^\beta(x,\vec{p}_1,\vec{p}_2,\vec{p}_3)[\mathrm{Tr}(U_1^{\eta\dagger} U_3^{\eta\dagger})\mathrm{Tr}(U_3^{\eta\dagger} U_2^{\eta\dagger}) - N_c\mathrm{Tr}(U_1^\eta U_2^{\eta\dagger})](\vec{p}_1,\vec{p}_2,\vec{p}_3)\right\}. \quad (11)$$

The explicit expressions for Φ_1 and Φ_2, given below in Eqs. (25)–(28), are the main results of the present Letter. Again, in the example of the scattering on a proton, the computation of the NLO amplitude $A^\eta_{\mathrm{NLO}} \equiv \langle P'|\mathcal{A}^\eta_{\mathrm{NLO}}|P\rangle$ will now involve, in addition to the amplitude (10), the nonforward double-dipole-proton scattering amplitude

$$\langle P'|[\mathrm{Tr}(U_1^{\eta\dagger} U_3^{\eta\dagger})\mathrm{Tr}(U_3^{\eta\dagger} U_2^{\eta\dagger})$$
$$- N_c\mathrm{Tr}(U_1^\eta U_2^{\eta\dagger})](\vec{p}_1,\vec{p}_2,\vec{p}_3)|P\rangle. \quad (12)$$

In order to get phenomenological predictions for the whole process (1) at NLO, one should combine the NLO impact factors Φ_1 and Φ_2 with the two scattering amplitudes (10) and (12), which are obtained by solving the NLO JIMWLK equation with initial conditions at rapidity η_0 with $p^+_{\mathrm{target}} = e^{\eta_0}p_\gamma^+$. Then,

$$\eta - \eta_0 = \ln\frac{s}{s_0}, \quad (13)$$

where the arbitrary scale $s_0 \sim p^+_{\rm target} p^-_{\rm target} \ll s$ is a typical target scale.

In Eq. (11), Φ_2 is obtained by computing diagrams 5 and 6 (and their $q \leftrightarrow \bar{q}$ symmetric counterparts) with $\vec{p}_3 \neq \vec{0}$, see Fig. 1. Φ_1 is the sum of the $\vec{p}_3 = \vec{0}$ contribution from the same two diagrams with the contribution from diagrams 2, 3 (and its $q \leftrightarrow \bar{q}$ symmetric counterpart), and 4. For readability we will now omit the dependence on the t-channel transverse momenta in the impact factors Φ_i. The QED gauge invariance relation

$$p_\gamma \cdot \Phi_i = 0 \tag{14}$$

for $i = 0, 1, 2$ allows one to reduce the computation to the only evaluation of Φ_i^+ and $\Phi_{i\perp}^\beta$. In the following we will work in the frame where the transverse momentum of the photon is $\vec{0}$. The contributions to the $\gamma_L^* \to V_L$ and $\gamma_T^{(*)} \to V_L$ transitions are then given by

$$\varepsilon_L \cdot \Phi_i = \frac{Q}{p_\gamma^+} \Phi_i^+ \quad \text{and} \quad \varepsilon_T \cdot \Phi_i = \varepsilon_\perp \cdot \Phi_{i\perp}. \tag{15}$$

Divergences and evolution equations.—First, let us note that in the shock-wave framework, contrary to the Balitsky-Fadin-Kuraev-Lipatov (BFKL) [45] approach, the coupling to the t-channel exchanged state does not involve the QCD coupling constant. As a consequence, the LO impact factor as defined in Eq. (9) is of order α_s^0, while the NLO impact factor in Eq. (11) is of order α_s. Thus, the running of α_s has to be considered as a next-to-next-to-leading order (NNLO) effect when computing an impact factor.

