Novel Role of Superfluidity in Low-Energy Nuclear Reactions

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We demonstrate, within symmetry unrestricted time-dependent density functional theory, the existence of new effects in low-energy nuclear reactions which originate from superfluidity. The dynamics of the pairing field induces solitonic excitations in the colliding nuclear systems, leading to qualitative changes in the reaction dynamics. The solitonic excitation prevents collective energy dissipation and effectively suppresses the fusion cross section. We demonstrate how the variations of the total kinetic energy of the fragments can be traced back to the energy stored in the superfluid junction of colliding nuclei. Both contact time and scattering angle in noncentral collisions are significantly affected. The modification of the fusion cross section and possibilities for its experimental detection are discussed.

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Introduction.—The dynamics of the pairing field during the nuclear reactions has rarely been investigated to date, although it is well known that the static pairing field is crucial for the description of the atomic nuclei, both in the ground state as well as in excited states (see, e.g., Refs. [1–5] and references therein). The reason is twofold: first, it is believed that the pairing field dynamics will produce only small corrections to the commonly accepted picture of low-energy nuclear reactions; second, the proper treatment of the pairing field dynamics requires us to use more advanced approaches resulting in a rapid increase in computational complexity. On the other hand, it is well known that the pairing correlations give rise to abundant fascinating phenomena, like topological excitations, observed with great details in superfluid helium [6] or ultracold atomic gases [7,8]. For example, in experiments with ultracold atomic gases, where two clouds of atomic Bose-Einstein condensates (BEC) are forced to merge, the interface between the two BECs may lose its superfluid character (solitonic excitation). This excitation is unstable, and decays through quantum vortices [9,10]. In this Letter, we investigate the possibility of creating similar excitations in nuclear reactions; see Fig. 1.

The pairing field in nuclear systems is small in a sense that the ratio of its magnitude to the Fermi energy does not exceed 5%. It implies that BCS treatment is regarded as a justified approximation and the size of the Cooper pair is of the same order as the size of a heavy nucleus. Although the pairing field is small as compared to, e.g., the unitary Fermi gas [11], it is important for the proper description of the nuclear systems: while it smears out shell effects responsible for static deformations, it also enables large-amplitude collective motion which otherwise would be strongly damped. Therefore, the description of nuclear fission requires us to take into account superfluidity as one of the crucial ingredients [12–14]. Recently, it has been pointed out that dynamic excitations of the pairing field, which is absent in the static treatment, affect significantly the induced fission process leading to much longer fission time scales than predicted by other simplified approaches [15].

The pairing field $\Delta(\boldsymbol{r})$ can be regarded as an order parameter that specifies whether the nucleus is superfluid or not [16]. The order parameter belongs to the U(1) universality class and it can be decomposed as $\Delta(\boldsymbol{r}) = |\Delta(\boldsymbol{r})|e^{i\varphi(\boldsymbol{r})}$. In the ground state the phase is uniform across the nucleus, and it can be absorbed by the gauge transformation. Then the only relevant quantity is its absolute value $|\Delta(\boldsymbol{r})|$, which is on the order of 1 MeV. The situation is different when two superfluid nuclei collide. Then the relative phase $\Delta\varphi$ between two pairing fields is well defined (see Fig. 1) and

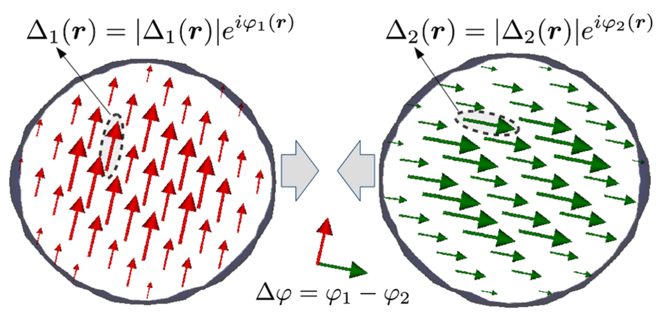

FIG. 1. Schematic picture of the situation we examine in the present Letter: a collision of two superfluid nuclei with different phases of the pairing fields. Each disc represents a cross section of a nucleus. The arrows inside the nucleus indicate the pairing field $\Delta_i(\boldsymbol{r})$, where the length of the arrow indicates its absolute value $|\Delta_i(\boldsymbol{r})|$, while the direction indicates its phase $\varphi_i(\boldsymbol{r})$ ($i = 1, 2$). In the ground state, the phase is uniform across each nucleus $\varphi_i(\boldsymbol{r}) = \varphi_i$ and the phase difference $\Delta\varphi$ ($\equiv \varphi_1 - \varphi_2$) is well defined. We will show how the phase difference affects the reaction dynamics.

cannot be removed by the gauge transformation. This difference will trigger various excitation modes of the pairing field as well as the particle flow between colliding nuclei. Although the phases of the pairing fields are not controlled in nuclear experiments, they will affect the reaction outcomes in an averaged way. The consequences of this effect turn out to be significant and are discussed in this Letter.

