Quasicontinuous-Variable Quantum Computation with Collective Spins in Multipath Interferometers

Tomáš Opatrný
Optics Department, Faculty of Science, Palacký University, 17. Listopadu 12, 77146 Olomouc, Czech Republic
(Received 10 February 2017; published 5 July 2017)

Collective spins of large atomic samples trapped inside optical resonators can carry quantum information that can be processed in a way similar to quantum computation with continuous variables. It is shown here that by combining the resonators in multipath interferometers one can realize coupling between different samples, and that polynomial Hamiltonians can be constructed by repeated spin rotations and twisting induced by dispersive interaction of the atoms with light. Application can be expected in the efficient simulation of quantum systems.

DOI: 10.1103/PhysRevLett.119.010502

Introduction.—Quantum computation with continuous variables (CVs) is an alternative to the computation based on qubits [1,2]. Efficient simulation of quantum processes with dynamical CVs is one of the main motivations for this approach [3–6]. A universal CV quantum computer would need (i) a sufficiently large set of single CV modes each of which can be initialized in a suitable quantum state, (ii) a suitable set of single-mode Hamiltonians capable of being combined into more complicated Hamiltonians realizing arbitrary polynomials of the CV, and (iii) a suitable interaction Hamiltonian of different CVs. As shown in Ref. [1], having in each mode k a conjugate pair of variables q_k and p_k commuting as $[q_k, p_k] = i$, any polynomial Hamiltonian of q_k and p_k can be constructed if a Hamiltonian of at least third power of q_k or p_k is available, as well as some simpler Hamiltonians realizing, e.g., displacements or rotations in the phase space. Hamiltonians containing higher powers of q and p are generated by cascaded application of commutators of lower power Hamiltonians. As a possible model of a CV quantum computer, one considers a set of optical modes where Kerr interaction $\propto (q^2 + p^2)^2$ plays the role of the higher order Hamiltonian, and beam splitters realize the interaction between different modes. Since the Kerr interaction is typically too weak to be practical for quantum CV operations, alternate schemes have been proposed. These include quantum computing with CV clusters [7,8], or measurement-based schemes for higher order Hamiltonians such as $\propto q^3$ [9,10].

Here, a scheme of quasi-CV quantum computation based on collective spins of large ($N \gtrsim 10^3$) atomic samples interacting via optical fields in multipath interferometers is proposed. Although spin is a discrete variable, for large N and nearly polarized atomic samples the spin components perpendicular to the polarization direction have similar properties as the CVs position Q and momentum P of a harmonic oscillator. Visualizing collective spin states on a Bloch sphere, the computationally relevant states are localized in a confined area where the geometry is close to that of a flat phase space. On the other hand, the curved geometry brings a special advantage in that already quadratic Hamiltonians typically used to generate spin squeezing are sufficient to generate higher power Hamiltonians by commutators. This is achieved by a sequence of rotations (linear Hamiltonian) and squeezing operations (quadratic) which can, in principle, realize Hamiltonians containing, among others, arbitrary powers of the computational variable. Moreover, if the atomic samples are placed in optical resonators mutually coupled to form an interferometer, the off-resonant atom-light interaction can mediate quantum nondemolition (QND) interaction between various samples. By changing resonator lengths and optical phases between the resonators, one can select the modes to interact. Thus, multimode polynomial Hamiltonians can be realized. To realize quantum computation, the system is initialized by squeezing the atomic spins in each resonator, and at the end the results are read-off by measuring the relevant spin components as in the cavity spin squeezing experiments [11,12].

