Can Tetraneutron be a Narrow Resonance?

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The search for a resonant four-neutron system has been revived thanks to the recent experimental hints reported in [1]. The existence of such a system would deeply impact our understanding of nuclear matter and requires a critical investigation. In this work, we study the existence of a four-neutron resonance in the quasistationary formalism using *ab initio* techniques with various two-body chiral interactions. We employ no-core Gamow shell model and density matrix renormalization group method, both supplemented by the use of natural orbitals and a new identification technique for broad resonances. We demonstrate that while the energy of the four-neutron system may be compatible with the experimental value, its width must be larger than the reported upper limit, supporting the interpretation of the experimental observation as a reaction process too short to form a nucleus.

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Introduction.—Whether the four-neutron system (^4n) exists or not is a long-standing question that rests on the fact that such a system would be the result of the subtle interplay between the various components of the nuclear interaction, the Pauli principle, and the coupling to the neutron continuum. The recent enthusiasm in the search for the ^4n system [1] started with the experimental claim [2] in 2002 that a bound ^4n system could be formed in the breakup reaction: $^{14}\text{Be}^* \to {}^{10}\text{Be} + {}^4n$. This result, though unconfirmed, stimulated theoretical investigations [3,4], but none of them concluded that a bound ^4n system can exist. The most compelling study against the existence of a bound ^4n system is given in Ref. [4] where a large range of modifications to the nuclear interaction (two-, three- and four-body components) was investigated in an *ab initio* framework without couplings to the continuum. However, the existence of a resonant ^4n system has not been ruled out.

The experimental hints of a ^4n resonance provided by the recent measurement [1] exacerbate the need for reliable nuclear interactions and *ab initio* methods able to cope with couplings to the continuum. According to those experimental results, if such a state exists it would have an energy $E = 0.83 \pm 0.65(\text{stat}) \pm 1.25(\text{syst})$ MeV above the ^4n threshold and a maximal width $\Gamma = 2.6$ MeV. This large width is unlikely to correspond to a nuclear state [5] and the question we want to address in this work is the following: Can a four-neutron system form a narrow resonance? In Ref. [6] the possibility of forming a ^4n resonance by adding a phenomenological $T = 3/2$ three-body force to a realistic two-body interaction was investigated. In that study, the continuum was included and it was shown that unrealistic modifications to the nuclear interaction would be necessary to obtain a ^4n system at the experimental value. Moreover, in Ref. [7], an *ab initio* study of ^4n was done in the harmonic oscillator (HO) basis using the realistic two-body JISP16 interaction [8]. The energy and width of a ^4n resonance were obtained in that approach from phase shifts [9] assuming that all four neutrons decay simultaneously, and lead to $E = 0.8$ MeV and $\Gamma = 1.4$ MeV. Previously mentioned results in Ref. [6] that are based on the uniform complex scaling method [10,11] fully include couplings to the continuum and suggest that if a ^4n system exists its energy and width should be much larger than in Ref. [7]. Recent Monte Carlo results [12] using two- and three-body chiral interactions suggest that (i) three-body forces are not important in the three- and four-neutron systems, (ii) if a three-neutron resonance exists it must be lower than that of the four-neutron system, and (iii) a four-neutron resonance might exist at $E = 2.1$ MeV. The contradictory results on the nature of the four-neutron system range from its existence as a narrow resonance to its nonexistence as a genuine nucleus. In the present study we investigate the conditions of existence of a ^4n system using *ab initio* methods for various two-body chiral interactions while including the continuum.

Models and formalism.—In the present work, two different techniques are used to describe ^4n in the continuum. These techniques are discussed below and both allow a consistent treatment of the couplings to the continuum by using the Berggren basis [13]. The Berggren single particle (SP) completeness relation includes explicitly the Gamow (resonant) states and the nonresonant scattering continuum. For each partial wave $c = (\ell, j)$ we have

$$\sum_i |u_c(k_i)\rangle\langle\tilde{u}_c(k_i)| + \int_{\mathcal{L}_c^+} dk |u_c(k)\rangle\langle\tilde{u}_c(k)| = \hat{1}_c, \quad (1)$$

where $|u_c(k_i)\rangle$ are the radial wave functions of resonant states and $|u_c(k)\rangle$ are the complex-energy scattering states along a contour $\mathcal{L}_c^+$ in the fourth quadrant of the complex-momentum plane that surrounds the poles $\{k_i\}$ and then extends to $k \to +\infty$. In Eq. (1), the tilde denotes the time-reversed states. The form of the contour is unimportant because of Cauchy's integral theorem, as long as the poles are all embedded between the real axis and the contour; details can be found in Ref. [14].

