Quantum Fluctuations in Quasi-One-Dimensional Dipolar Bose-Einstein Condensates

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Recent experiments have revealed that beyond-mean-field corrections are much more relevant in weakly interacting dipolar condensates than in their nondipolar counterparts. We show that in quasi-one-dimensional geometries quantum corrections in dipolar and nondipolar condensates are strikingly different due to the peculiar momentum dependence of the dipolar interactions. The energy correction of the condensate presents not only a modified density dependence, but it may even change from attractive to repulsive at a critical density due to the surprising role played by the transversal directions. The anomalous quantum correction translates into a strongly modified physics for quantum-stabilized droplets and dipolar solitons. Moreover, and for similar reasons, quantum corrections of three-body correlations, and hence of three-body losses, are strongly modified by the dipolar interactions. This intriguing physics can be readily probed in current experiments with magnetic atoms.

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Introduction.—Quantum fluctuations introduce a shift of the ground-state energy of a Bose gas, which at first order is given by the well-known Lee-Huang-Yang (LHY) correction [1]. However, in the weakly interacting regime, experiments on Bose-Einstein condensates are well described within the mean-field approximation. The situation may be crucially different in the presence of competing interactions, as recently discussed in the context of Bose-Bose mixtures [2]. In that scenario, the interplay between inter- and intra-species interactions results, at the verge of mean-field instability, in a dominant LHY correction well within the weakly interacting regime. The LHY correction may stabilize a collapsing condensate, resulting in the formation of quantum droplets, a novel ultradilute liquid whose surface tension is provided by purely quantum effects.

Dipolar condensates, formed by particles with large magnetic or electric dipolar moments, are also characterized by competing interactions, in this case short-range and dipole-dipole interactions. Indeed, recent experiments on highly magnetic atoms have revealed the crucial role played by quantum fluctuations at the mean-field instability, showing for the first time the formation of quantum droplets [3], which may remain self-bound even in the absence of external trapping [4]. Quantum stabilization and droplet formation have attracted wide theoretical and experimental attention [5–12], being a general phenomenon that is expected to characterize not only condensates of magnetic atoms, but the whole rapidly developing field of strongly dipolar gases [13,14].

In Bose-Bose mixtures and in dipolar condensates, quantum stabilization stems from the compensation between the attractive residual mean-field interaction, proportional to the 3D density n_{3D}, and the repulsive LHY correction, which in both systems is proportional to $n_{3D}^{3/2}$ [2,15]. As a result, there is a critical density at which both contributions compensate. Quantum fluctuations play an even more intriguing role in lower dimensions. In particular, droplets are stabilized for a sufficiently low density in 1D Bose-Bose mixtures [16], against melting rather than collapse, by the competition of a residual repulsive mean-field term, proportional to the 1D density n_{1D}, and the attractive LHY correction, proportional to $-n_{1D}^{1/2}$.

Whereas beyond-mean-field effects in 3D Bose-Bose mixtures and dipolar condensates are very similar due to the almost identical density dependence of the quantum correction, we show in this Letter that quantum fluctuations lead in quasi-1D dipolar condensates to a strikingly different physics compared to their nondipolar counterparts. This difference stems from the peculiar momentum dependence of the dipole-dipole interactions in quasi-1D geometries [17]. As a result, not only is the density dependence of the quantum corrections very different, but even its sign may change due to the remarkable role played by transversal directions in dipolar gases well within the 1D regime. The anomalous quantum corrections change the nature of quantum stabilization and strongly influence the physics of solitons. We also show that, whereas three-body correlations present the same density dependence in 3D dipolar and nondipolar condensates [18], they display in 1D a radically different dependence.

Dipolar interaction in 1D.—We consider bosons with mass M and magnetic moment $\vec{\mu}_D$, although our results also apply for electric dipoles. The system is strongly confined on the xy plane by an isotropic harmonic trap of frequency $\omega_\perp$, but it is untrapped along z. We assume that the chemical potential $|\mu| \ll \hbar\omega_\perp$, and hence the condensate, remains kinematically 1D such that its wave function splits as $\Psi(\vec{r}) = \psi(x,y)\phi(z)$, with $\psi(x,y) = e^{-(x^2+y^2)/2l_\perp^2}/\sqrt{\pi}l_\perp$ the ground state of the transversal trap, with $l_\perp^2 = \hbar/M\omega_\perp$. After integrating over x and y, the interaction between particles in the condensate acquires a momentum dependence k of the form

