# Test of Special Relativity Using a Fiber Network of Optical Clocks

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Phase compensated optical fiber links enable high accuracy atomic clocks separated by thousands of kilometers to be compared with unprecedented statistical resolution. By searching for a daily variation of the frequency difference between four strontium optical lattice clocks in different locations throughout Europe connected by such links, we improve upon previous tests of time dilation predicted by special relativity. We obtain a constraint on the Robertson-Mansouri-Sexl parameter $|\alpha| \lesssim 1.1 \times 10^{-8}$, quantifying a violation of time dilation, thus improving by a factor of around 2 the best known constraint obtained with Ives-Stilwell type experiments, and by 2 orders of magnitude the best constraint obtained by comparing atomic clocks. This work is the first of a new generation of tests of fundamental physics using optical clocks and fiber links. As clocks improve, and as fiber links are routinely operated, we expect that the tests initiated in this Letter will improve by orders of magnitude in the near future.

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Special relativity (SR), one of the cornerstones of modern physics, assumes that Lorentz invariance (LI) is a fundamental symmetry of nature. The search for a violation of LI is motivated by two factors: (i) theoretical suggestions that LI may not be an exact symmetry at all energies and (ii) the tremendous advances in the precision of experimental tests. Indeed, a strong violation of LI at the Planck scale is likely to yield a small amount of violation at low energy, which could be measured with precise experiments [1].

Optical clocks are now the most precise measurement devices. They reach systematic uncertainties of a few $10^{-18}$, which can be resolved after a mere few hours of measurement with optical lattice clocks based on trapped neutral atoms [2–6]. Thanks to these unparalleled performances, comparing the resonance frequencies of optical clocks has led to new tests of fundamental physics, such as bounding the time variation of fundamental constants [7,8].

In this Letter, we perform a test of SR using a network of distant optical lattice clocks located in France, Germany, and the United Kingdom. By exploiting the difference between the velocities of each clock in the inertial geocentric frame, due to their different positions on the surface of Earth, we are able to improve upon previous tests of time dilation. The connection between these clocks, achieved with phase-compensated optical fibers, allows for an unprecedented level of statistical resolution for the comparison of remote atomic clocks [9], making such a test possible for the first time.

LI violations are predicted by several theoretical frameworks, categorized as kinematical and dynamical frameworks (see Ref. [1] for a review). In this Letter we use the Robertson-Mansouri-Sexl (RMS) kinematical framework [10–13] that contains only three parameters. It assumes the existence of a preferred frame $\Sigma$ where light propagates rectilinearly and isotropically in free space with constant speed $c$. The ordinary Lorentz transformations from $\Sigma$ to the observer frame $S$ with relative velocity $\vec{w}$ are generalized to allow for violations of SR:

$$T = a^{-1}(t - c^{-1}\vec{\epsilon}\cdot\vec{x}), \tag{1}$$

$$\vec{X} = d^{-1}\vec{x} - (d^{-1} - b^{-1})(\vec{w}\cdot\vec{x})\vec{w}/w^2 + \vec{w}T, \tag{2}$$

where $a$, $b$, and $d$ are functions of $w^2$, and $\vec{\epsilon}$ is a $w$ dependent vector specifying the clock synchronization procedure in $S$. In the low-velocity limit:

$$a(\vec{w}) = 1 + c^{-2}(\alpha - 1/2)w^2 + \mathcal{O}(c^{-4}w^4), \tag{3}$$

 

where $\alpha$ is an arbitrary parameter quantifying the LI violation, the value of which is zero in SR.

First-order tests in $\vec{w}/c$ are based on the comparison of clocks [12]. Until recently, they gave the best constraints on the LI violating parameter $\alpha$ (see Ref. [14] for a review) with, e.g., $|\alpha| \leq 10^{-6}$ obtained by comparing atomic clocks' onboard GPS satellites with ground atomic clocks [15].