In this work we used the no-core Gamow shell model (NCGSM), which is a generalization of the no-core shell model [15], in the complex-energy plane [14,16] through the replacement of the usual HO SP basis by the Berggren basis. While the Hamiltonian operator is Hermitian, the Hamiltonian matrix in the NCGSM is complex symmetric and has complex eigenvalues. An advantage of this approach based on the quasistationary formalism is that it fully includes the couplings to the continuum while still solving the many-body problem using configuration interaction techniques.

The second approach used in this work, the density matrix renormalization group (DMRG) method [17–19], is an alternative way to solve the nuclear many-body problem in the continuum. In this approach, instead of building the Hamiltonian matrix in the full space and diagonalizing it as in the NCGSM, one starts from an approximate eigenstate of the Hamiltonian obtained in a small space and gradually includes the nonresonant continuum in the configuration mixing, while keeping only configurations corresponding to the largest eigenvalues of the density matrix.

In order to reduce the cost of our calculations the DMRG method has been supplemented with natural orbitals (NO) [20,21], which are eigenstates of the SP density matrix associated with the targeted Hamiltonian eigenstate. A first standard DMRG run is performed to approach the final eigenstate of the Hamiltonian; then the corresponding NO are calculated and a new DMRG run is performed where the NO replace the Berggren basis states. This technique leads to an impressive gain in time while efficiently incorporating many-body correlations with the number of DMRG iterations. Following this development, the NCGSM has also been augmented by a similar technique where NO are generated from a truncated space large enough to have a decent approximation of the targeted Hamiltonian eigenstate. Then one selects the NO with an occupation in the density matrix η larger than some chosen value $0 < \eta_{\min} < 1$ in order to replace the Berggren basis in a second run with fewer or no truncations.

In general, in both methods, the full spectrum of the Hamiltonian contains some many-body bound states and decaying resonances as well as a large number of many-body complex-energy scattering states. The proper extraction of many-body resonant states from the nonresonant background is usually achieved in these methods by selecting the eigenstates that have the maximal overlap with the solutions obtained in a truncated space only made of poles of the Berggren basis (resonant states) [22]. This method works as long as the truncated space gives a decent approximation of the final eigenstate; however, when couplings to the continuum are dominant it is not the case anymore. To circumvent this limitation, the interaction is multiplied by a factor $f > 1$ so that the targeted state is bound and its identification immediate, and then the obtained eigenstate is used to identify the eigenstate for a smaller scaling factor $f' < f$. This process is repeated until the factor equals 1. In practice this new technique preserves the unambiguous identification of resonances and extends the range of applicability of the overlap method to broader resonances.

Results.—In the present work we considered a model space made of the $0s_{1/2}$ and $0p_{3/2}$ resonant shells (pole space), and associated nonresonant partial waves whose energies are selected along the contours in the complex-momentum plane defined by the points (0,0), (0.15, −0.05), (0.3,0), and (4.0,0) (all in fm^{-1}), each segment being discretized by 15 points. The Woods-Saxon potential generating the SP basis was defined by the diffuseness $a = 0.67$ fm, the radius $R_0 = 1.9$ fm, the depth $V_0 = -27.0$ MeV, and the spin-orbit term $V_{\rm so} = 9.5$ MeV. Except for the depth, these parameters provide a decent basis for the description of light systems such as ^{3}H, ^{3}He, or ^{4}He. We checked that changing these parameters by about 15% had almost no effect on results for the four-neutron system. The model space is augmented by 30 nonresonant $p_{1/2}$ partial waves along a real contour going up to 4.0 fm^{-1}, and seven HO shells for each higher partial wave (d, f, and g). The oscillator length for HO wave functions is set at $b = 2.0$ fm and we verified that the effect of this parameter on our results is negligible. Also, removing the $g_{7/2}$ HO waves has almost no effect on our results.