Atoms in a resonator.—Based on the idea of atomic spin squeezing by cavity feedback [11,13], we first consider a scheme in Fig. 1(a). An incoming laser beam of electric intensity $E_{\rm in}$ is partially reflected from the left cavity mirror and partially enters the cavity. The laser is tuned close to the cavity resonance where the field intensity inside the cavity strongly depends on the optical phase. A large collection of nearly resonant atoms is optically trapped inside the cavity by an additional field at antinodes of the standing wave $E_{\rm cav}$. The relevant atomic states are the hyperfine-split states g_1 and g_2 of the electronic ground state and an electronically excited state e. The laser frequency is tuned halfway between the transitions g_1e and g_2e such that the field is detuned by Δ from each of them. The presence of an atom in state g_1 (g_2) changes the optical phase by $\pm\delta\varphi$, respectively, where $\delta\varphi = (6/\pi^2)(\lambda/w)^2(\Gamma/\Delta)$ [14]. Here λ is the wavelength, w is the beam waist, and Γ is the optical decay rate from state e.

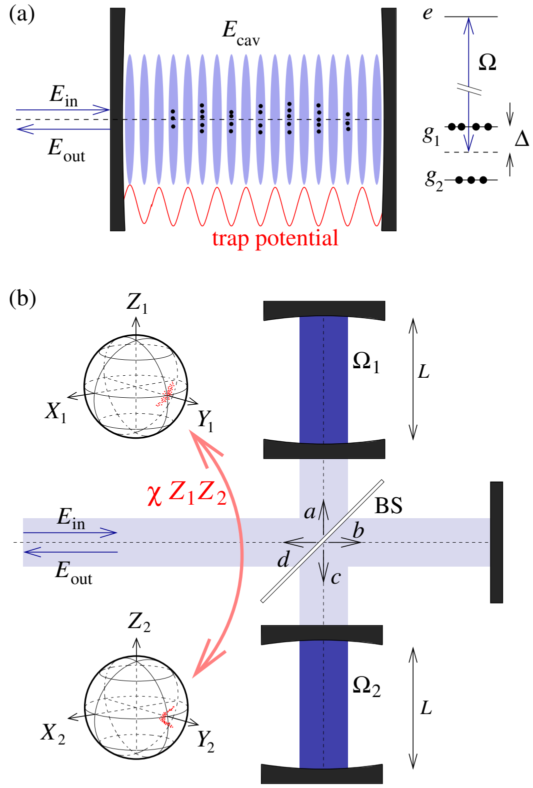

FIG. 1. (a) Scheme of the resonator with trapped atoms. Red-detuned standing wave holds the atoms trapped at locations coinciding with the antinodes of the field $E_{\rm cav}$ interacting with the atoms with Rabi frequency Ω. The cavity field frequency is tuned halfway between the transitions eg_1 and eg_2 such that the phase shift in the cavity is proportional to the difference of atomic numbers in states g_1 and g_2. The phase in the resonator influences the field intensity inside. (b) Michelson-like interferometer with two resonators. Difference of atomic numbers in states g_1 and g_2 corresponds to the spin coordinate Z. Phase of each resonator influences the intensity in both of them. The QND interaction rotates sphere 1 around the Z_1 axis in dependence on the value of Z_2 and vice versa.

A collective spin state of the atoms can be expressed in terms of operators $\hat{a}_{1,2}$ and their Hermitian conjugates, where $\hat{a}_j^\dagger$ ($\hat{a}_j$) creates (annihilates) an atom in state g_j, respectively. The total number of atoms is $N=\hat{a}_1^\dagger\hat{a}_1+\hat{a}_2^\dagger\hat{a}_2$, and the commutation rules are $[\hat{a}_j,\hat{a}_k^\dagger]=\delta_{jk}$. We construct angular momentumlike operators X, Y, and Z as $X=\frac{1}{2}(\hat{a}_1^\dagger\hat{a}_2+\hat{a}_1\hat{a}_2^\dagger)$, $Y=(1/2i)(\hat{a}_1^\dagger\hat{a}_2-\hat{a}_1\hat{a}_2^\dagger)$, and $Z=\frac{1}{2}(\hat{a}_1^\dagger\hat{a}_1-\hat{a}_2^\dagger\hat{a}_2)$ satisfying the commutation relations $[X,Y]=iZ$, $[Y,Z]=iX$, and $[Z,X]=iY$. Note that for simplicity we use X, Y, Z rather than the more common notation $J_{x,y,z}$. For states with $|X|,|Z|\ll N$ and $Y\approx N/2$ [near the equator, as in Fig. 1(b)], the operators $\tilde{q}\equiv\sqrt{2/N}Z$ and $\tilde{p}\equiv\sqrt{2/N}X$ commute as $[\tilde{q},\tilde{p}]\approx i$ and can be used to simulate the CVs q and p.