The three classical LI tests are the Michelson-Morley, Kennedy-Thorndike, and Ives-Stillwell experiments [10]; they are second-order tests as the LI violating signal depends on $w^2/c^2$ [13]. With the advent of heavy-ion storage rings, Ives-Stillwell type experiments now give the best constraint on $\alpha$ [16,17]. A limit of $|\alpha| \leq 8.4 \times 10^{-8}$ was found using $^7Li^+$ ions prepared in a storage ring to 6.4% and 3.0% of the speed of light [16]. The experiment described in Ref. [17] uses $^7Li^+$ ions confined at a velocity of 33.8% of the speed of light. When neglecting higher order RMS parameters, the constraint on the LI violating parameter is $|\alpha| \lesssim 2.0 \times 10^{-8}$.

In this Letter, we improve upon this best previous constraint on the LI violating parameter $\alpha$ by a factor of around 2. Our test is based on four optical lattice clocks using Sr atoms, two located at LNE-SYRTE, Observatoire de Paris, France [18,19], one at PTB, Braunschweig, Germany [20,21], and one at NPL, Teddington, United Kingdom [22]. These clocks are connected by two fiber links, one running from SYRTE to PTB operated in June 2015 [9], and one from SYRTE to NPL operated in June 2016. This Letter exclusively uses the stability of the frequency comparisons between the clocks by looking for a periodic variation.

In a simplified setup, an optical clock comparison using a phase noise compensated fiber link can be described as a two-way frequency transfer between two observers $A$ and $B$ [23–27]. Observer $A$ emits an electromagnetic signal (e.g., an IR laser) with proper frequency $\nu_0$, received by observer $B$ at a proper frequency $\nu_1$, and partly reflected back to observer $A$, where it is received with a proper frequency $\nu_2$. The "redshift signal" or desyntonization is

$$\Delta = \frac{\nu_1 - \nu_0}{\nu_0} - \frac{\nu_2 - \nu_0}{2\nu_0}. \tag{4}$$

The first term contains the relativistic redshift between the two observer locations as well as the first order Doppler shift, while the second term contains only the first order Doppler shift, realizing a well-known "Doppler cancellation" scheme (see, e.g., Refs. [14,25]). The first term is measured by locally beating an optical clock with the electromagnetic signal at each end of the link, while the second term is fixed at a known value.

In the low-velocity limit the desyntonization can be written as

$$\Delta = \Delta_{cl} + \Delta_\alpha, \tag{5}$$

where $\Delta_{cl}$ contains the relativistic redshift due to the static part of the gravity potential as well as temporal variations. During the considered dates of clock comparisons, peak-to-peak fractional frequency variations up to $1.3 \times 10^{-17}$ between PTB and SYRTE, and up to $5 \times 10^{-18}$ between NPL and SYRTE are due to variations of the gravity potential induced by tides. Solid Earth and ocean tides are taken into account (see Ref. [28]).

The LI violating term signal is

$$\Delta_\alpha = \alpha c^{-2}[2\vec{w} \cdot (\vec{v}_A - \vec{v}_B) + (v_A^2 - v_B^2)] + \mathcal{O}(c^{-3}), \tag{6}$$

where $\vec{v}_A$ and $\vec{v}_B$ are, respectively, the velocities of clocks $A$ and $B$ in the nonrotating geocentric celestial reference system (GCRS). They are obtained by transforming the terrestrial coordinates of the clocks, considered as constant, with the SOFA routines [29]. $\vec{w}$ is the velocity of the Earth with respect to a preferred frame, taken as the rest frame of the cosmological microwave background (CMB). It is the sum of the Earth velocity with respect to the Solar System Barycenter (SSB), and the SSB velocity with respect to the CMB. The celestial coordinates of the SSB velocity with respect to the CMB in galactic coordinates are 263.99° (longitude) and 48.26° (latitude) [30], which transformed to the GCRS give 11 h 11 m 36 s (right ascension) and −6° 54' 00" (declination) [31] with a norm of 369 $\text{km} \cdot \text{s}^{-1}$. In June 2015 and 2016 the norm of $\vec{w}$ was $w \simeq 340\ \text{km} \cdot \text{s}^{-1}$.