Collision of two superfluid nuclei.—Let us first focus on the energy scale of the possible effect which may appear during a collision of two superfluid nuclei at a fixed pairing phase difference $\Delta\varphi$. One would naively expect that it is governed by the pairing energy which is proportional (for protons or neutrons) to $\frac{1}{2}g(\varepsilon_F)|\Delta|^2$, where $g(\varepsilon_F)$ represents the density of states per one spin projection at the Fermi level, and Δ is the pairing gap. Such a quantity for nuclei is on the order of MeV, and thus one may infer that the possible effects would be too weak to be observed in nuclear reactions. However, this is not the case, since during the collision a junction between two superfluids is created, where the phase varies rapidly. The energy stored in the junction depends both on the phase difference and the size of the junction. One may estimate the energy of the junction from phenomenological theory of superfluids, namely, the Ginzburg-Landau (GL) approach:

$$E_j = \frac{S}{L}\frac{\hbar^2}{2m}n_s \sin^2\frac{\Delta\varphi}{2}, \quad (1)$$

where S is the area of the junction, L is the length scale over which the phase varies, and n_s is the superfluid density (for derivation, see Ref. [17]). Note that neither the pairing energy, nor the pairing gap enters this formula explicitly. For a collision of two heavy nuclei at energies close to the Coulomb barrier, one can show that the energy stored in the junction can vary by several tens of MeV depending on the phase difference [17]. Such a drastic energy change may significantly alter the dynamics of the collision. Clearly in order to determine those quantities in Eq. (1) (S, L, n_s) one needs to perform microscopic simulations, since they are in general dependent on the actual reaction dynamics. Note that the situation described here is markedly different from the Josephson effect encountered in solids, ultracold atomic gases, or heavy ion collisions [29–33]. The Josephson effect involves tunneling between weakly coupled pairing condensates. Here we focus on the strong-coupling limit: the nature of the junction is entirely different even though its decay will also involve a Josephson-like current. In this Letter, we show that the associated pairing field dynamics has a significant impact on the fusion cross section and the total kinetic energy (TKE) of the fragments.

TDSLDA for nuclear reactions.—Presently, the most accurate microscopic approaches to the dynamics of superfluid systems are based on the density functional theory [34,35]. Here we utilize an approximated formulation known as the time-dependent superfluid local density approximation (TDSLDA), which is formally equivalent to the time-dependent Hartree-Fock-Bogoliubov theory (TDHFB). The approach has been proved to be very accurate for describing dynamics of strongly correlated fermionic systems, like ultracold atomic gases [9,10,36–40] and nuclear systems [15,41–43]. We solve the TDSLDA equations numerically on a 3D spatial lattice (without any symmetry restrictions) with periodic boundary conditions. We use a box of size 80 fm × 25 fm × 25 fm for head-on collisions and 80 fm × 60 fm × 25 fm for noncentral collisions. The lattice spacing is set to 1.25 fm. For the energy density functional, we use the FaNDF^0 functional [44,45] without the spin-orbit term.

Although it is well known that the spin-orbit interaction is crucial for a proper description of nuclear static properties as well as energy dissipation in low-energy nuclear reactions, it does not induce qualitative change in the pairing field dynamics. In this Letter, we investigate the possible impact of the phase difference on the reaction dynamics and address the following questions: what observables are affected by the phase difference, and for each affected quantity what is the predicted size of the effect? In order to answer these questions one needs to set correctly the scales of the problem, which are determined in the present context by the average magnitude of the pairing gap and the ratio of the coherence length to the size of the system. None of the meaningful scales in our problem is affected by the spin-orbit interaction. However, in order to provide quantitative results that can be compared directly with experimental data, one needs to perform calculations with a full nuclear density functional. We defer these extremely numerically expensive studies to future works. This simplification allows us to construct a highly efficient solver of the TDSLDA equations (for details, see Supplemental Material of Ref. [18]). Nevertheless, the problem is still numerically demanding and requires usage of supercomputers. Very recently, the first attempt was reported in Ref. [33], where the effects of the phase difference in head-on collisions of $^{20}O + ^{20}O$ were investigated based on TDHFB, including the spin-orbit contribution. In case of reactions with light systems, the impact of the phase difference on various observables was found to be very small [33,46].