The optical phase shift in the cavity due to the collective atomic state can be expressed as $\Delta\varphi=2\delta\varphi Z$. The cavity phase shift influences the inside field intensity as follows. Assume the left mirror of the cavity has transmissivity $T\ll 1$, whereas the right mirror is perfectly reflecting. Assume the loss per one round trip in the cavity is $\epsilon\ll T$. If α describes the optical phase deviation from the center of the resonance line, the field intensity at the antinodes in the cavity can be expressed as $E_{\rm cav}^2=E_{\rm in}^2(4/T)[1+(2\alpha/T)^2]^{-1}$ (see [14] for the derivation). Expressing the wave number $k=2\pi/\lambda$ as $k=k_0+\Delta k$ where $k_0L=n\pi$ with L being the cavity length and n integer, the phase deviation is $\alpha=2(L\Delta k+\delta\varphi Z)$.

The cavity field induces an ac Stark shift of the atomic states so that the levels $g_{1,2}$ move apart by $\omega_{\rm ac}=2\Omega^2/\Delta$ with the Rabi frequency being $\Omega=|E_{\rm cav}|\wp/\hbar$, where $\wp$ is the electric dipole moment of the optical transition. The dipole moment is related to the spontaneous decay rate by $\Gamma=(1/4\pi\epsilon_0)(4\omega_0^3\wp^2/3\hbar c^3)$ [15], where ϵ_0 is the vacuum permittivity and $\omega_0=k_0c$. The energy of the whole atomic sample is thus changed by $H=2\hbar\omega_{\rm ac}Z$. Expressing the intensity of the incoming field in terms of the incoming power $P_0=(\pi/2)\epsilon_0cw^2E_{\rm in}^2$ which can be expressed by means of the rate $\mathcal{R}$ of incoming photons $P_0=\mathcal{R}\hbar\omega_0$, we can write

$$H=\hbar\frac{24}{\pi^2T}\frac{1}{1+(\frac{2\alpha}{T})^2}\left(\frac{\lambda}{w}\right)^2\frac{\Gamma}{\Delta}Z\mathcal{R}. \tag{1}$$

Linearizing the dependence of $[1+(2\alpha/T)^2]^{-1}$ on Z for sufficiently large detuning $|\Delta k|\gg\delta\varphi|Z|/L$ one finds

$$H=\hbar(\omega Z+\chi Z^2), \tag{2}$$

where

$$\omega=\frac{24}{\pi^2T}\frac{1}{1+(\frac{4L\Delta k}{T})^2}\left(\frac{\lambda}{w}\right)^2\frac{\Gamma}{\Delta}\mathcal{R}, \tag{3}$$

$$\chi=-\frac{2^7\times3^2}{\pi^4}\frac{1}{T^2}\frac{\frac{4L\Delta k}{T}}{[1+(\frac{4L\Delta k}{T})^2]^2}\left(\frac{\lambda}{w}\right)^4\left(\frac{\Gamma}{\Delta}\right)^2\mathcal{R}. \tag{4}$$

Note that a suitable choice of parameters T, L, and Δk is needed so as to ensure both that the interaction is strong enough and that χ is independent of Z with a reasonable precision. Hamiltonian Eq. (2) realizes the one-axis twisting (OAT) scenario of spin squeezing [16]. The sign of the quadratic term χZ^2 can be switched by switching the sign of detuning Δk. Recently, application of this twist-untwist feature for quantum metrological purposes was proposed [17].