The first term of the LI violation in Eq. (6) varies with a period of one sidereal day as the Earth rotates around its axis. It is therefore possible to bound the LI violating parameter $\alpha$ by looking for daily variations in the relative frequency difference $y$ between remote clocks, located at different longitudes (i.e., different orientation of $\vec{v}$) and/or different latitudes (i.e., different norms of $\vec{v}$). The second term of Eq. (6) is constant, and, considering an upper bound of $2 \times 10^{-8}$ on the parameter $\alpha$ [17], is lower than $4 \times 10^{-20}$, which is significantly below the accuracy of the clocks. Therefore we do not take it into account in our model.

We first analyze the result of the comparison between the clocks at SYRTE and NPL. Between June 10 and 15 2016, we accumulated about 60 h of clock comparison data between SYRTE's Sr2 and SrB lattice clocks, and NPL's Sr clock. These clocks are connected by a 812 km long cascaded optical fiber link using infrared lasers operated at 1542 nm. The first span of 769 km connects NPL to Laboratoire de Physique des Lasers (LPL) in the north of Paris with the use of a repeater laser station at LPL [32]; the second link connects SYRTE to LPL [33,34]. The frequency ratio of the infrared lasers and the Sr clock lasers at NPL and SYRTE are measured using optical frequency combs [35,36]. The propagation noise in the fibers is actively compensated. At LPL, a beat note is generated with light from the two stabilized links and recorded using a GPS-disciplined ultrastable quartz oscillator and a

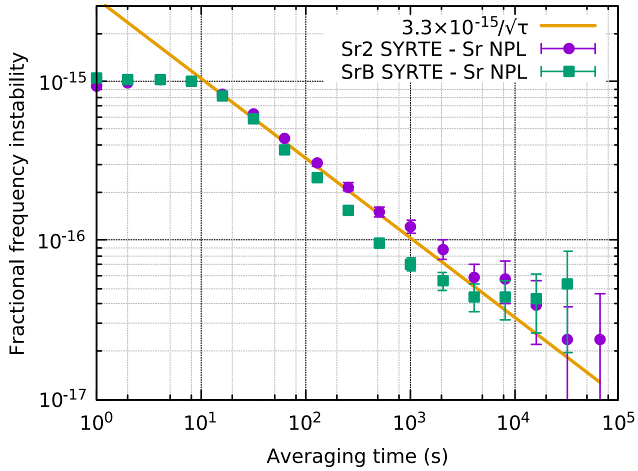


FIG. 1. Allan deviation of the fractional frequency difference between SYRTE's clocks Sr2 and SrB and NPL's Sr clock. After less than one day, the instability of the fractional frequency difference averages down to a few $10^{-17}$. This frequency instability is solely limited by the performances of the clocks, as the fiber link between SYRTE and NPL shows a fractional frequency instability of $1 \times 10^{-18}$ at 1000 s.

dead-time-free frequency counter with a similar approach to the setup described in Ref. [9]. The frequency counters at NPL, LPL, and SYRTE are synchronized to UTC(NPL), GPS time, and UTC(OP), respectively, with an accuracy well below 1 ms. Figure 1 shows the relative frequency instability of the comparison.

To search for a violation of LI in the clock comparisons, we consider three different data subsets: $A$: Sr2 data only; $B$: SrB data only; $C$: Sr2 and SrB data combined. The relative frequency difference $y_{\text{NPL-SYRTE}}$ between the NPL Sr clock and the SYRTE Sr clock is corrected from the term $\Delta_{cl}$. The model used to fit the data contains two (for $A$ and $B$ subsets) or three parameters (for $C$ subset):

$$y_{\text{NPL-SYRTE}}(t) = \bar{y}^{\text{i}}_{\text{NPL-SYRTE}} + 2\alpha c^{-2}\vec{w}\cdot[\vec{v}_{\text{SYRTE}}(t) - \vec{v}_{\text{NPL}}(t)], \quad (7)$$

where $\bar{y}^{\text{i}}_{\text{NPL-SYRTE}}$ allows for one or two fractional frequency offsets, depending on the chosen data subset: $A$: $i = \{\text{Sr2}\}$; $B$: $i = \{\text{SrB}\}$; $C$: $i = \{\text{Sr2}, \text{SrB}\}$, and $\alpha$ is the LI violating parameter. All parameters are determined in the fitting procedure, along with correlations and uncertainties. The mean frequency offsets were removed from each comparison data subset as we are looking only for daily variations. The second line of Eq. (7) is the LI violation; it is very similar to a sinusoid: $Q_0 \sin[2\pi(t - t_0)/T]$, where $T$ is one sidereal day, $Q_0 = 1.60 \times 10^{-10}$ for $\alpha = 1$, and $t_0 = 57549.130$ (MJD).

