Out-of-Time-Ordered Density Correlators in Luttinger Liquids

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Information scrambling and the butterfly effect in chaotic quantum systems can be diagnosed by out-of-time-ordered (OTO) commutators through an exponential growth and large late time value. We show that the latter feature shows up in a strongly correlated many-body system, a Luttinger liquid, whose density fluctuations we study at long and short wavelengths, both in equilibrium and after a quantum quench. We find rich behavior combining robustly universal and nonuniversal features. The OTO commutators display temperature- and initial-state-independent behavior and grow as t^2 for short times. For the short-wavelength density operator, they reach a sizable value after the light cone only in an interacting Luttinger liquid, where the bare excitations break up into collective modes. This challenges the common interpretation of the OTO commutator in chaotic systems. We benchmark our findings numerically on an interacting spinless fermion model in 1D and find persistence of central features even in the nonintegrable case. As a nonuniversal feature, the short-time growth exhibits a distance-dependent power.

Introduction.—Spectacular experimental progress in the study of coherent quantum dynamics has focused much attention on the question how many-body systems evolve in real time [1,2]. Particularly interesting is the study of ergodic versus nonergodic behavior. This builds on previous work under the heading of classical and quantum chaos, where ideas such as the Lyapunov exponents (and the concomitant "butterfly effect") have turned out to be useful, together with level statistics of the Hamiltonian. This has been supplemented by questions about how information—in the form of correlations or entanglement—spreads [3–6]. Here, the concept of scrambling encodes the loss of information under time evolution, in particular, asking the question to what extent different initial states can be distinguished at later times [7,8].

Recently, out-of-time-ordered (OTO) correlation functions [9] have been identified as quantities providing insights into quantum chaos and information scrambling. The OTO commutator is defined as

$$C(t) = \langle [V, W(t)][W^+(t), V^+] \rangle \geq 0, \qquad (1)$$

where V and W are usually local operators, possibly separated by some spatial distance and $W(t) = \exp(iHt)W \times \exp(-iHt)$. It contains terms of the form $-\langle VW(t) \times V^+W^+(t) \rangle$, coined the OTO correlator due to its unusual temporal structure. It probes the spread of information, in particular, signaling the presence of quantum chaos, with a growth bounded by a thermal Lyapunov exponent [10]. Much effort, including experiments [11–18], has been devoted to its study, with intriguing connections to black hole physics and the random matrix theory [10,19] appearing, while it has turned out that a simple "mesoscopic," Sachdev-Ye-Kitaev model [9,20–23] captures many interesting phenomena.

The quantum butterfly effect (sensitivity to small perturbations) occurs in chaotic systems. It is diagnosed by the OTO commutator via an exponential growth before becoming of the order of $2\langle VV^+ \rangle \langle WW^+ \rangle$ at late times [7,8,10]. While this in itself is an interesting phenomenon, it is equally important to determine what universal features characterize OTO commutators in well-studied yet non-trivial models of condensed matter, even though these are expected not to saturate any scrambling bounds as they reside on finite-dimensional lattices.

In order to advance this program, we consider Luttinger liquids (LLs), realized in a variety of settings [24,25]. LLs describe the low-energy physics of both integrable (i.e., nonergodic) and nonintegrable (ergodic) critical [26] 1D systems, predicting universal behavior for the long-time dynamics, irrespective of the (non)integrability of the system [27]. LLs thus represent an ideal setting to gain universal information about the quantum butterfly effect and to disentangle chaotic from regular behavior. Moreover, in the presence of interactions, the original noninteracting quasiparticle description breaks down as excitations fractionalize into collective bosonic modes. This, in particular, turns a Fermi gas into a non-Fermi liquid. How these effects combine in the OTO commutator in LLs is the main subject of our work.

Here, we focus on Eq. (1) in LLs [31] and find that, in the OTO density commutator, an initial rise $\propto t^2$ builds up to a strong signal upon the arrival of the light cone, beyond which it saturates. This saturation is reminiscent of the quantum butterfly effect in chaotic models. However, we suspect that it occurs here due to the replacement of the original excitations by new bosonic collective excitations. Indeed, in the noninteracting case, both features are absent

together. The saturation is due to short-wavelength degrees of freedom, while the long-wavelength contribution—which exhibits a large degree of universality—vanishes at large times. Our findings are insensitive to the choice of initial thermal or ground states, as well as whether we time-evolve an eigenstate or follow a quantum quench protocol [1,2]. These central features are reproduced by exact diagonalization studies of the XXZ model. Remarkably, the short-time t^2 behavior gives way to a t^{2x} rise for spatially separated local densities (with separation x), reflecting the prominent role of microscopic details in the model under study.