$$\tilde{V}_{\mathrm{1D}}(k) = g_{\mathrm{1D}}\{1 + \epsilon_{\mathrm{dd}}[3F(0, k^2 l_\perp^2/2) - 1]\}, \qquad (1)$$

with $F(j,\sigma) \equiv \sigma^{j+1}e^{\sigma}\Gamma(-j,\sigma)$ [17], where $\Gamma(-j,\sigma)$ is the incomplete Gamma function. Short-range interactions are characterized by the 1D coupling constant $g_{\mathrm{1D}} = g_{\mathrm{3D}}/2\pi l_\perp^2$, where $g_{\mathrm{3D}} = 4\pi\hbar^2 a/M$, with $a > 0$ the s-wave scattering length. Assuming $\vec{\mu}_D$ along z, $\epsilon_{\mathrm{dd}} = \mu_0\mu_D^2/3g_{\mathrm{3D}}$ is the ratio between the strengths of the dipolar and contact interactions [19], with μ_0 the vacuum permeability. This 1D condition $|\mu|/\hbar\omega_\perp \ll 1$ demands $|1-\epsilon_{\mathrm{dd}}| \ll 1/2n_{\mathrm{1D}}a$, a condition satisfied in all the calculations in this Letter [20].

LHY correction.—Single particle excitations, (n_r, m, k), are characterized by their radial quantum number n_r, angular momentum m, and axial linear momentum k. In 1D contact-interacting systems, transversal excitations, with $(n_r, m) \neq (0,0)$, play a negligible role in beyond-mean-field corrections. This may be crucially different in dipolar gases. In the weakly interacting regime, the main processes involving condensed and excited particles are sketched in Fig. 1. A collision between a particle in (n_r, m, k) and one in the condensate $(0,0,0)$ (left) preserves both m and k but may change the radial number into n_r'. On the other hand two condensed particles may collide (right) and create excitations in (n_r, m, k) and $(n_r', -m, -k)$. Both processes are characterized by the interaction energy [21]

$$[\hat{U}_m(k)]_{n_r,n_r'} = g_{\mathrm{1D}}n_{\mathrm{1D}}C_{n_r,n_r',m}F\left(n_r + n_r' + m, \frac{k^2 l_\perp^2}{2}\right), \qquad (2)$$

where $C_{n_r,n_r',m} = 6(-1)^{n_r+n_r'}/2^{n_r+n_r'+m+1}\times\sqrt{\binom{n_r+n_r'+m}{n_r}\binom{n_r+n_r'+m}{n_r'}}$, and we have considered for simplicity $\epsilon_{\mathrm{dd}} = 1$ [22]. It is crucial that, although for $\epsilon_{\mathrm{dd}} = 1$ the compensation of dipolar and contact interactions results in an ideal 1D condensate $\{\tilde{V}_{\mathrm{1D}}(0) = [\hat{U}_0(0)]_{0,0} = 0\}$, $[\hat{U}_m(k)]_{n_r,n_r'}$ may be of the order of $g_{\mathrm{1D}}n_{\mathrm{1D}}$. Because of this peculiar feature, which stems from the momentum dependence of the dipolar interactions, transversal excitations play in dipolar gases a key role in quantum corrections if $g_{\mathrm{1D}}n_{\mathrm{1D}} \gtrsim \hbar\omega_\perp$ despite the 1D character of the condensate.

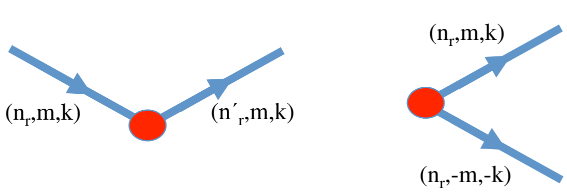

FIG. 1. Dominant collisions between particles in the condensate and in excited states (see text).