In the NCGSM, when allowing only two neutrons into the continuum, the predicted energies and widths of the ^4n system for $J^\pi = 0^+$ and for various two-body chiral interactions (N3LO [23], N2LO$_{\rm opt}$ [24], and N2LO$_{\rm sat}$ [25]) with different renormalization cutoffs ($\lambda = 1.7$–2.1 fm^{-1}) in $V_{{\rm low}\,k}$ [26,27] and the realistic two-body JISP16 interaction [8] for $\hbar\Omega = 20$ MeV are all consistent as shown in Table I. Strictly speaking, the N2LO$_{\rm sat}$ interaction has three-body components and we only used its two-body part for a qualitative comparison. Moreover, the negligible influence

TABLE I. Energies and widths (in brackets) of the $J^\pi = 0^+$ pole of the four-neutron system (in MeV) for various two-body interactions. The asterisk means that only the two-body part of the interaction was considered.

	$\lambda = 1.7$ fm^{-1}	$\lambda = 1.9$ fm^{-1}	$\lambda = 2.1$ fm^{-1}
N3LO	7.27 (3.69)	7.28 (3.67)	7.28 (3.69)
N2LO$_{\rm opt}$	7.32 (3.74)	7.33 (3.78)	7.34 (3.95)
N2LO$_{\rm sat}^{*}$	7.24 (3.48)	7.22 (3.58)	7.27 (3.55)
JISP16		7.00 (3.72)	

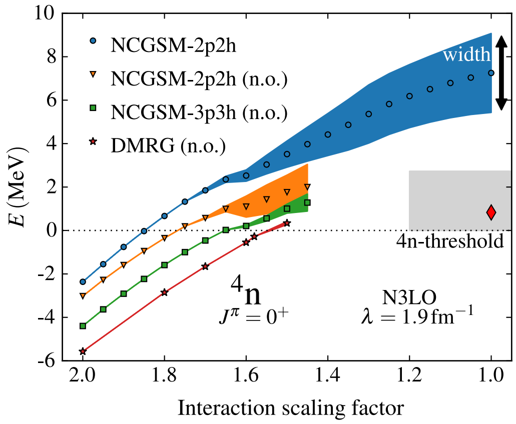

FIG. 1. Evolution of the energy and width (shaded area) of the four-neutron system with the scaling of the N3LO interaction from 2.0 to 1.0. The circles represent the NCGSM results with two neutrons in the continuum, which is used to generate the NCGSM results based on natural orbitals with two (triangles) and three (squares) neutrons in the continuum. The DMRG results without truncations are represented by stars. The experimental energy is indicated by a diamond and the gray area shows the maximal experimental uncertainties. This area is extended up to an interaction 20% more attractive to guide the reader.

of the renormalization cutoff of the interaction on the results indicates a weak influence of the missing three- and four-body forces.

The large widths obtained with only two neutrons in the continuum ($\Gamma \approx 3.7$ MeV) already discredit the existence of the four-neutron system as a narrow resonance. However, in these calculations the width of the four-neutron system is mostly controlled by the occupation of the p waves. Below, we show that additional neutrons in the continuum lower the energy but do not reduce the width.

In the following, we use the N3LO two-body chiral interaction with a renormalization cutoff of $\lambda = 1.9\ \mathrm{fm}^{-1}$. The role of the continuum in shaping the ${}^4n$ system is illustrated by scaling the interaction by a factor $f = 2.0$ so that the system is artificially bound, and then one follows the evolution of the energy and width of the $J^\pi = 0^+$ state when $f \to 1.0$ by step of 0.05 and for a total of 20 points as shown in Fig. 1.