The intermediate steps of the calculation involve various types of divergences, namely, ultraviolet, soft, collinear, and the spurious light-cone gauge pole (to which we will refer as the rapidity divergence). These divergences are controlled by dimensional regularization in transverse space $d \equiv 2 + 2\epsilon$, and by an infinitesimal cutoff αp_γ^+ in longitudinal space. In particular the rapidity divergence, which is regularized by the α cutoff, is canceled via the BK-JIMWLK evolution equation for the dipole operator, which allows one to get rid of the dependence on α. In momentum space it reads [22]

$$\begin{aligned}\frac{\partial}{\partial \eta}&[\mathrm{Tr}(U_1^\eta U_2^{\eta\dagger}) - N_c](\vec{p}_1, \vec{p}_2)\\ &= \alpha_s \mu^{2-d} \int \frac{d^d\vec{k}_1 d^d\vec{k}_2 d^d\vec{k}_3}{(2\pi)^{2d}} \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{p}_1 - \vec{p}_2)\\ &\quad \times \mathcal{H}(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{p}_1, \vec{p}_2)[\mathrm{Tr}(U_1^\eta U_3^{\eta\dagger})\\ &\quad \times \mathrm{Tr}(U_3^\eta U_2^{\eta\dagger}) - N_c \mathrm{Tr}(U_1^\eta U_2^{\eta\dagger})](\vec{k}_1, \vec{k}_2, \vec{k}_3),\end{aligned} \tag{16}$$

where the kernel $\mathcal{H}$ reads

$$\begin{aligned}&\mathcal{H}(\vec{p}_1, \vec{p}_2, \vec{p}_3, \vec{k}_1, \vec{k}_2)\\ &\quad = 4 \frac{(\vec{k}_1 - \vec{p}_1) \cdot (\vec{k}_2 - \vec{p}_2)}{(\vec{k}_1 - \vec{p}_1)^2 (\vec{k}_2 - \vec{p}_2)^2} + \frac{\Gamma(1 - \frac{d}{2}) \Gamma^2(\frac{d}{2})}{\Gamma(d-1)}\\ &\quad \times \left(\frac{2\pi^{\frac{d}{2}} \delta(\vec{k}_2 - \vec{p}_2)}{[(\vec{k}_1 - \vec{p}_1)^2]^{1-\frac{d}{2}}} + \frac{2\pi^{\frac{d}{2}} \delta(\vec{k}_1 - \vec{p}_1)}{[(\vec{k}_2 - \vec{p}_2)^2]^{1-\frac{d}{2}}} \right).\end{aligned} \tag{17}$$

Evolving the Wilson lines from α to η creates a counterterm to the double dipole contribution as follows:

$$\begin{aligned}\tilde{\Phi}_2^\beta(\eta, \alpha, \vec{p}_1, \vec{p}_2, \vec{p}_3) &= -\frac{\mu^{2-d}}{\Gamma(1-\epsilon)\pi^{1+\epsilon}} \ln\left(\frac{e^\eta}{\alpha}\right)\\ &\quad \times \int d^d\vec{k}_1 d^d\vec{k}_2 \delta(\vec{p}_V - \vec{p}_\gamma - \vec{k}_1 - \vec{k}_2)\\ &\quad \times \mathcal{H}(\vec{p}_1, \vec{p}_2, \vec{p}_3, \vec{k}_1, \vec{k}_2) \Phi_0^\beta(x, \vec{k}_1, \vec{k}_2).\end{aligned} \tag{18}$$

This counterterm allows one to get rid of the dependence on α in the NLO contribution. By similar arguments one can cancel the overall dependence of the impact factor on the rapidity divide η up to NNLO terms: indeed changing η to η' and evolving the LO amplitude from η to η' gives rise to $\tilde{\Phi}_2^\beta(\eta', \eta, \vec{p}_1, \vec{p}_2, \vec{p}_3)$, whose dependence on η is canceled by combining it with the NLO impact factor.

The collinear divergence is canceled via the Efremov-Radyushkin-Brodsky-Lepage (ERBL) evolution equation for the twist 2 DA φ. In the $\overline{\rm MS}$ scheme it reads

$$\frac{\partial \varphi(x, \mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s C_F}{2\pi} \frac{\Gamma(1-\epsilon)}{(4\pi)^\epsilon} \left(\frac{\mu_F^2}{\mu^2}\right)^\epsilon \int_0^1 dz \varphi(z, \mu_F) \mathcal{K}(x, z), \tag{19}$$

where $C_F \equiv (N_c^2 - 1)/(2N_c)$ is the eigenvalue of the Casimir operator in the fundamental representation of $SU(N_c)$ and $\mathcal{K}(x, z)$ is the well known ERBL evolution kernel [32]

$$\begin{aligned}\mathcal{K}(x, z) &= \frac{1-x}{1-z} \left(1 + \left[\frac{1}{x-z}\right]_+\right) \theta(x-z)\\ &\quad + \frac{x}{z} \left(1 + \left[\frac{1}{z-x}\right]_+\right) \theta(z-x) + \frac{3}{2} \delta(z-x).\end{aligned} \tag{20}$$