The elementary excitations may be obtained for each m and k from the Bogoliubov–de Gennes equations

$$\xi_\nu\begin{pmatrix}\vec{u}_\nu\\ \vec{v}_\nu\end{pmatrix} = \begin{pmatrix}\hat{E}_m(k)+\hat{U}_m(k) & \hat{U}_m(k)\\ -\hat{U}_m(k) & -\hat{E}_m(k)-\hat{U}_m(k)\end{pmatrix}\begin{pmatrix}\vec{u}_\nu\\ \vec{v}_\nu\end{pmatrix}, \qquad (3)$$

where $[\hat{E}_m(k)]_{n_r,n_r'} = E_{n_r m}(k)\delta_{n_r,n_r'}$, with $E_{n_r m}(k) = \hbar k^2/2M + \hbar\omega_\perp(2n_r + m)$. Following a similar procedure as in Ref. [23], the LHY energy correction ΔE_{LHY} may be obtained from the differential equation [21]

$$\frac{\Delta E_{\mathrm{LHY}}}{L} - \frac{1}{2}n_{\mathrm{1D}}\frac{d}{dn_{\mathrm{1D}}}\left(\frac{\Delta E_{\mathrm{LHY}}}{L}\right) = \frac{1}{2}\sum_m\int_{-\infty}^{\infty}\frac{dk}{2\pi}\sum_\nu\sum_{n_r}[E_{n_r,m}(k) - \xi_\nu](\vec{v}_\nu)^2_{n_r}, \qquad (4)$$

with L the quantization length [24]. Figure 2 shows the LHY correction of the chemical potential, $\Delta\mu_{\mathrm{LHY}} = (d/dn_{\mathrm{1D}})(\Delta E_{\mathrm{LHY}}/L)$, for different $g_{\mathrm{1D}}n_{\mathrm{1D}}/2\hbar\omega_\perp = n_{\mathrm{1D}}a$.

For $n_{\mathrm{1D}}a \ll 1$, the effect of the transversal modes is, as expected, negligible, and the LHY correction remains attractive. However, whereas for contact interacting systems $\Delta\mu_{\mathrm{LHY}} \propto -n_{\mathrm{1D}}^{1/2}$ [16], the density dependence in dipolar condensates is radically different. For $n_{\mathrm{1D}}a \to 0$, $\Delta\mu_{\mathrm{LHY}} \propto -n_{\mathrm{1D}}$, whereas for growing $n_{\mathrm{1D}}a$, $\Delta\mu_{\mathrm{LHY}}$ departs from the linear dependence (top inset of Fig. 2). This is crucial for the physics of 1D droplets, as discussed below.

For $n_{\mathrm{1D}}a \gtrsim 0.1$, transversal excitations become significant. The LHY correction reaches a maximal negative value at $n_{\mathrm{1D}}a \simeq 0.2$ and then increases, becoming repulsive for

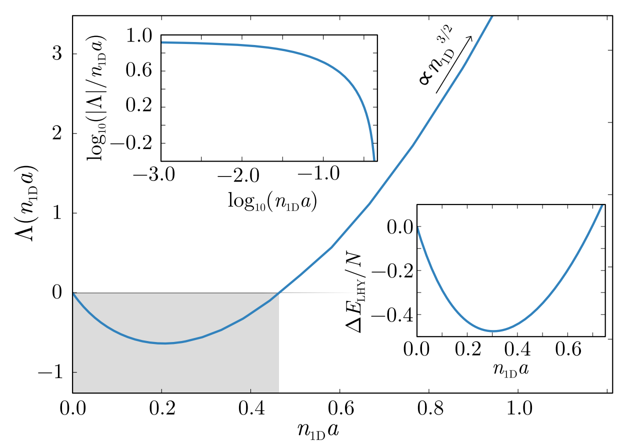

FIG. 2. $\Lambda \equiv (\Delta\mu_{\mathrm{LHY}}/\hbar\omega_\perp)(l_\perp/a)$ as a function of $n_{\mathrm{1D}}a$ for a homogeneous 1D dipolar condensate. The top inset depicts $\log_{10}(|\Lambda|/n_{\mathrm{1D}}a)$ against $\log_{10}(n_{\mathrm{1D}}a)$, showing that $\Delta\mu_{\mathrm{LHY}} \propto n_{\mathrm{1D}}$ for $n_{\mathrm{1D}} \to 0$. The bottom inset shows $\Delta E_{\mathrm{LHY}}/N$ as a function of $n_{\mathrm{1D}}a$.