One may raise a question regarding the adequacy of the description of the finite system using the theoretical framework admitting the broken particle-number symmetry. It gives rise to the Nambu-Goldstone (NG) modes related to the rotation of the phase of the pairing field [47,48]. The phase can be traced back to the phase of the Cooper-pair wave function, which can be defined as the eigenfunction corresponding to the dominant eigenvalue of the two-body density matrix, and, thus, is independent of a particular approximation in the treatment of the pairing correlations. The particle-number projected (symmetry-restored) wave function would imply averaging over the phase. The natural question is whether this averaging needs to be performed before the collision. The answer to this question is related to the time scale of the associated NG mode, which is governed by the nuclear chemical potentials [49]. Since phases of both projectile and target nuclei rotate during the time evolution, what matters is the difference of the periods of the phase rotations. If it is long enough, as compared to the collision time, the use of the framework with broken particle-number symmetry is validated [50]. In the case of nuclear collision it is determined by the difference of (one nucleon) separation energies of the projectile and target nuclei $\Delta S = |S_1 - S_2|$. Thus, the description will be valid if one limits to the collision of nuclei whose difference between the separation energies does not exceed 1 MeV that leads $T = (2\pi\hbar/\Delta S) > 1200$ fm$/c$, which is longer than the collision time. The most clean case corresponds to the symmetric collision where the phase difference does not depend on time.

Kinetic energy and Josephson current.—As a first example, let us consider symmetric collisions of two heavy nuclei, ^{240}Pu (since the spin-orbit term is neglected the nucleus does not exhibit a prolate deformation). In such a case two nuclei do not fuse and reseparate shortly after collision. In Fig. 2, we show pairing fields and densities of the colliding nuclei at various times in two extreme cases, $\Delta\varphi = 0$ and $\Delta\varphi = \pi$. It is clearly visible that in the $\Delta\varphi = \pi$ case a narrow solitonic structure is created; i.e., inside the structure the order parameter vanishes, the density is suppressed, and the phase changes rapidly from one value to another when one crosses the structure. It stays there until the composite system splits. This produces a significant impact on the resulting TKE of the fragments. In Fig. 3(a), we show the TKE as a function of the relative phase for various collision energies. The TKE clearly shows the $\sin^2(\Delta\varphi/2)$ pattern (gray solid curves), which exactly recovers the dependence of the energy of the junction given by the GL approach, Eq. (1). The dominating contribution comes from the neutron pairing field. The contribution from the proton pairing field is less than 30% of the neutron effect, due to Coulomb repulsion [17]. These results indicate that the phase difference hinders the energy transfer from the relative motion to internal degrees of freedom. We emphasize that the observed change of TKE cannot be attributed to the Josephson effect. For example, for extreme cases $\Delta\varphi = 0$ and $\Delta\varphi = \pi$, there is no Josephson current (as it scales like $\sin\Delta\varphi$) while the dynamics of the reaction is altered.

The situation becomes qualitatively different when the energy is further increased. Namely, at energies about 30% above the barrier, the departure from this simple pattern is