For a function $F(z)$ that behaves as $F_0 + F_1 \ln(z - z_0)$ for $z \to z_0$ we defined the $+$ prescription as

$$\int_0^1 dz \left[\frac{1}{z - z_0}\right]_+ F(z) \equiv \int_0^1 dz \frac{F(z) - F_0 - F_1 \ln(z - z_0)}{z - z_0}. \tag{21}$$

Evolving the DA in the LO contribution from 0 to μ_F gives rise to a counterterm to the NLO dipole contribution, which reads

$$\tilde{\Phi}_1^{\beta}(x,\mu_F) = -\int_0^1 dz\mathcal{K}(z,x)\left[\frac{1}{\epsilon}+\ln\left(\frac{\mu_F^2}{\mu^2}\right)\right]\Phi_0^{\beta}(z). \tag{22}$$

Infrared finiteness and final results.—The leading order impact factor reads

$$\Phi_0^{+}(x) = \frac{2x\bar{x}(p_V^{+})^2}{[(\bar{x}\vec{p}_1 - x\vec{p}_2)^2 + x\bar{x}Q^2]}, \tag{23}$$

$$\Phi_{0\perp}^{\beta}(x) = \frac{(x-\bar{x})p_V^{+}(\bar{x}p_{1\perp}^{\beta} - xp_{2\perp}^{\beta})}{[(\bar{x}\vec{p}_1 - x\vec{p}_2)^2 + x\bar{x}Q^2]}, \tag{24}$$

where $\bar{x} \equiv 1-x$.

Let us consider separately the NLO dipole contribution Φ_1 and the double dipole contribution Φ_2 since they are independently gauge invariant and infrared finite, and the mechanisms for the cancellation of their divergences are different.

The sum of the dipole contribution from each diagram with the contribution in Eq. (22) from the ERBL evolution of the DA is finite. It reads

$$\begin{aligned}\Phi_1^{+}(x) = {}& \int_0^x dz\left(\frac{x-z}{x}\right)\left[1+\left(1+\left[\frac{1}{z}\right]_+\right)\ln\left(\frac{([(\bar{x}+z)\vec{p}_1-(x-z)\vec{p}_2]^2+(x-z)(\bar{x}+z)Q^2)^2)}{\mu_F^2(x-z)(\bar{x}+z)Q^2}\right)\right]\Phi_0^{+}(x-z)\\ &+\frac{1}{2}\Phi_0^{+}(x)\left[\frac{1}{2}\ln^2\left(\frac{\bar{x}}{x}\right)+3-\frac{\pi^2}{6}-\frac{3}{2}\ln\left(\frac{((\bar{x}\vec{p}_1-x\vec{p}_2)^2+x\bar{x}Q^2)^2)}{x\bar{x}\mu_F^2Q^2}\right)\right]\\ &+\frac{(p_\gamma^{+})^2}{2x\bar{x}}\int_0^x dz[(\phi_5)_{\mathrm{LL}}|_{\vec{p}_3=\vec{0}}+(\phi_6)_{\mathrm{LL}}|_{\vec{p}_3=\vec{0}}]_+ + (x\leftrightarrow\bar{x},\vec{p}_1\leftrightarrow\vec{p}_2)\end{aligned} \tag{25}$$