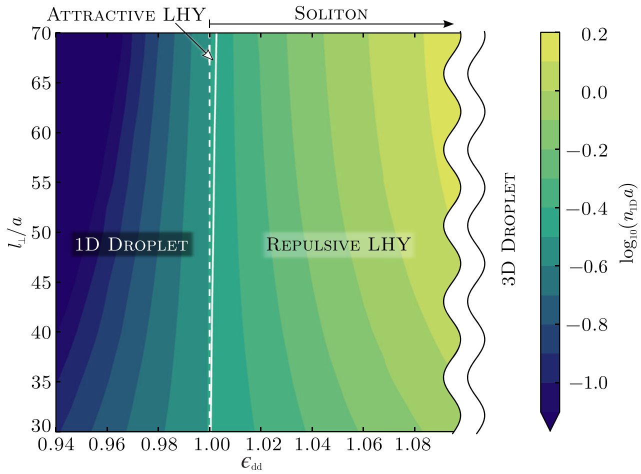

FIG. 3. Peak density for $N = 5000$ atoms as a function of $l_\perp/a$ and ϵ_{dd}. At $\epsilon_{dd} > 1$ (dashed vertical line) the droplet regime smoothly crosses over into the soliton regime. When $n_{1D}a \ll 1$, the anomalous density dependence of $\Delta\mu_{LHY}$ results in droplet melting. The solid line marks the point at which the LHY becomes in average repulsive.

$n_{1D}a > 0.42$. For $(n_{1D}a) \gg 1$, $\Delta\mu_{LHY} \propto n_{1D}^{3/2}$, i.e., the LHY correction becomes that expected for a 3D condensate [15]. This radical change in the nature of the quantum correction for a condensate well within the 1D regime constitutes a striking qualitatively novel feature of quasi-1D dipolar gases.

Phase diagram.—We consider at this point an axially untrapped, but possibly self-bound, condensate with an axial width $R \gg l_\perp$. In that case the use of the local density approximation, i.e., substituting in Eq. (4) n_{1D} by $n_{1D}(z)$, is well justified since the momenta contributing most to the LHY correction fulfill $kR \gg 1$. The resulting generalized Gross-Pitaevskii equation is

$$\mu\phi(z) = \frac{-\hbar^2}{2M}\frac{d^2\phi}{dz^2} + \{\mu_{MF}[n_{1D}(z)] + \Delta\mu_{LHY}[n_{1D}(z)]\}\phi(z), \tag{5}$$

with μ the chemical potential, and $\mu_{MF}[n_{1D}(z)] = \int(dk/2\pi)\tilde{V}_{1D}(k)\tilde{n}_{1D}(k)e^{ikz}$ the mean-field interaction, with $\tilde{n}_{1D}(k)$ the Fourier transform of $n_{1D}(z)$.

Figure 3 depicts the peak density for $N = \int_{-\infty}^{\infty} dz\, n_{1D}(z) = 5000$ particles as a function of ϵ_{dd} and $l_\perp/a$ (which must be $\gg 1$ to guarantee the 3D nature of the scattering [25]). Neglecting quantum corrections, the interactions are repulsive for $\epsilon_{dd} < 1$ preventing any self-bound solution (see Fig. 4), whereas for $\epsilon_{dd} > 1$ the attractive interactions lead to the formation of a soliton.

For sufficiently low densities, the effective LHY attraction results as in Bose-Bose mixtures [16] in the formation for $\epsilon_{dd} \leq 1$ of self-bound droplets (see Figs. 3 and 4) that present a flattop profile (inset of Fig. 4). Note that at $\epsilon_{dd} = 1$, the mean-field contribution vanishes. The droplet acquires, however, a finite peak density, $n_{1D}^{peak}a \simeq 0.3$ (inset of Fig. 4), at which $\Delta E_{LHY}/N$ is minimal (bottom inset of Fig. 2). Note that this minimum, and hence universal, peak droplet density at $\epsilon_{dd} = 1$ also results from the nontrivial role played by the transversal degrees of freedom.

When ϵ_{dd} is lowered, the density decreases and the system enters in the regime in which $\Delta\mu_{LHY} \propto -n_{1D}$. Since the LHY correction and the mean-field energy have an equal density dependence, the competition between both energies crucial for quantum stabilization is absent, and the system undergoes an abrupt *droplet inflation* into the unbound solution. The latter must be compared to the case of nondipolar Bose-Bose mixtures, which are characterized by a fixed dependence $\Delta\mu_{LHY} \propto -n_{1D}^{1/2}$. As a result, the competition between mean-field energy and LHY correction remains efficient in binary mixtures even at very low densities, and hence the peak density smoothly decreases within the mean-field unbound region without any droplet inflation.

