Possible Precise Measurement of Delbrück Scattering Using Polarized Photon Beams

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The advent of high-flux-polarized γ-ray sources makes possible the nearly isolated precise measurement of the vacuum contribution, Delbrück scattering, to the elastic scattering of these photons off nuclei. Because of the fact that the elastic scattering of the photons is a coherent summation of four processes and that up to now unpolarized sources have been used, the isolated measurement of Delbrück scattering has not been performed. We show that for the appropriate choice of scattering angles, photon polarization, and energies this scattering can be measured nearly independently of other scattering processes. This makes possible the precise measurement of the vacuum contribution to scattering and the possibility of the detection of new physics.

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Quantum electrodynamics (QED) has been one of the most successful theories in physics. Predictions from lowest-order perturbation calculations have been found to agree with experimental results to very high precision. Further confirmation of QED requires precisely measuring higher-order perturbative predictions where deviations could indicate the need for new physics. One inherently higher-order process is the polarization of vacuum by electromagnetic fields due to the formation of virtual particle pairs [1]. The lowest-order processes, which involve four interacting electromagnetic fields, are photon-photon scattering, photon splitting, photon merging, and Delbrück scattering [1]. Photon-photon scattering has a small cross section and at low energies has been measured only with an upper bound [2–4]. Photon splitting has been observed at high photon energies but is difficult to observe because of the smallness of its cross section relative to other similar processes [5]. Photon merging has not been observed, although there are proposals to measure it using petawatt lasers and protons [6]. Among the four processes, only Delbrück scattering, which is the elastic scattering of γ rays off the Coulomb field of nuclei due to the formation of virtual electron-positron pairs from a vacuum [7], has been extensively experimentally measured (for example, see Refs. [8,9]). This is because its cross section (scattering amplitude) is enhanced with nuclear charge Z as Z^4 (Z^2) [1]. There are two main drawbacks in measuring Delbrück scattering. One is that the scattering cross section is so difficult to calculate numerically that the lowest-order scattering amplitudes exist in only tabular form over a fairly coarse grid in photon energies and scattering angles [10]. The second drawback is that the elastic scattering of photons off nuclei is the coherent summation of four different scattering processes [8,9,11]: Rayleigh, nuclear Thomson, giant dipole resonance, and Delbrück. As a result, the measurement of the Delbrück scattering is obtained by calculating each contribution to the cross section and showing that incorporating the Delbrück component is necessary for an agreement with the experimental results.

Previous experimental results were obtained with unpolarized γ rays [8,9]. As pointed out in Ref. [12], by using linear polarized photons in the scattering plane with the scattering angle of 90°, a more isolated measurement could be achieved, because two of the processes, the nuclear Thomson and giant dipole resonance scattering amplitudes, go to zero at that angle. Until recently, high peak brightness monoenergetic tunable photon sources above 100 keV did not exist. By scattering laser light off relativistic electron beams, linearly polarized tunable high flux sources, laser Compton scattered (LCS) γ-ray sources [13], have been developed and used for the study of various types of science at HIγS [14], NewSUBARU [15], and UVSOR [16]. Furthermore, extremely high-flux LCS sources will soon be available such as ELI-NP-GBS [17] and MEGa-ray [18]. The energy recovery linac-LCS source has been proposed [19]. Through the technological development of LCS γ-ray sources, high-precision measurements of processes such as Delbrück scattering using such polarized γ rays will be possible. In this Letter, we show, using such LCS sources, that Delbrück scattering can be nearly isolated from the other contributions to elastic scattering at angles shifted away from 90° due to the contributions of Rayleigh scattering.

The total elastic differential scattering cross section of photons from atoms is expressed as [20]

$$\frac{d\sigma_{\perp \atop \parallel}}{d\Omega} = r_e^2 |A_{\perp \atop \parallel}|^2, \tag{1}$$

where the subscripts $\parallel$ and $\perp$ refer to photon polarizations parallel and perpendicular to the scattering plane, respectively, and $r_e = e^2/m_e c^2$ is the classical electron radius with e being the electron charge, m_e being its mass, and c being the speed of light. The scattering amplitude is [20]

$$A_{\substack{\perp\\ \parallel}} = A^T_{\substack{\perp\\ \parallel}} + A^{\mathrm{GDR}}_{\substack{\perp\\ \parallel}} + A^R_{\substack{\perp\\ \parallel}} + A^D_{\substack{\perp\\ \parallel}} \tag{2}$$

with the superscripts T, GDR, R, and D referring to the nuclear Thomson, giant dipole resonance, Rayleigh, and Delbrück scattering, respectively. The first two amplitudes are expressed in terms of relatively simple formulas.