Apart from a quadratic Hamiltonian, one needs also a suitable set of operators linear in the variables X, Y, Z. A microwave field off-resonantly coupling states $g_{1,2}$ realizes Hamiltonians proportional to Z. A resonant microwave field can realize Hamiltonians proportional to $X\cos\gamma+Y\sin\gamma$ where γ is the mutual phase between the microwave and the atomic sample [11,18]. Alternately, one can also use optical Raman transitions between the spin states.

Coupling between different atomic samples.—First consider a scheme with two cavities in a Michelson-like setup as in Fig. 1(b). Various modifications are possible, but for concreteness, let us assume cavities in branches a and c with resonant path lengths $2L_a k_0 = 2L_c k_0 = 2\pi n$ and a mirror in path b with $2L_b k_0 = (2n+1)\pi$. In this case a phase shift in one cavity strongly influences the intensity in both cavities. Assuming a sufficiently large detuning $|\Delta k| \gg \epsilon/(2L)$, linearization of the phase dependence leads to the Hamiltonian of the form

$$H = \hbar[\omega(Z_1 + T_B Z_2) + \chi(Z_1 - Z_2)^2], \tag{5}$$

where

$$\omega = \frac{2^6 \times 3}{\pi^2} \frac{R_B}{(1+T_B)^2} \frac{1}{T} \left(\frac{\lambda}{w}\right)^2 \frac{\Gamma}{\Delta} \mathcal{R}, \tag{6}$$

$$\chi = -\frac{2^8 \times 3^2}{\pi^4} \frac{R_B T_B}{(1+T_B)^3} \frac{1}{TL\Delta k} \left(\frac{\lambda}{w}\right)^4 \left(\frac{\Gamma}{\Delta}\right)^2 \mathcal{R}, \tag{7}$$

with $T_B = 1 - R_B$ being the transmissivity of the interferometer beam splitter (see Ref. [14] for the detailed derivation). Hamiltonian Eq. (5) can be straightforwardly used to generate evolution corresponding to the QND Hamiltonian

$$H_{\rm QND} = -\hbar 2\chi Z_1 Z_2. \tag{8}$$

This is achieved by a four-step sequence in which rotations of the Bloch spheres change $Z_j \to -Z_j$ and the sign of χ is changed to the opposite value in the two steps when exactly one of the coordinates Z_j is changed. This sequence eliminates the linear terms $\propto Z_j$ as well as the quadratic terms $\propto Z_j^2$ of (5).

The strength of the interaction χ can be increased by decreasing the detuning Δk. This is illustrated in Fig. 2 where the power inside one of the cavities as well as the resulting interaction Hamiltonian are shown for two different values of Δk. For small Δk [Fig. 2(a)] the atoms may tune the system close to resonance where the power increases dramatically. This leads to strong interaction, but also to the deformation of the dependence of H on $Z_{1,2}$ beyond the approximation of Eqs. (5) and (8). For larger Δk [Fig. 2(b)] the Hamiltonian is closer to the bilinear form, Eq. (8), but the interaction is weaker. The optimal choice of Δk will depend on the particular task to be achieved with the interacting atoms.

The scheme can be scaled up to contain more cavities. The simplest generalization is a three-cavity scheme where a cavity is placed in each of the a, b, c branches of the interferometer in Fig. 1(b). By shifting the cavity mirrors, some cavities can be brought sufficiently near to resonance whereas others will be far off-resonant. In the resulting