At $\epsilon_{dd} = 1$, the system smoothly crosses over into the soliton regime. For $\epsilon_{dd} > 1$, the soliton density grows smoothly for increasing ϵ_{dd}, and the LHY correction changes eventually from attractive to repulsive. When this occurs, the soliton density is significantly lower than that expected from mean-field theory (up to a factor of 2 in Fig. 4). Moreover, since for large-enough densities, $\Delta\mu_{LHY} \propto n_{1D}^{3/2}$, the effect of the LHY correction remains relevant even far from the mean-field instability. This must be compared to the case of Bose-Bose mixtures, where the $\Delta\mu_{LHY} \propto -n_{1D}^{1/2}$ dependence renders the LHY correction

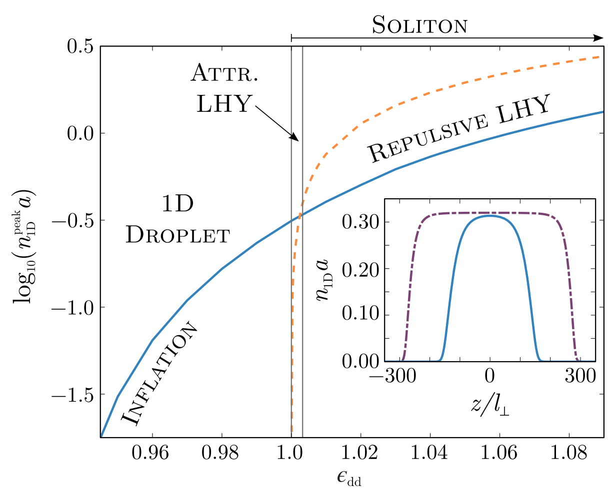

FIG. 4. Peak density, n_{1D}^{peak}, of a 1D self-bound dipolar condensate of $N = 5000$ particles with $l_\perp/a = 65$ as a function of ϵ_{dd}. The dashed and solid curves depict, respectively, the mean-field results and those taking into account the LHY correction. For a discussion of the different regions, see text. In the inset we depict a typical flattop droplet profile for $\epsilon_{dd} = 1$ with $l_\perp/a = 65$ (solid) and 30 (dot-dashed).

basically negligible within the soliton regime. Note that for sufficiently large $\epsilon_{dd} > 1$, eventually $\mu \gtrsim \hbar\omega_\perp$, and the condensate crosses over into the 3D regime, where the repulsive LHY prevents collapse. This would correspond to the elongated 3D macro-droplet regime recently explored experimentally [12]. The description of this crossover, however, lies beyond the scope of this Letter.

Three-body correlations.—Whereas in mean-field approximation three-body correlations fulfill $g^{(3)} = [1/n(\vec{r})^3]\langle\hat{\Psi}^\dagger(\vec{r})^3\hat{\Psi}(\vec{r})^3\rangle = 1$, quantum corrections may significantly correct its value, $g^{(3)} = 1 + \Delta g^{(3)}$, and hence in turn the three-body loss rate. For homogeneous 3D nondipolar condensates with density n_{3D}, $\Delta g^{(3)} \simeq (64/\sqrt{\pi})(n_{3D}a^3)^{1/2}$ [18], as confirmed in recent experiments [26]. As for the LHY correction, in 3D homogenous dipolar condensates, the correction of $g^{(3)}$ is very similar: $\Delta g^{(3)} \simeq (64/\sqrt{\pi})(n_{3D}a^3)^{1/2}(1 + C\epsilon_{dd}^2)$, with $C \simeq 0.3$ [27]. Dipolar interactions hence introduce corrections that may be sizable in current experiments with magnetic atoms, but the density dependence of $g^{(3)}$ is identical to that of nondipolar condensates.