for a longitudinal photon, and

$$\begin{aligned}\Phi_{1\perp}^{\beta}(x) = {}& \frac{1}{4}\left[\ln^2\left(\frac{\bar{x}}{x}\right)-\frac{\pi^2}{3}+6-3\ln\left(\frac{(\bar{x}\vec{p}_1-x\vec{p}_2)^2+x\bar{x}Q^2}{\mu_F^2}\right)+3\frac{x\bar{x}Q^2}{(\bar{x}\vec{p}_1-x\vec{p}_2)^2}\ln\left(\frac{(\bar{x}\vec{p}_1-x\vec{p}_2)^2+x\bar{x}Q^2}{x\bar{x}Q^2}\right)\right]\Phi_{0\perp}^{\beta}(x)\\ &+\int_0^x dz\left(\frac{x-z}{x}\right)\Phi_{0\perp}^{\beta}(x-z)\left[1+\left(1+\left[\frac{1}{z}\right]_+\right)\ln\left(\frac{[(\bar{x}+z)\vec{p}_1-(x-z)\vec{p}_2]^2+(x-z)(\bar{x}+z)Q^2}{\mu_F^2}\right)\right.\\ &\left.-\left(1+\left[\frac{1}{z}\right]_+\right)\frac{(x-z)(\bar{x}+z)Q^2}{[(\bar{x}+z)\vec{p}_1-(x-z)\vec{p}_2]^2}\ln\left(\frac{[(\bar{x}+z)\vec{p}_1-(x-z)\vec{p}_2]^2+(x-z)(\bar{x}+z)Q^2}{(x-z)(\bar{x}+z)Q^2}\right)\right]\\ &+\frac{p_\gamma^{+}}{2x\bar{x}}\int_0^x dz[(\phi_5)_{\mathrm{TL}}^{\beta}|_{\vec{p}_3=\vec{0}}+(\phi_6)_{\mathrm{TL}}^{\beta}|_{\vec{p}_3=\vec{0}}]_+ + (x\leftrightarrow\bar{x},\vec{p}_1\leftrightarrow\vec{p}_2)\end{aligned} \tag{26}$$

for a transverse photon. The quantities $(\phi_{5,6})_{\mathrm{LL}}$ and $(\phi_{5,6})_{\mathrm{TL}}^{\beta}$ can be extracted from Ref. [22], with the change of variables $(\vec{p}_q,\vec{p}_{\bar{q}})\to(x\vec{p}_V,\bar{x}\vec{p}_V)$ [46]. One should understand $[\phi]_+$ in Eqs. (25) and (26) as the finite term that results from the replacement of the $1/z$ pole in ϕ by the + prescription as defined in Eq. (21). The total dependence on the dimensional regulator μ cancels as expected, and the absence of a renormalization scale is due to the absence of running coupling contributions in the impact factor at this order. In the final expressions, ϕ_6 terms are evaluated by replacing μ by μ_F in Ref. [22].

The sum of the double dipole contributions with the contribution (18) from the BK-JIMWLK evolution of the dipole operator is finite and reads

$$\begin{aligned}\Phi_2^{+} = {}& -\frac{x\bar{x}(p_\gamma^{+})^2((x\vec{p}_V-\vec{p}_1)^2+(\bar{x}\vec{p}_V-\vec{p}_2)^2-\vec{p}_3^{\,2}+2x\bar{x}Q^2)}{((x\vec{p}_V-\vec{p}_1)^2+x\bar{x}Q^2)((\bar{x}\vec{p}_V-\vec{p}_2)^2+x\bar{x}Q^2)-x\bar{x}\vec{p}_3^{\,2}Q^2}\\ &\times\ln\left(\frac{x\bar{x}}{e^{2\eta}}\right)\ln\left(\frac{((x\vec{p}_V-\vec{p}_1)^2+x\bar{x}Q^2)((\bar{x}\vec{p}_V-\vec{p}_2)^2+x\bar{x}Q^2)}{x\bar{x}\vec{p}_3^{\,2}Q^2}\right)\\ &-\frac{4x\bar{x}(p_\gamma^{+})^2}{(x\vec{p}_V-\vec{p}_1)^2+x\bar{x}Q^2}\ln\left(\frac{\bar{x}}{e^{\eta}}\right)\ln\left(\frac{\vec{p}_3^{\,2}}{Q^2}\right)+\frac{(p_\gamma^{+})^2}{2x\bar{x}}\int_0^x dz[(\phi_5)_{\mathrm{LL}}+(\phi_6)_{\mathrm{LL}}]_+ + (x\leftrightarrow\bar{x},\vec{p}_1\leftrightarrow\vec{p}_2)\end{aligned} \tag{27}$$