The nuclear Thomson scattering (T) amplitudes parallel, $A^T_\parallel$, and perpendicular, $A^T_\perp$, to the scattering plane are calculated for a rigid spin-zero nucleus using Eqs. (3) and (4) in [20]:

$$A^T_\perp = -\frac{Z^2 e^2 m_e}{Mc^2}\left(1 - \frac{1}{3}k^2\langle r^2\rangle\right), \tag{3}$$

$$A^T_\parallel = A^N_\perp \cos\theta, \tag{4}$$

where M is the mass of the nucleus, k is the wave number of the photon, r is the nuclear charge radius which is taken to be zero, and θ is the scattering angle in the scattering plane formed by the directions of the initial and final photon. Note that in the case of scattering perpendicular to the scattering plane the amplitude is independent of θ.

For the giant dipole resonance (GDR) perpendicular, $A^{\mathrm{GDR}}_\perp$, and parallel, $A^{\mathrm{GDR}}_\parallel$, scattering amplitudes, we use Eq. (11) in [20]:

$$A^{\mathrm{GDR}}_\perp = \frac{E^2}{4\pi\hbar c r_e}\sum_{j=1}^{2}\sigma_j\Gamma_j\frac{E_j^2 - E^2}{(E_j^2 - E^2)^2 + E^2\Gamma_j^2}, \tag{5}$$

$$A^{\mathrm{GDR}}_\parallel = A^{\mathrm{GDR}}_\perp \cos\theta, \tag{6}$$

which was calculated using the optical theorem and dispersion relation where E is the γ-ray energy and $(\sigma_j, E_j, \Gamma_j)$ are GDR parameters.

The Rayleigh and Delbrück scattering amplitudes do not have a simple form like the above T and GDR scattering amplitudes. The Rayleigh scattering amplitudes $A^R_\parallel$ and $A^R_\perp$ have been previously calculated using the relativistic second-order S matrix and form factors [21] and can be found in the Rayleigh scattering database (RTAB) [22]. The Delbrück scattering amplitudes $A^D_\parallel$ and $A^D_\perp$ have been previously calculated from the lowest-order vacuum polarization tensor [1], and the values can be found in Ref. [10]. For our study, we have found that a finer resolution is necessary.

We have recalculated the Rayleigh (R) scattering matrix amplitudes using the code "ENTING," which uses the relativistic second-order S matrix (see [21] and references cited therein). We have confirmed the agreement over the range of energies and angles in which we are interested with the RTAB database [22].

Delbrück scattering (D) represented by the lowest-order Feynman diagrams is shown in Fig. 1 [23]. Compact expressions for the real and imaginary parts of the scattering amplitudes expressed in terms of three- and four-dimensional integrals with relatively simple irrational functions of the arguments have been obtained and calculated in Refs. [12,24,25]. We have written a code to calculate the Delbrück scattering amplitudes using the expressions and numerical techniques of Refs. [12,24,25], which are valid for γ-ray energies above $2m_e c^2$, where m_e is the electron mass. The differential scattering cross section is expressed in terms of the right- and left-handed polarization of the photons, $+$ and $-$ subscripts, respectively [12,24,25]:

$$\frac{d\sigma_{\substack{++\\+-}D}}{d\Omega} = (Z\alpha)^4 r_e^2 |a_{\substack{++\\+-}}|^2. \tag{7}$$

The Delbrück scattering amplitudes perpendicular, $A^D_\perp$, and parallel, $A^D_\parallel$, to the scattering plane are given by [25]

$$A^D_\perp = a_{++} - a_{+-}, \tag{8}$$

$$A^D_\parallel = a_{++} + a_{+-}. \tag{9}$$

The imaginary part is [12,24,25]

$$\mathrm{Im}[a_{\substack{++\\+-}}(d,p)] = \frac{1}{\pi p}\int_1^{k^2/4} dy \int_{x_-}^{x^+} dx \times \int_0^{b(y)} dz A_\pm(x,y,z;d,p), \tag{10}$$

where $k = \omega/m_e c^2$, $d = k\sin(\theta/2)$, $p = k\cos(\theta/2)$, $k^2 = d^2 + p^2$ with the conditions $k \geq 2$, $x_\pm = (p \pm \sqrt{k^2 - 4y})^2$, $b(y) = \sqrt{1 - 1/y}$, and $\mathrm{Im}[a_{\substack{++\\+-}}(d,p)] = 0$ for $k \leq 2$, and the functions $A_\pm(x,y,z;d,p)$ are given in Refs. [12,24,25]. The real part is [12,24,25]

$$\mathrm{Re}[a_{\substack{++\\+-}}(d,p)] = C_\pm(d) + \frac{2p^2}{\pi}P\int_{\alpha(d)}^{\infty}\frac{dp'}{p'}\frac{D_\pm(p',d)}{p'^2 - p^2} \tag{11}$$

where $\alpha(d) = \sqrt{4 - d^2}$, when $d \leq 2$; $\alpha(d) = 0$, when $d > 2$. The function $D_\pm(p',d)$ is [12,24,25]