The situation is radically different in 1D. For a 1D nondipolar condensate $\Delta g^{(3)} = -(6/\pi)\sqrt{\gamma}$ [28], with $\gamma = 2a/n_{1D}l_\perp^2 \ll 1$. Three-body correlations are hence reduced by quantum effects, and the correction increases for a decreasing density, since, counterintuitively, 1D systems are more strongly interacting the more dilute they are. As for the LHY correction, the momentum dependence of the dipolar interactions leads to a markedly different density dependence in dipolar condensates. The correction of $g^{(3)}$ averaged over the transversal degree of freedom [29] may be evaluated from the LHY correction using the Hellmann-Feynman theorem [21],

$$\Delta g^{(3)} \equiv \int \frac{d^3r}{L} \frac{\psi(x,y)^4}{\int dx'dy'\psi(x',y')^4} \left(\frac{\langle\hat{\Psi}^\dagger(\vec{r})^3\hat{\Psi}(\vec{r})^3\rangle}{n(\vec{r})^3} - 1 \right)$$
$$= \frac{6}{n_{1D}^2 L} \frac{\partial \Delta E_{LHY}}{\partial g_{1D}} = -\frac{6}{\pi}\sqrt{\gamma}\beta(\epsilon_{dd}, n_{1D}a), \qquad (6)$$

where $\beta(\epsilon_{dd}, n_{1D}a)$ is depicted in Fig. 5. For small $n_{1D}a$, $\Delta g^{(3)} \propto -n_{1D}^\lambda$, with $-1/2 < \lambda < 0$. As for nondipolar condensates, $\Delta g^{(3)}$ remains negative and increases with decreasing n_{1D}, albeit with a significantly modified power law. In contrast, when $n_{1D}a > 0.42$, the growing role of the transversal modes results in a change in the sign of $\Delta g^{(3)}$; i.e., three-body correlations are enhanced rather than reduced by quantum effects despite the fact that the condensate remains in the 1D regime. For $n_{1D}a \gg 1$, $\Delta g^{(3)} \propto n_{1D}^{1/2}$, as expected for 3D condensates. This nontrivial behavior of three-body correlations in quasi-1D dipolar condensates may be probed in on-going experiments with magnetic atoms using similar techniques as those applied in nondipolar quasi-1D condensates [26].

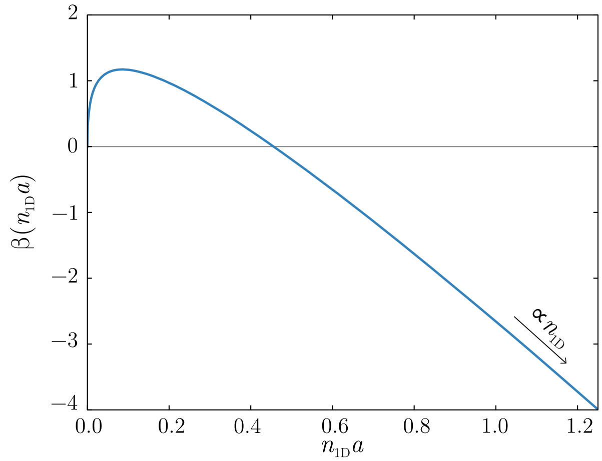

FIG. 5. Correction of the three-body correlations, $\beta(\epsilon_{dd} = 1, n_{1D}a) = -\Delta g^{(3)}(\pi/6\sqrt{\gamma})$.

Conclusions.—The momentum dependence of the dipolar interactions leads to strikingly different quantum effects in quasi-1D dipolar condensates compared to their nondipolar counterparts. In contrast to Bose-Bose mixtures, quantum stabilization is disrupted in dipolar condensates at low densities due to the modified density dependence of the LHY correction. As a result quantum droplets only exist in a window of density values. Moreover, although the condensate remains one dimensional, the LHY may be crucially affected by transversal modes, which induce a change from attractive to repulsive LHY correction at a critical density. This change of character results in a significant reduction of the peak density of the soliton, as well as a modification of its shape. Hence quantum corrections should be carefully considered in future studies of dipolar solitons. Furthermore, the peculiar nature of quantum fluctuations is also reflected in the beyond-mean-field correction of three-body losses, which also changes its sign within the 1D regime for growing density. Our results open intriguing questions about 2D dipolar condensates, where we expect a similar nontrivial density dependence of the quantum corrections, as well as about the role of transverse modes in anharmonic transversal confinements. This surprising physics of low-dimensional dipolar condensates can be readily probed in current experiments with magnetic atoms.

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[20] In a homogeneous quasi-1D dipolar condensate, $\mu = g_{1\mathrm{D}}n_{1\mathrm{D}}(1 - \epsilon_{\mathrm{dd}})$. Hence $\mu/\hbar\omega_\perp = 2n_{1\mathrm{D}}a(1 - \epsilon_{\mathrm{dd}})$.
[21] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.119.050403 for derivation of Eq. (2), (4) and (6).
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