For each data subset we used an affine invariant Markov Chain Monte Carlo ensemble sampler (MCMC) fitting method with $10^5$ points [37]. As can be seen in Fig. 1, the Allan deviation is flat up to around 10 s averaging time. Indeed, in the short term the laser probing the narrow transition is not yet locked to the atoms; therefore, the flicker floor of the free running laser is visible. This temporal correlation is taken into account in the MCMC method fit by using a nondiagonal covariance matrix. For the combined data set $C$ the correlation between both data sets is also taken into account in the covariance matrix.

TABLE I. Fitting results using the MCMC fitting method with $10^5$ points. Fits $A$ to $C$ use the NPL-SYRTE comparison data with $A$: Sr2 data only; $B$: SrB data only; $C$: Sr2 and SrB data combined. Fit $D$ uses the PTB-SYRTE comparison data.

| | $\delta t^{\text{SYRTE}}$ (hours) | $\alpha_T^{\text{SYRTE}}$ ($10^{-16}$ K$^{-1}$) | $\alpha$ ($10^{-8}$) |
|---|---|---|---|
| $A$ | ... | ... | $+3.81 \pm 8.41$ |
| $B$ | ... | ... | $-5.87 \pm 7.78$ |
| $C$ | ... | ... | $-2.83 \pm 6.19$ |
| $D$ | $4.81 \pm 0.25$ | $1.76 \pm 0.12$ | $-0.38 \pm 1.06$ |

Fitting results are given in Table I for the three cases $A$ to $C$. The correlations between the parameters are of the order or below 0.2. The best result is found when combining the two sets of data:

$$\alpha_C = (-2.83 \pm 6.19) \times 10^{-8}. \quad (8)$$

Correlations and histograms of the parameters for data set $C$ are shown in Fig. 2.

The PTB-SYRTE comparison took place between June 4 and 24 2015. This comparison, involving SYRTE's Sr2 clock and PTB's stationary Sr clock, is reported in Ref. [9]. We use in this Letter the data of the second of the two

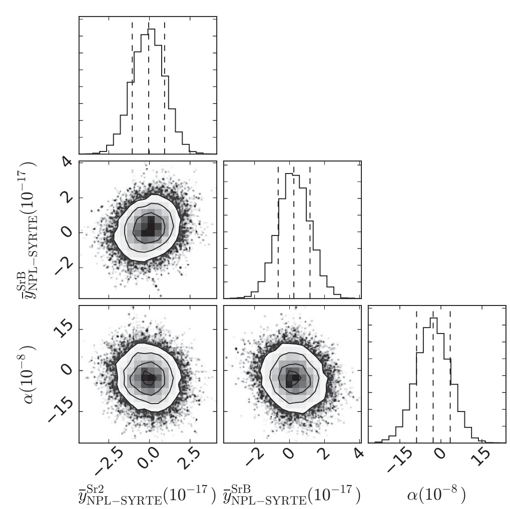


FIG. 2. Correlations and histograms of the parameters of model (7) in fit $C$ of Table I, corresponding to the NPL-SYRTE comparison data from Sr2 and SrB combined, using the MCMC fitting method with $10^5$ points.

campaigns reported in Ref. [9], representing around 150 h of clock comparison data.