Luttinger model.—The low-energy description of LLs is in terms of bosonic soundlike collective excitations with Hamiltonian [24,33]

$$H = \sum_{q \neq 0} v|q|b_q^+ b_q + \frac{g(q)}{2}[b_q b_{-q} + b_q^+ b_{-q}^+], \quad (2)$$

where b_q is the annihilation operator of a bosonic density wave, $g(q) = g_2|q|$, with g_2 the interaction strength, and v the sound velocity of the noninteracting system. The interaction is also characterized by the dimensionless Luttinger parameter $K = \sqrt{(v-g_2)/(v+g_2)}$. Equation (2) is diagonalized by a Bogoliubov rotation, and the dispersion relation is $\omega_q = v_f|q|$ with the renormalized final velocity $v_f = \sqrt{v^2 - g_2^2}$ in the interacting system. The transformation gives $b_q = \cosh(\theta)B_q + \sinh(\theta)B_{-q}^+$ with B_q the new boson operators, $\cosh(\theta) = (K+1)/2\sqrt{K}$, and $\sinh(\theta) = (K-1)/2\sqrt{K}$, which are q independent for the present case.

The time dependence of the original boson field is

$$b_q(t) = u_q(t)b_q + v_q(t)^* b_{-q}^+ \quad (3)$$

with $u_q(t) = \cos(\omega_q t) - i\sin(\omega_q t)\cosh(2\theta)$ and $v_q(t) = -i\sin(\omega_q t)\sinh(2\theta)$. Any expectation value is taken using a Bogoliubov rotation into the B_q basis. One can consider both a quantum quench and equilibrium time evolution. In the former case, the LL parameter K in the time evolution, Eq. (3) differs from those in the B_q operators. For the sake of simplicity, we assume quenching from a noninteracting, $K = 1$ state; therefore, the B_q operators are identical to the b_q's. In the equilibrium case, the same LL parameter is used for both time evolution and Bogoliubov rotation.

We focus on the long-wavelength ($q \sim 0$) density fluctuations and a vertex operator [33]. Their bosonized versions are

$$n_0(x) = -\frac{1}{\pi}\frac{\partial\phi(x)}{\partial x}, \qquad V_n(x) = \exp[in\phi(x)], \quad (4)$$

respectively, where n is an integer and $V_{\pm 2}/2\pi\alpha$ corresponds to the $\pm 2k_F$ Fourier component of the short-wavelength density operator with k_F the Fermi wave number and α the short distance cutoff [24]. This operator is responsible for, e.g., the density wave phase transitions

[24,34,35] and Friedel oscillations. A similar expression describes the phase fluctuations in a 1D quasicondensate [24,25]. The bosonic field in Eqs. (4) is expressed in terms of the canonical Bose operators as

$$\phi(x) = \sum_{q \neq 0} \sqrt{\frac{\pi}{2|q|L}}[\exp(iqx)b_q + \text{H.c.}]. \quad (5)$$

Long-wavelength density.—The OTO commutator of the long-wavelength field builds on the density response function $\chi(t) = \langle[n_0(x,t), n_0(0,0)]\rangle$, where the bare commutator $[n_0(x,t), n_0(0,0)]$ is already a c number [24,36] due to the linear dispersion in Eq. (2); therefore, $\chi(t) = \frac{iK}{\pi^2}[\{\alpha(v_f t + x)/[\alpha^2 + (v_f t + x)^2]^2\} + (x \to -x)]$. The OTO commutator then yields

$$C_0(t) = \langle|[n_0(x,t), n_0(0,0)]|^2\rangle = |\chi(t)|^2. \quad (6)$$

It has several interesting consequences: (a) From Eq. (6) and the fact that $\chi(t)$ does not contain the OTO correlator, this OTO commutator is not influenced by the OTO correlator either. (b) Since the bare commutator is already a temperature-independent c number, the expectation value in $C_0(t)$ becomes independent of both the temperature [10] and the wave functions—it depends only on the time evolution operator and is completely independent of the initial state. Therefore, there is no distinction between a quantum quench and equilibrium evolution. (c) Putting all this together, $C_0(t)$ grows in a t^2 manner initially, exhibits double peaks at around the light cone, and decays as $1/t^6$ for long times. The t^2 is the lowest possible power at short times of the OTO commutator, with prefactor $\langle|[[H,W],V]|^2\rangle$, unless this expectation value vanishes. As we show later for the *short-wavelength* density fluctuations, this can also occur.