for a longitudinal photon, and

$$\Phi^{\beta}_{2\perp}(x) = p_\gamma^+ (x p^{\beta}_{V\perp} - p^{\beta}_{1\perp})(\bar{x} - x)\left(\frac{-2}{(x\vec{p}_V - \vec{p}_1)^2 + x\bar{x}Q^2}\ln\left(\frac{\vec{p}_3^{\,2}}{Q^2}\right)\ln\left(\frac{\bar{x}}{e^{\eta}}\right)\right.$$
$$+\ln\left(\frac{x\bar{x}}{e^{2\eta}}\right)\left[\frac{1}{(x\vec{p}_V - \vec{p}_1)^2}\ln\left(\frac{(x\vec{p}_V - \vec{p}_1)^2 + x\bar{x}Q^2}{x\bar{x}Q^2}\right) - \frac{((\bar{x}\vec{p}_V - \vec{p}_2)^2 + x\bar{x}Q^2)\ln\left(\frac{((x\vec{p}_V - \vec{p}_1)^2 + x\bar{x}Q^2)((\bar{x}\vec{p}_V - \vec{p}_2)^2 + x\bar{x}Q^2)}{x\bar{x}\vec{p}_3^{\,2}Q^2}\right)}{[(x\vec{p}_V - \vec{p}_1)^2 + x\bar{x}Q^2][(\bar{x}\vec{p}_V - \vec{p}_2)^2 + x\bar{x}Q^2] - x\bar{x}\vec{p}_3^{\,2}Q^2}\right]\Bigg)$$
$$+\frac{p_\gamma^+}{2x\bar{x}}\int_0^x dz[(\phi_5)^{\beta}_{\mathrm{TL}} + (\phi_6)^{\beta}_{\mathrm{TL}}]_+ + (x \leftrightarrow \bar{x}, \vec{p}_1 \leftrightarrow \vec{p}_2) \quad (28)$$

for a transverse photon.

Discussion.—An explicit check shows that Eqs. (26) and (28) are still valid in the $Q^2 = 0$ (photoproduction) limit despite the presence of $\ln Q^2$ terms since they cancel one another.

None of the results in the present Letter contains endpoint singularities ($x \to 0$ or 1), even in the photoproduction limit: the presence of nonzero transverse momenta in the t channel allows one to regularize such singularities.

Our small-x treatment of the target resums contributions to the amplitude of all twists that are proportional to the leading power of $1/x$ and that are enhanced by the leading $\alpha_s^n \ln^n 1/x$ and first subleading $\alpha_s^n \ln^{n-1} 1/x$ powers of energy logarithms. What we neglect are the contributions that, in comparison to those considered by us, are suppressed by additional powers of Λ^2/Q^2, with Λ the nonperturbative scale describing the vector meson.

The remaining dependence of the amplitude on the factorization and renormalization scales μ_F and μ_R and on the arbitrary parameter s_0 is only of next-to-next-to-leading logarithmic orders.

Our results were obtained for arbitrary kinematics, in the shock-wave approach. It would be interesting to compare them (in the linear limit for the double dipole contribution) with the result of Ref. [47], which was obtained with forward kinematics and for a longitudinally polarized photon, in the usual k_t-factorization framework of the linear BFKL approach. Still, a detailed comparison is not straightforward since the distribution of radiative corrections between the kernel and the impact factor is different in the BK and in BFKL frameworks [48]. Nontrivial kernel and impact factor transformations are required for such a comparison, which is left for further studies [39].

Conclusion.—In this Letter, we have obtained for the first time the complete NLO impact factor for the $\gamma^{(*)}_{L,T} \to V_L$ transitions in the shock-wave framework. This study contributes to an understanding of the physics of gluonic dynamics not only in the small-x regime but also permits us to discriminate between a concurrent description of the same process based on the so called transverse momentum dependent partonic distributions in the common kinematic domain.

The obtained result, when combined with solutions to the NLO BK-JIMWLK evolution and to the leading twist NLO ERBL equation, allows for the very first complete NLO study of exclusive meson production at asymptotic energies with the inclusion of saturation effects.

It paves the way for precision studies of small-x QCD and saturation physics of nucleon and nuclei with a diverse range of phenomenological applications for present and future colliders.

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