An analysis of the PTB-SYRTE comparison data with a model similar to Eq. (7) (replacing the NPL clock velocity with the PTB clock velocity, and $i = \{\mathrm{Sr2}\}$) results in a significant bias for the parameter $\alpha$, 5 times larger than the $1\sigma$ uncertainty on $\alpha$. Indeed, the power spectral density distribution of the raw data shows a significant peak at a frequency of 1 $\mathrm{day}^{-1}$, which is around the frequency of the LI violating signal. Although this signal could be interpreted as a violation of LI, a detailed analysis shows that this effect is probably due to temperature variations in the SYRTE clock laboratory. We analyzed two independent local clock comparisons: Sr against $\mathrm{Yb}^+$ at PTB, and Sr against Hg at SYRTE, which are not affected by a LI violation, and we used a simple model of the effect of temperature on the relative frequency difference:

$$y_{T,X}(t) = \alpha_T[T_X(t-\delta t) - \bar{T}_X], \tag{9}$$

where $T_X(t)$ is a function that interpolates the temperature at time $t$ at some location $X$, $\bar{T}_X$ is the mean of the temperature function $T_X(t-\delta t)$ evaluated for the comparison data times, $\alpha_T$ is a temperature coefficient and $\delta t$ a lag, both to be determined in the fitting procedure. A significant variation was found at 1 $\mathrm{day}^{-1}$ frequency in the local SYRTE comparison, while the comparison at PTB did not show any significant variation at this frequency.

Therefore, in addition to the LI violation, we included the effect of temperature in the SYRTE clock room, leading to the following model:

$$y_{\text{PTB-SYRTE}}(t) = \bar{y}^{\text{Sr2}}_{\text{PTB-SYRTE}} + y_{T,\text{SYRTE}}(t) + 2\alpha c^{-2}\vec{w}\cdot[\vec{v}_{\text{SYRTE}}(t) - \vec{v}_{\text{PTB}}(t)], \tag{10}$$

where $\bar{y}^{\text{Sr2}}_{\text{PTB-SYRTE}}$ allows for a fractional frequency offset, $y_{T,\text{SYRTE}}$ is the temperature effect model given in Eq. (9) and $\alpha$ is the LI violating parameter. The relative frequency difference $y_{\text{PTB-SYRTE}}$ between the PTB Sr clock and the SYRTE Sr clock is corrected from the term $\Delta_{cl}$. As for the model in Eq. (7), here the LI violating term is similar to a sinusoid with a period of one sidereal day, an amplitude $Q_0 = 3.54\times10^{-10}$ for $\alpha = 1$, and $t_0 = 57177.421$ (MJD). As this is a nonlinear model, we use the MCMC method with $10^5$ points for the fitting procedure. Note that $Q_0$ for the PTB—SYRTE link is more than twice the value for the NPL—SYRTE link such that it is more sensitive to a violation of LI.

The detailed result of this analysis is given in Table I—line $D$. It shows a significant effect of the temperature on the frequency comparison of the order of $10^{-16}\ \mathrm{K}^{-1}$, with a lag of around 4.8 h. Correlations and distributions of parameters can be seen in Fig. 3. The lag $\delta t^{\text{SYRTE}}$ is not well constrained and its distribution is non-Gaussian.

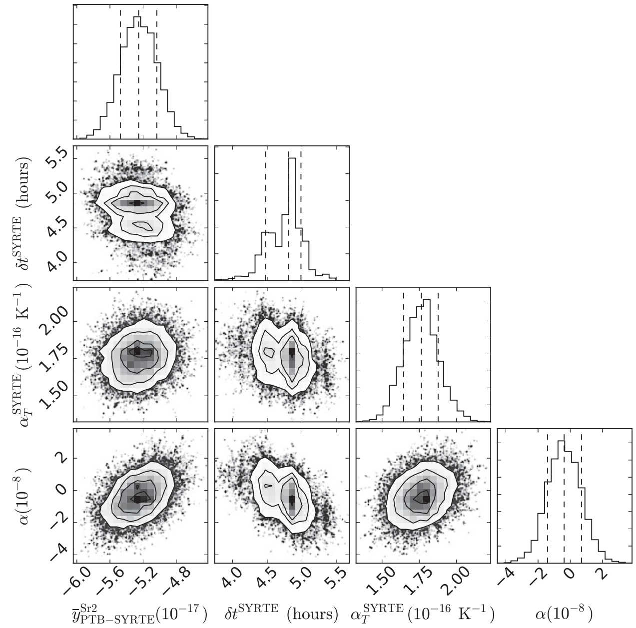


FIG. 3. Correlations and histograms of the parameters of fit $D$ of Table I, corresponding to the fit of the PTB-SYRTE comparison data, using the MCMC fitting method with $10^5$ points.