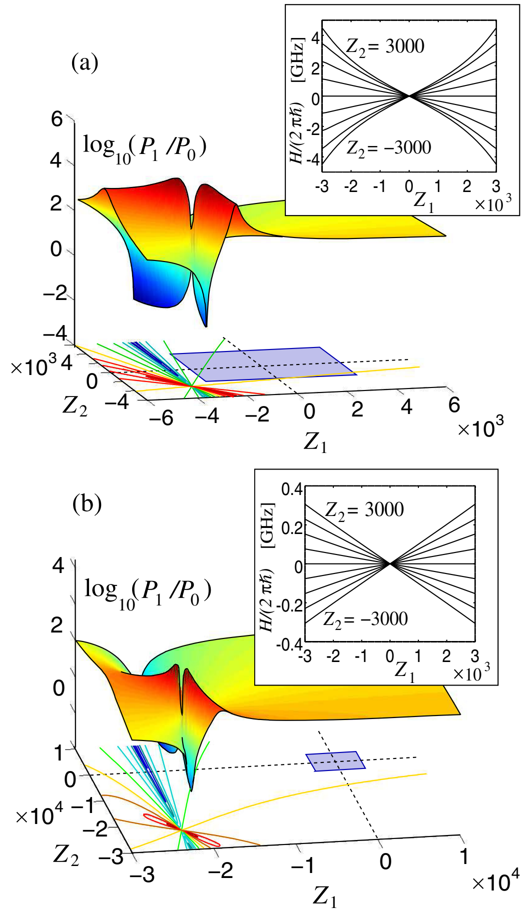

FIG. 2. Dependence of power P_1 in cavity 1 on the atomic spins $Z_{1,2}$. The lines at the bottom are contours of equal power, the shaded rectangular area of $-3\times10^3 \le Z_{1,2} \le 3\times10^3$ shows the accessible values with $N = 6\times10^3$ atoms. Insets: Resulting interaction Hamiltonian after the four-step sequence described in the text. The lines show dependence of the Hamiltonian on Z_1 for 9 equidistant values of Z_2 between $\pm3\times10^3$. The setup corresponds to that in Fig. 1(b) with $w/\lambda = 100$, cavity mirror transmissivity $T = 5\times10^{-3}$, absorption $\epsilon = 1.2\times10^{-6}$, cavity length $L = 26$ mm, and input power $P_0 = 12$ nW. (a) $L\Delta k = 0.08T$, (b) $L\Delta k = 0.5T$.

Hamiltonian, only the atomic samples of the nearly resonant cavities will interact. A five-cavity scheme is illustrated in Fig. 3. As checked by numerical simulation of the interferometer, two arbitrary cavities can be brought to interaction whereas the rest of them are switched off.

Construction of higher power Hamiltonians and functions of the CVs.—Having the Hamiltonian $\propto Z^2$ with both signs, as well as rotations of the Bloch sphere by linear Hamiltonians, one can construct any quadratic Hamiltonian of X, Y, Z. In particular, the two-axis countertwisting (TACT) [16] Hamiltonians $X^2 - Y^2$ or $XY + YX = \frac{1}{2}[(X+Y)^2 - (X-Y)^2]$ are built by rotating the Bloch sphere by $\pm\pi/2$ or $\pm\pi/4$ and applying $\pm Z^2$ (for a general treatment of spin squeezing by quadratic Hamiltonians,

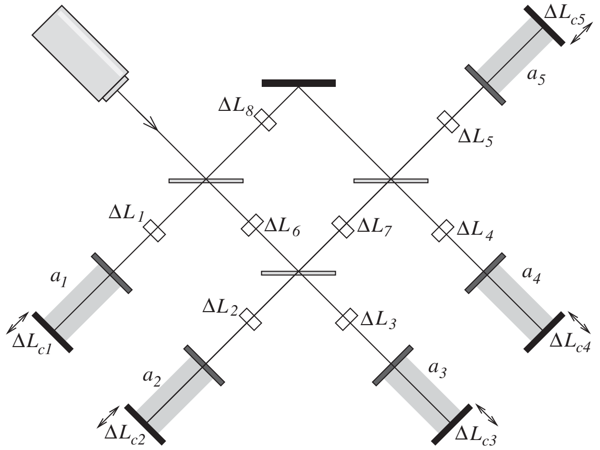

FIG. 3. Scheme of an interferometer with five cavities with atomic samples. The interaction between various cavities is switched on and off by changing ΔL_{cj} to bring the cavities near to resonance or far off-resonance, and by varying the phases in the paths.