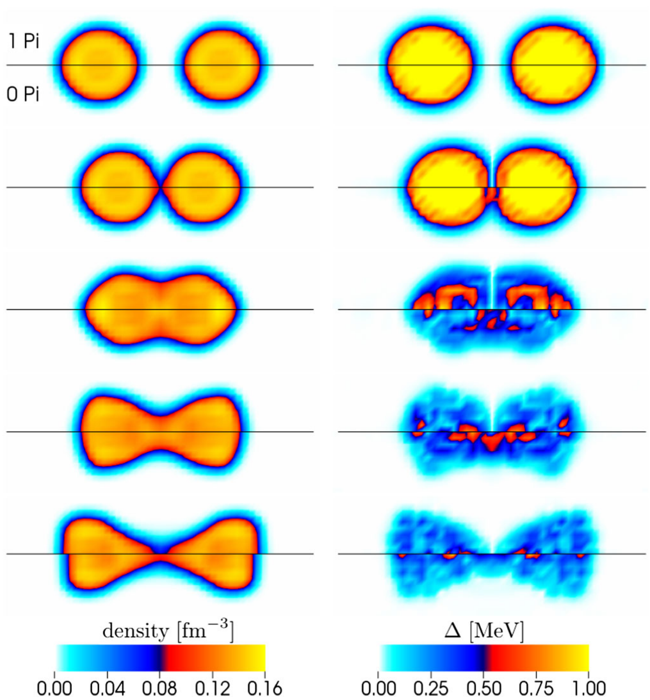

FIG. 2. Snapshots from the collision of ^{240}Pu + ^{240}Pu for two extreme values of the relative phase differences ($\Delta\varphi = 0$ and π) at the energy $E \simeq 1.1 V_{\mathrm{Bass}}$, where V_{Bass} represents the phenomenological fusion barrier [23]. Left panels show the total density distribution, whereas the right panels show the neutron paring field of two colliding nuclei. Top half of each panel corresponds to the phase difference $\Delta\varphi = \pi$ case, while the bottom half corresponds to the case without phase difference $\Delta\varphi = 0$. Contact time is about 550–600 fm$/c$ depending on the phase difference. For movies and plots showing the phase evolution see Ref. [17].

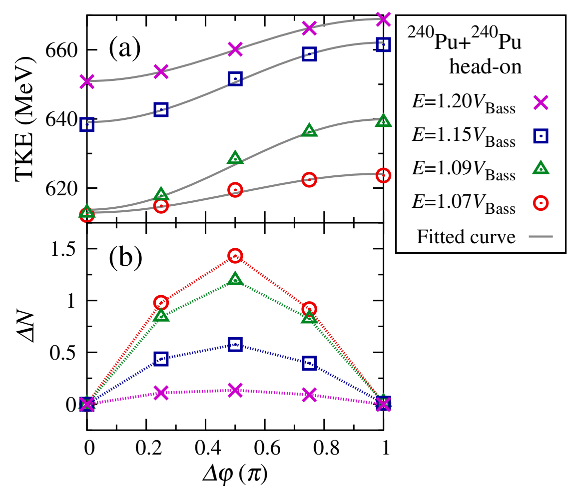

FIG. 3. Results of the TDSLDA simulations for ^{240}Pu + ^{240}Pu head-on collisions at various collision energies. (a) Total kinetic energy (TKE) of the outgoing fragments is shown. Line shows fit to the data by a formula $\alpha + \beta\sin^2(\Delta\varphi/2)$ with respect to parameters α and β. (b) The average number of transferred neutrons from the left nucleus to the right nucleus due to the Josephson current is shown. The horizontal axis is the relative pairing phase $\Delta\varphi$. Note that the change of TKE has different phase dependence, and cannot be explained by the Josephson effect.

observed. It corresponds to the energies at which a third light fragment is generated [17]. The appearance of the third light fragment in the quasifission process is understood as a consequence of the density and charge excesses in the neck region [24]. However, the solitonic excitation effectively reduces the density in the neck region. Consequently, for the energy range $1.3V_{\mathrm{Bass}} < E < 1.5V_{\mathrm{Bass}}$ the number of fragments depends on the phase difference and smaller phase differences favor the creation of the third fragment [17]. For $E > 1.5V_{\mathrm{Bass}}$ the ternary quasifission is observed for all phase differences.

The stability of the solitonic excitation described here depends on the possibility of phase transfer between the pairing fields of the colliding nuclei, which manifests itself as particle transfer. Even though the reaction is symmetric, it can cause nucleon transfer from one nucleus to the other. Indeed, after reseparation the fragments are not symmetric. However, the amount of nucleon transfer does not exceed 1.5 for neutrons and 0.5 for protons during the collision [see Fig. 3(b) and Ref. [17] for more details]. This result is consistent with earlier studies [29–33]. Note that this particle transfer resembles a Josephson current, even though the solitonic excitation itself has an entirely different origin.

Energy threshold for fusion.—Results for the heavy system indicate that the phase difference effectively works as a potential barrier, and, consequently, it will affect the fusion cross section. In order to investigate this issue, we examine collisions of two medium mass nuclei, ^{90}Zr, that can fuse. Note that when the spin-orbit term is dropped this is an open-shell nucleus for neutrons and thus neutrons are superfluid, whereas protons occupy a closed shell. In Fig. 4, we show the minimum energy required for the system to merge in head-on collisions and stay in contact for times longer than 12 000 fm/c (40 zs). The results clearly demonstrate that the fusion reaction is effectively hindered by the dynamic excitations of the pairing field. The energy threshold as a function of the angle does not have $\sin^2(\Delta\varphi/2)$ dependence, since we consider now collisions varying both the phase difference and the collision energy.