However, this does not affect the Gaussianity of the other parameters. The correlations between the parameter $\alpha$ and the parameters $\bar{y}^{\text{Sr2}}_{\text{PTB-SYRTE}}$, $\delta t^{\text{SYRTE}}$, and $\alpha_T^{\text{SYRTE}}$ are, respectively, −0.48, 0.44, and 0.30. These correlations slightly degrade the uncertainty on the determination of $\alpha$. The bias of the parameter $|\alpha|$ is below the parameter uncertainty:

$$\alpha_D = (-0.38 \pm 1.06)\times10^{-8}. \tag{11}$$

It is interesting to note that the bias found for the parameter $\alpha$ is $-0.93\times10^{-8}$ if tides are not taken into account.

In order to check further if the choice of this temperature is significant, we repeated the same analysis with several other temperature series: (i) the SYRTE exterior, (ii) the PTB exterior, (iii) the PTB clock room, (iv) the PTB clock laser room, (v) the PTB comb room, and, finally, (vi) a simulated sinusoid with a 1 day period. For each temperature model we fit an amplitude and a lag, as in Eq. (9). None of them are able to explain the residual signal, leading for the series (i–v) to biases in the parameter $\alpha$ ranging from $2\sigma$ up to $5.5\sigma$, and for (vi) to a complete degeneracy of all parameters with no unique solution, which shows that the residual effect cannot be well represented by a simple sinusoidal model, as parameters are completely degenerated with parameters from the LI violating model.

The effect of temperature on the PTB-SYRTE comparison data is not yet fully understood. Further comparisons will help improve our understanding of this, thereby allowing reduction of the bias and, hence, the uncertainty on the determination of the parameter $\alpha$. The existence of the temperature effect is, however, evident by the fact that it can be seen both on the distant PTB-SYRTE and the local SYRTE clock comparisons. A detailed analysis of the NPL-SYRTE comparisons did not show any significant systematic effects above the noise level. Consistently, a simulation of the temperature model (9) for the NPL-SYRTE comparisons, using the parameters determined in the PTB-SYRTE comparison, does not produce any signal above the noise level. This justifies the fact that the temperature model (9) was not used for the NPL-SYRTE comparisons.

A combination of the three data sets $A$, $B$, and $D$ has been evaluated but gives no improvement on the uncertainty of the determination of the LI violating parameter. This is due to the fact that the absolute value of the bias on $\alpha$ obtained in combination $C$ $(2.83 \times 10^{-8})$ is larger that the uncertainty obtained with data set $D$ $(1.06 \times 10^{-8})$.

As noted in Ref. [14], one major limitation of the RMS framework is that it is purely kinematical, and our results cannot be simply mapped to dynamical frameworks. The constraints that can be derived from distant optical clock comparisons on dynamical frameworks such as the standard model extension (see, e.g., Refs. [38–41]) or dark matter models (see, e.g., Refs. [42,43]) will be tackled in future publications.

In conclusion, by using clock comparisons between four optical clocks at NPL (United Kingdom), PTB (Germany), and SYRTE (France), linked by a leading-edge optical fiber network, we are able to put a more stringent bound on the LI violating parameter $\alpha$ of the RMS framework. With $1.1 \times 10^{-8}$, $\alpha$ is now by around a factor of 2 better constrained compared to the best previous determination of this parameter, which was obtained with accelerated ions, and by 2 orders of magnitude with respect to the best constraint previously obtained by comparing atomic clocks. Moreover, this bound is purely limited by technical noise sources on the clock systems, which will improve in future comparisons. Projecting the comparison of distant clocks with an instability of $10^{-16}/\sqrt{\tau}$ over several weeks, a reduction in uncertainty of more than 1 order of magnitude for $\alpha$ is within reach. This shows the significant potential for tests of fundamental physics with networks of optical clocks connected by optical fiber links.

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