Note that the Fourier transform of $\chi(t)$ gives the dynamical structure factor $S(\omega, q) \sim K|q|\delta(\omega - v_f|q|)$ in a LL, indicating that bosonic excitations have an infinite lifetime due to the linearized dispersion [36]. In a nonlinear LL picture with finite curvature from the noninteracting band structure [37], the correlation function changes and develops additional tails. How curvature and other higher-energy features are manifested in the OTO commutator is an intriguing open question but is beyond the scope of the present work [38].

Short-wavelength density.—The OTO commutator of the general vertex operator is more involved. The simple commutator is rewritten [24,33] as

$$[V_n(x_1), V_{-m}(x_2)] = 2e^{i(n\phi_1 - m\phi_2)}\sinh\left(\frac{nm}{2}[\phi_1, \phi_2]\right) \quad (7)$$

with $\phi_{1,2} = \phi(x_{1,2})$ and the commutator in Eq. (7) yields a vertexlike operator, in contrast to the long-wavelength case.

The commutator of the ϕ fields is a temperature-independent c number and is evaluated as

$$[\phi(x,t),\phi(0,0)] = -i\frac{K}{2}\arctan\left(\frac{v_f t + x}{\alpha}\right) + (x \to -x). \quad (8)$$

Now, we calculate the OTO commutator from Eq. (7). The two exponential fields are Hermitian conjugates of each other, $\exp[i(n\phi_1 - m\phi_2)]\exp[i(m\phi_2 - n\phi_1)]$, and need to be contracted. In the properly regularized theory, the exponential fields are point split [33] and the exponents then merged. After taking the expectation value, we get 1, independent of the temperature. Putting all this together, only the commutator in Eq. (8) remains and

$$C_{dw}(t) = \langle|[V_2(x,t), V_{-2}(0,0)]|^2\rangle$$
$$= 4\sin^2\left[K\arctan\left(\frac{v_f t + x}{\alpha}\right) + (x \to -x)\right]$$
$$\overset{t \to \infty}{\to} 4\sin^2(K\pi). \quad (9)$$

This expression grows with t^2 for $v_f t \ll |x|$ and rises sharply to $4\sin^2(K\pi)$ on hitting the light cone. Its long-time value vanishes in the noninteracting limit [41] but can give a sizable contribution to the commutator for a range of K values. Surprisingly, $C_{dw}(t) \leq 4$ follows from the Cauchy-Schwarz inequality ($\|ab\| \leq \|a\|\|b\|$) and $\|V_n(x)\| = 1$, and its maximal value from Eq. (9) is reached at late times for, e.g., $K = 1/2$. Note that, even in a suitably chaotic system, the late time value is expected to be 2 [10]. This is in sharp contrast to the expectation value of the simple commutator in Eq. (7), which, for $n = m = 2$, gives the retarded short-wavelength charge susceptibility in a LL and vanishes in a power law fashion in the long-time limit [24]. Equation (9) is thus strongly influenced by the OTO propagator, as it differs from the square of the short-wavelength charge susceptibility. In getting the $C_{dw}(t)$ OTO commutator, Eq. (7) itself is a vertex-operator-like quantity and becomes independent of both the temperature and initial state after taking its expectation value. Even though the Luttinger model is not chaotic, this commutator still exhibits some characteristics of the butterfly effect in the sense that the late time limit of the OTO commutator reaches a sizable value, indicating the effect of the OTO propagator.

Finally, the simple square of Eq. (7) includes phase information into the OTO commutator, which "degrades" the signal at long times, in particular, replacing the above saturation with a decaying form sensitive to the temperature (power law versus exponential) as well as evolution protocol [32].