see Ref. [19]). Other Hamiltonians can be constructed as commutators of the available operators by the sequence $e^{-iA\Delta t}e^{-iB\Delta t}e^{iA\Delta t}e^{iB\Delta t} = e^{[A,B]\Delta t^2} + \mathcal{O}(\Delta t^3)$ and by the Suzuki-Trotter expansion [20–22]. Using the commutation relations of X, Y, Z, one finds, e.g.,

$$X^3 = \frac{i}{4}[(Z^2 - Y^2), (YZ + ZY)] + \frac{i}{4}[(XZ + ZX), (XY + YX)] + \frac{1}{4}X, \tag{9}$$

or a two-mode Hamiltonian

$$X_1^3 Z_2 = \frac{1}{4}X_1 Z_2 + \frac{1}{4}[(Z_1^2 - Y_1^2), [Z_1^2, X_1 Z_2]] - \frac{1}{4}[X_1 Z_1 + Z_1 X_1, [X_1^2, Z_1 Z_2]], \tag{10}$$

that can be useful to construct functions mapping the variables as $(X_1, X_2) \to (X_1, f(X_1) + X_2)$. By cascading the commutators one can construct Hamiltonians of arbitrary power. More efficient ways of producing various Hamiltonians by using the fact that limited area of the Bloch sphere is used can be found; optimization of the process is in focus of further research.

Decoherence and losses.—Several challenges have to be addressed to fully utilize the scheme. Losses are inherently connected with the dispersive interaction as $\epsilon \sim N(\lambda/w)^2(\Gamma/\Delta)^2$ [14]. Decreasing losses thus means also decreasing the strength of the Hamiltonians and thus making the process longer. Therefore, optimization of the interaction strength should be applied to make the process useful. Also, the optical field becomes entangled with the atomic system leading to decoherence: the phase of the outgoing light is influenced by the atomic number inside, and phase of the atomic spins is influenced by the fluctuating light intensity. This problem was studied in detail in Ref. [23] and one can anticipate various scenarios to solve it: recycling the light pulses to disentangle them from the atoms, using sub-shot-noise squeezed pulses, or detecting the energy of the outgoing light and considering the atomic state conditioned on the result.

Discussion and conclusion.—The essential features of the proposed scheme are the possibility to vary the sign of the nonlinearity, to build Hamiltonians of higher powers of the computational CVs out of the quadratic Hamiltonian, and the possibility to couple multiple resonators in interferometric schemes. The seeming contradiction between the possibility to generate higher power Hamiltonians out of quadratic ones and the fact that at least cubic nonlinearity is required in schemes as in Ref. [1] is resolved by considering that the spin operators X, Y, Z themselves are quadratic in the creation and annihilation operators. Thus, the Hamiltonian Z^2 contains terms like $\hat{a}_1^\dagger \hat{a}_1 \hat{a}_2^\dagger \hat{a}_2$, i.e., of the cross-Kerr type.

The approach is fully compatible with the scheme of quantum computing with CV clusters [7,8] as all its ingredients are present here: multimode squeezed states can be initially prepared by the QND Hamiltonians, and non-Gaussian operations are generated either by X^3 and higher order Hamiltonians, or by projective measurements of a suitable non-Gaussian variable. Here, such a measurement can be done by rotating the states close to the pole of the Bloch sphere and then measuring Z which would be analogous to photon counting in optical CV schemes. Note that non-Gaussian features of the detected statistics in atomic spin systems have been used recently for metrology improvement [24].

The potential of collective spins of atoms in optical resonators for CV quantum computation seems promising taking into account the huge squeezing recently achieved [12]. The scheme is expected to be useful especially for the simulation of quantum systems [3–6].

This work was supported by the Czech Science Foundation, Grant No. 17-20479S.

[1] S. Lloyd and S. L. Braunstein, Quantum Computation over Continuous Variables, Phys. Rev. Lett. **82**, 1784 (1999).

[2] S. L. Braunstein and P. van Loock, Quantum information with continuous variables, Rev. Mod. Phys. **77**, 513 (2005).