The first set of results denoted NCGSM-$2p2h$ corresponds to the NCGSM calculations with only two neutrons in the continuum and shows a rapid increase of the width when the scaling of the interaction is gradually decreased. The small variation of the energy with the scaling factor around $f = 1.6$ is due to the use of several different bases where the $0p_{3/2}$ shell is bound for $f \geq 1.6$ and unbound for $f < 1.6$, to acknowledge the opening of the ${}^4n \to {}^3n + n$ channel. In practice, the depth V_0 of the potential generating the p waves was changed from -32.0 to -27.0 MeV. We used the NCGSM-$2p2h$ results to generate NO for each scaling factor and kept only the NO having an occupation $\eta > 10^{-7}$ in the SP density matrix, reducing the size of the basis by a factor ≈ 2.9. The first $s_{1/2}$ and $p_{3/2}$ NO are treated as pole states while remaining orbitals are considered as continuum shells. Then we considered two and three neutrons in the NO continuum shells in the results denoted NCGSM-$2p2h$ (NO) and NCGSM-$3p3h$ (NO), respectively. This technique allows us to reduce the computational cost and to include additional many-body correlations in a more efficient way. The NCGSM-$2p2h$ (NO) results shown in Fig. 1 illustrate this point clearly, as they are all lower in energy than the initial NCGSM-$2p2h$ results obtained in the Berggren basis. However, the quality of the NO for the description of the targeted state depends on how close the generating eigenstate is to the final eigenstate. The NCGSM-$2p2h$ (NO) calculations could only be performed for $f > 1.45$ for that reason. Another advantage of the NO is the possibility to remove some of the truncations as compared to the generating calculations (NCGSM-$2p2h$). However, the generating calculations need to include enough correlations in the continuum for the removal of truncations in the calculations with NO to be meaningful. In the NCGSM-$3p3h$ (NO) calculations we allowed three neutrons in the NO outside the pole space and thus included more correlations. The NCGSM-$3p3h$ (NO) calculations were limited to $f > 1.45$ as well. It was not possible to completely remove the truncations in the NCGSM and hence one had to rely on the DMRG method.

The DMRG results are without truncations on the number of particles (same shells as in the NCGSM) and the convergence criterion of the method has been fixed by the parameter $\varepsilon = 10^{-8}$ [18,19]. These results are about 1 MeV lower than the NCGSM-$3p3h$ (NO) at a scaling factor of $f = 2.0$, which indicates important missing correlations in the NCGSM calculations. This shows that configurations with four neutrons in the continuum shells have a large contribution to the wave function even when the system is artificially bound. In fact, the opening of new decay channels and the presence of continuum states in the configuration mixing above the threshold is expected to make the width explode when $f \to 1$, especially in the DMRG results where all decay channels are open. This is in qualitative agreement with the results in Ref. [6], which show a rapid increase of the width when the strength of the phenomenological $T = 3/2$ three-body force decreases. This explosion of the width is already visible in the NCGSM results with NO where, comparatively, the width increases faster than in the NCGSM-$2p2h$ results. Another hint of the explosion of the width is the impossibility to perform the DMRG calculations far above the four-neutron threshold, even when using the improved identification technique for broad resonances. This was due to the strong couplings to the continuum, resulting in large overlaps between complex-energy scattering states and the targeted decaying resonance, making them indistinguishable. Finally, while the energy position of the four-neutron system may be compatible with the experimental value when $f \to 1$, calculations including more than two particles in the continuum as in Table I suggest that the width of ^4n is larger than $\Gamma \approx 3.7$ MeV.

Conclusions.—In this work, we investigated the existence of the ^4n system in the continuum using the no-core Gamow shell model and the density matrix renormalization group method and realistic two-body chiral interactions. Two new ingredients have been added to these approaches in order to make this study. First, the introduction of natural orbitals has been a key element for improving the convergence of the calculations in the continuum, and second, the progressive rescaling of the interaction to produce starting eigenstates as a technique to identify unambiguously broad resonances into the continuum was critical.

While three-body forces were not included in this work, the important role of the Pauli principle in shaping the many-body structure of the ^4n system as well as its low density suggest that their exclusion yields a reasonable approximation as recently confirmed in Ref. [12]. Interestingly, the results we obtained for various two-body chiral interactions were all consistent and were mostly dependent on the number of neutrons in the continuum. We confirm the existence of a pole of the scattering matrix associated with the spin and parity $J^\pi = 0^+$ in this system as shown in previous studies; however, the proper inclusion of the couplings to the continuum shows that this pole must be a feature in scattering experiments and not a genuine nuclear state. Physically this can be interpreted as a reaction process involving four neutrons, which is too short to form a nucleus. However, the description of such a broad state is at the limit of the quasistationary formalism and it is clear that any conclusion on the existence of a light nucleus solely based on the width is speculative to some extent. A measurement of a resonance with a half-life greater than 10^{-22}s would provide a strong case for the existence of the ^4n system as a nucleus.

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