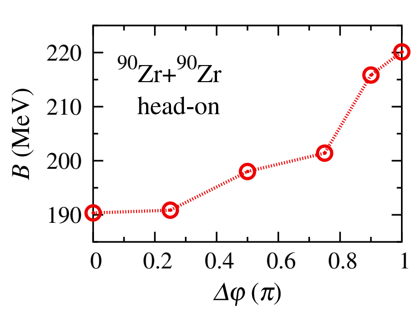

FIG. 4. Results of the TDSLDA simulations for $^{90}\mathrm{Zr} + {}^{90}\mathrm{Zr}$ head-on collisions. Fusion threshold energy B is shown as a function of the relative pairing phase $\Delta\varphi$. For this reaction the barrier height is $V_{\mathrm{Bass}} \simeq 192$ MeV. The phase difference prevents fusion for energies up to 15% above the barrier.

The fusion hindrance phenomenon associated with pairing dynamics may likely be observed by studies of the fusion cross section for symmetric systems at the vicinity of the barrier, in a similar way to experimental detection of the so-called extra-push energy [51,52], which is the energy introduced by Swiatecki to explain the experimental fusion cross sections for collisions of medium and heavy nuclei at energies above the Coulomb barrier [53–55]. As a good candidate we suggest symmetric collisions of different Zr isotopes. For these reactions the extra-push energy is negligible. ^{90}Zr is neutron magic ($N = 50$) and the pairing correlations are absent. As the neutron number increases neutrons become superfluid, which hinders the fusion reaction. Based on our results the extra energy for fusion is expected to be about $E_{\mathrm{extra}} = (1/\pi)\int_0^{\pi}[B(\Delta\varphi) - V_{\mathrm{Bass}}]d(\Delta\varphi) \approx 10$ MeV.

Another possibility is to investigate asymmetric reactions like $^{90-96}\mathrm{Zr} + {}^{124}\mathrm{Sn}$. Despite the fact that the extra-push model predicts that the extra-push energy becomes smaller with increasing the neutron excess, the experimental data suggest the opposite trend [51]. TDHF calculations also show similar disagreement [56]. The measured trend is consistent with the results presented here, as the fusion reaction is hindered as the system departs from the neutron magic ^{90}Zr. The chemical potentials for colliding nuclei are fairly similar admitting the description within broken particle-number symmetry. We have performed exploratory simulations for asymmetric reactions, and we have found that, similarly to the symmetric case, the phase difference can hinder the fusion for energies around the barrier; however, no clear solitonic structure was observed [17].

Finally, we have also performed simulations of noncentral collisions. If we are in the energy window where the phase difference can hinder the fusion, we find that it affects the contact time, and, consequently, the scattering angle is affected (see Ref. [17] for movies demonstrating this effect).

Summary.—We have investigated collisions of medium and heavy nuclei at energies around the Coulomb barrier, taking into account the pairing field dynamics with TDDFT for superfluid systems. We have found that during collision a stable solitonlike structure appears when two superfluid nuclei collide with phase difference of the pairing fields close to $\Delta\varphi = \pi$. The solitonic excitation suppresses the neck formation and hinders energy dissipation as well as the fusion reaction, leading to significant changes in reaction dynamics. It implies that the pairing field dynamics effectively increases the barrier height for fusion resembling a "thud wall" in the extra-push model, although at much smaller energies. The Josephson current between two colliding nuclei turns out to be small, does not exceed 2 particles. The effects on the kinetic energy of the fragments and fusion cross section may likely be observed experimentally. Last but not least, it is to be noted that the effects studied in this Letter are clearly beyond the commonly used TDHF + BCS approach [27,28,57–59] (see Ref. [17] for a detailed discussion).

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The contribution of each one of the authors has been significant and the order of the names is alphabetical.

Note added in proof.—Recently, in a related work [60], it has been shown that the phase difference can influence the outcome of the collision only in the case of systems characterized by the weak pairing correlations (like systems discussed here). In the strong pairing limit, the role of the initial phase difference is erased.

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