OTO density commutator for interacting fermions.—In order to test our results on the OTO commutator, we have studied one-dimensional spinless fermions in a tight-binding chain with nearest-neighbor repulsion at half filling and a periodic boundary condition using numerical exact diagonalization (ED) of a finite-size system. This problem is equivalent to the 1D Heisenberg XXZ chain after a Jordan-Wigner transformation [24,35]. The Hamiltonian is

$$H = \sum_{m=1}^{N} \frac{J}{2}(c_{m+1}^+ c_m + \text{H.c.}) + J_z n_{m+1}n_m, \quad (10)$$

where c's are fermionic operators, $c_{N+1} = c_1$, and J_z denotes the nearest-neighbor repulsion. This model realizes a LL for $J_z < J$ with LL parameter $K = \pi/2[\pi - \arccos(J_z/J)]$. We have evaluated the OTO commutator of the local charge density, $n_1 = c_1^+ c_1$, and its time-evolved counterpart to be able to access directly the late time behavior after the light cone. System sizes up to $N = 22$ are considered, the number of electrons being $N/2$.

According to Ref. [10], this should approach $2\langle n_1\rangle^2 = 2(\frac{1}{2})^2 = \frac{1}{2}$ at late times when the butterfly effect occurs. In suitably chosen chaotic systems, this occurs through the exponential growth of the OTO correlator of local operators under time evolution as is the case in the Sachdev-Ye-Kitaev model [9,20–23]. In the bosonized form, the local charge density [24] is $n(0) = -\partial_x\phi(0)/\pi + n_{2k_F}\cos[2\phi(0)]$, where n_{2k_F} depends on the short-range properties of Eq. (10) available only from its exact solution [42]. The OTO commutator contains both short- and long-wavelength operators, but its long-time limit will be dominated by Eq. (9); i.e., it should approach a nonzero, constant value.

The numerical evaluation of its OTO commutator is shown in Fig. 1, together with the predicted behavior. We investigate both equilibrium time evolution from an interacting ground state as well as quantum quenches from a noninteracting, $J_z = 0$ ground state to an interacting system. We find very satisfactory agreement between bosonization and ED. In particular, (a) both equilibrium and sudden quench OTO commutators stay very close to each other, (b) the commutator reaches a time-independent value after a transient time, and (c) the short-time behavior is t^2. The agreement on the steady state for the OTO commutator with Eq. (9) is surprisingly good, given it contains the unknown prefactor n_{2k_F}, which can in principle also depend [42] on J_z. The OTO commutator is indeed of the same order as expected for the quantum butterfly effect and reaches its maximal value at $J_z = J$ ($\Leftrightarrow K = 1/2$) from Eq. (9), even though the model in Eq. (10) is not chaotic but integrable. Any finite J_z in Eq. (10) destroys the Fermi gas and induces non-Fermi liquid (LL) behavior, and the bare fermionic excitations do not persist but give way to collective bosonic modes. The density operator in the OTO commutator naturally decomposes into collective modes during the time evolution, which could explain the large late time value. As evident from Fig. 1, absent fractionalization ($J_z = 0$) implies $C_n(t \to \infty) \to 0$.

We have also investigated the OTO commutator between n_1 and n_{1+x} with x positive integer. The late time behavior after hitting the light cone agrees with our previous findings
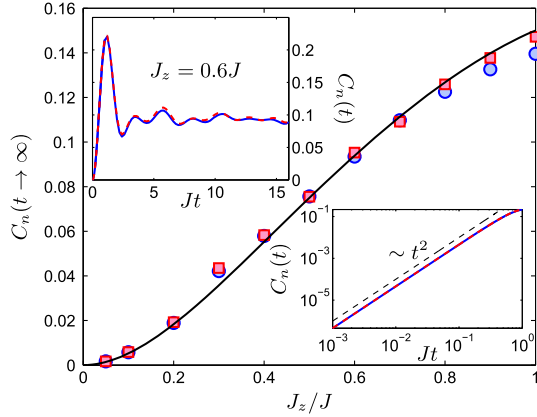
FIG. 1.   ED result for $N = 22$ for the OTO commutator for the local charge density in the interacting spinless fermion model. Red squares and blue circles denote the late time limit in equilibrium and after a sudden quench from the noninteracting limit, respectively. The solid line is $f \sin(K\pi)^2$ from Eq. (9) with only the parameter $f = 0.15$ adjustable. Top inset: Numerical time evolution of the OTO commutator with the representative value $J_z/J = 0.6$ in equilibrium (red dashed line) and after a sudden quench (blue solid line) from the noninteracting case, $J_z = 0$. For a short time ($t < 1/J$), it follows the predicted t^2 (bottom inset).