[3] V. M. Kendon, K. Nemoto, and W. J. Munro, Quantum analogue computing, Phil. Trans. R. Soc. A **368**, 3609 (2010).

[4] I. M. Georgescu, S. Ashhab, and F. Nori, Quantum simulation, Rev. Mod. Phys. **86**, 153 (2014).

[5] K. Marshall, R. Pooser, G. Siopsis, and C. Weedbrook, Quantum simulation of quantum field theory using continuous variables, Phys. Rev. A **92**, 063825 (2015).

[6] X. Deng, S. Hao, H. Guo, C. Xie, and X. Su, Continuous variable quantum optical simulation for time evolution of quantum harmonic oscillators, Sci. Rep. **6**, 22914 (2016).

[7] N. C. Menicucci, P. van Loock, M. Gu, C. Weedbrook, T. C. Ralph, and M. A. Nielsen, Universal Quantum Computation with Continuous-Variable Cluster States, Phys. Rev. Lett. **97**, 110501 (2006).
[8] M. Gu, C. Weedbrook, N. C. Menicucci, T. C. Ralph, and P. van Loock, Quantum computing with continuous-variable clusters., Phys. Rev. A **79**, 062318 (2009).
[9] P. Marek, R. Filip, and A. Furusawa, Deterministic implementation of weak quantum cubic nonlinearity, Phys. Rev. A **84**, 053802 (2011).
[10] K. Marshall, R. Pooser, G. Siopsis, and C. Weedbrook, Repeat-until-success cubic phase gate for universal continuous-variable quantum computation, Phys. Rev. A **91**, 032321 (2015).
[11] I. D. Leroux, M. H. Schleier-Smith, and V. Vuletić, Implementation of Cavity Squeezing of a Collective Atomic Spin, Phys. Rev. Lett. **104**, 073602 (2010).
[12] O. Hosten, N. J. Engelsen, R. Krishnakumar, and M. A. Kasevich, Measurement noise 100 times lower than the quantum-projection limit using entangled atoms, Nat. Phys. **529**, 505 (2016).
[13] M. H. Schleier-Smith, I. D. Leroux, and V. Vuletić, Squeezing the collective spin of a dilute atomic ensemble by cavity feedback, Phys. Rev. A **81**, 021804(R) (2010).
[14] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.119.010502 for detailed derivations of the formulas.
[15] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, England, 1997).
[16] M. Kitagawa and M. Ueda, Squeezed spin states, Phys. Rev. A **47**, 5138 (1993).
[17] E. Davis, G. Bentsen, and M. H. Schleier-Smith, Approaching the Heisenberg Limit without Single-Particle Detection, Phys. Rev. Lett. **116**, 053601 (2016).
[18] M. H. Schleier-Smith, I. D. Leroux, and V. Vuletić, States of an Ensemble of Two-Level Atoms with Reduced Quantum Uncertainty, Phys. Rev. Lett. **104**, 073604 (2010).
[19] T. Opatrný, Twisting tensor and spin squeezing, Phys. Rev. A **91**, 053826 (2015).
[20] H. F. Trotter, On the product of semi-groups of operators, Proc. Am. Math. Soc. **10**, 545 (1959).
[21] M. Suzuki, Generalized Trotter's formula and systematic approximants of exponential operators and inner derivations with applications to many-body problems, Commun. Math. Phys. **51**, 183 (1976).
[22] N. Hatano and M. Suzuki, Finding exponential product formulas of higher orders, arXiv:math-ph/0506007.
[23] I. D. Leroux, M. H. Schleier-Smith, H. Zhang, and V. Vuletić, Unitary cavity spin squeezing by quantum erasure, Phys. Rev. A **85**, 013803 (2012).
[24] H. Strobel, W. Muessel, D. Linnemann, T. Zibold, D. B. Hume, L. Pezzè, A. Smerzi, and M. K. Oberthaler, Fisher information and entanglement of non-Gaussian spin states, Science **345**, 424 (2014).