and takes an order one value as in Eq. (9). For shorter times, on the other hand, the OTO commutator grows as $t^{2x}/(2x)!$, shown in Fig. 2. This follows from a Baker-Campbell-Hausdorff expansion of the $W(t)$ in Eq. (1), with the nested commutators [8]

$$W(t) = W + it[H, W] + \frac{(it)^2}{2!}[H, [H, W]] + \cdots. \quad (11)$$

For $W = n_{x+1}$, the coefficient of the t^2 term in the OTO commutator arises from the second term in Eq. (11) as $\langle [[H, n_{x+1}], n_1]^2 \rangle$. However, this vanishes for $x > 1$, since
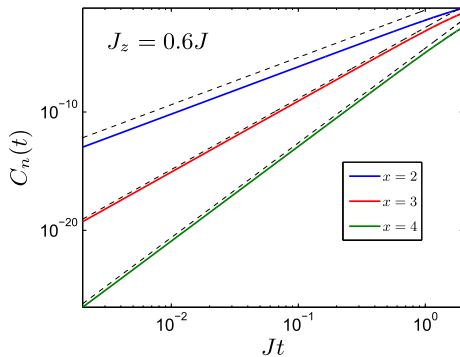

FIG. 2.   ED result for $N = 10$ for the OTO commutator between the local charge densities n_1 and n_{1+x} in the interacting spinless fermion model with $x = 2$ (blue line), 3 (red line), and 4 (green line) in equilibrium for $J_z/J = 0.6$; the quench data are indistinguishable in this time window. The thin black dashed lines denote $t^{2x}/(2x)!$.

the $[H, n_{x+1}]$ commutator contains fermionic operators with indices from x to $x + 2$, which commute with n_1. At distance x, the commutators start contributing at xth-order nesting, yielding a leading power t^{2x}.

This illustrates that the short-time growth of the OTO commutator is dictated by the short-range properties of the model—not accounted for by the low-energy theory captured by bosonization—such as the range of the hopping processes or interactions. The short-time growth, before reaching the light cone, is thus seen to depend on the high-energy (ultraviolet) part of the spectrum as well [8]. We emphasize that, for $x = 0$, the t^2 prediction is also confirmed numerically, shown in Fig. 1.

The XXZ Heisenberg chain in Eq. (10) is integrable but contains high-energy features not accounted for by the LL model. These dominate only the transient response around $tJ \sim 1$ in Fig. 1. Integrability is destroyed by adding a second nearest-neighbor density-density (i.e., $J_z' \sum_m n_{m+2} n_m$) or Ising interaction [43] in the spin language, which we have also studied numerically. It reproduces the central features found for the integrable case, in particular, the t^2 initial growth of the OTO commutator of the n_1 local density as well as the saturation of the OTO commutator after the light cone [44].

Concluding remarks.—We have investigated the OTO correlator in equilibrium and after a quantum quench in one of the canonical low-dimensional model systems in the thermodynamic limit, i.e., in a Luttinger liquid. The OTO commutators display robust behavior, independent of the temperature, initial state, and protocol (equilibrium time evolution vs quantum quenches).

In general, the quantum butterfly effect is manifest in an exponential growth and a large late time value of the OTO commutator [7,8]. Based on our results, the first feature is not realized [45] during the time evolution in LL. The short-time OTO commutator grows with t^2, i.e., the lowest possible power, before reaching the light cone as opposed to the exponential growth in chaotic systems. LLs are thus slow information scramblers, with information encoded in local operators lost slowly.

Surprisingly, the second feature of the butterfly effect appears also in LLs: The OTO commutators of vertex operators, which incorporate the phase fluctuation in a quasicondensate and the $2k_F$ density fluctuations, are often enhanced significantly in a strongly interacting theory after hitting the light cone. Thus, the identification of the large late time value of the OTO commutator as a signature of chaotic behavior [7,8,10] can be misleading. The time at which this enhancement occurs defines the scrambling time, which, in our case, is simply the position of the light cone, i.e., $t = x/v_f$. This large late time value, growing with interaction, occurs probably due to the replacement of the original quasiparticles by collective bosonic modes, being absent in the noninteracting case. It will be interesting to investigate what the minimal ingredients, and

alternative settings, are for such a transmutation of the underlying degrees of freedom to yield a characteristic OTO commutator signal.

Many of these features are benchmarked by exact diagonalization calculations on an interacting spinless fermion model, and they are remarkably robust. We also find that the short-time dynamics is extremely sensitive to the microscopic details of the model under study, yielding a much steeper initial growth with a power t^{2x} depending linearly on the distance x of the densities in the OTO commutator.

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