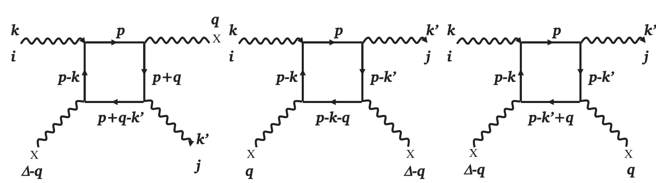

FIG. 1. Delbrück scattering represented by lowest-order Feynman diagrams. k and k' are the 4-vector momenta of the incoming and outgoing photons, respectively, and i and j are their polarization directions, respectively. $\Delta = k' - k$ is the momentum transfer. The X represents the Coulomb field of the nucleus in which q is the field momentum.

$$D_{\pm}(p',d) = \frac{1}{\pi p}\int_1^{k^2/4} dy \int_{x_-}^{x^+} dx \times \int_0^{b(y)} dz\epsilon(\bar{l})A_{\pm}(x,y,z;d,p), \quad (12)$$

where $k \geq 2$, $\epsilon(|x|) = 1$, $\epsilon(-|x|) = -1$, and the notation is the same as Eq. (10). The function $C_{\pm}(d)$ is [12,24,25]

$$C_{\pm}(d) = \{\mathrm{Re}[a_{\substack{++\\+-}}(d,p)]\}_{k=d,\theta=\pi}, \quad (13)$$

which is the amplitude for backscattering implying $C_+(d) = 0$. The nonzero component $C_-(d)$ is given by [25]

$$C_-(d) = \{\mathrm{Re}[a_{+-}(d,p)]\}_{k=d,\theta=\pi} = \frac{1}{\pi^2}\int_0^{\infty} dq \int_{-1}^{+1} d\xi \frac{1}{q^2+2d^2-2dq\xi}P \times \int_1^{\infty} dy\left(\frac{B_1(y,q,\xi;d)}{y-s} + B_2(y,q,\xi;d)\right), \quad (14)$$

where

$$4s = 4t = -q^2 + 2dq\xi, \quad 4\mu_1 = q^2,$$
$$4\mu_2 = q^2 + 4d^2 - 4dq\xi, \quad v = 4u = -d^2q^2(1-\xi^2), \quad (15)$$

and the other variables, functions $B_1(y,q,\xi;d)$ and $B_2(y,q,\xi;d)$, are given in Refs. [12,24,25]. We have confirmed the agreement with previously published data at energies lower than 10 MeV.

In the lowest order, the Delbrück scattering amplitude increases with Z as $(\alpha Z)^2$ and α is the fine structure constant; however, the next-higher-order contribution to the scattering also increases as $\alpha(\alpha Z)^4$ and with photon energy and large scattering angles [9]. Because the next-higher-order correction has not been theoretically calculated, we choose a regime where these higher-order corrections, also known as Coulomb corrections, are negligible. For targets with Z less than 50, the Coulomb correction contributions have been shown to be lower than 5% [26]. The lowest-order scattering amplitudes have been shown to be sufficient for γ-ray energies less than 2 MeV for uranium ($Z = 92$) (see references cited in Ref. [26]). For energies less than 1.115 MeV, experiments have shown that the lowest order is sufficient for $Z = 74$ and 82 within experimental uncertainties of ~5% [27]. At 2.754 MeV, experimental results with $Z = 82$, 83, 90, and 92 have shown differences from the lowest-order calculations by as much as a factor of 2 [27]. Considering the above information, we have chosen tin ($Z = 50$) as the target material and γ rays in the range from 1.050 to 1.150 MeV, where the lower bound has been chosen to be above 1.022 MeV, where the expressions we have chosen for the Delbrück scattering are valid [12,24,25] and the energies available in the RTAB database [22]. Although the full order amplitude has been analytically formulated [28] and numerically calculated [29], few data points exist. In addition, the large number of terms has prevented the calculation of the next-higher order [23].

To locate where the Delbrück scattering amplitudes are nearly isolated, it is first necessary to find regions where the other three contributions to the scattering are small. For the GDR scattering amplitude in Eq. (5) for tin, we use the fitting parameters for the most abundant isotope having an atomic number, $A = 120$, from the Reference Input Parameter Library (RIPL-3) database [30,31], where $j = 1$ is the only term in the summation in Eq. (5) with $E_1 = 15.37$ MeV, $\sigma_1 = 284.1$ mb, and $\Gamma_1 = 5.08$ MeV [30]. The Rayleigh scattering (R) amplitudes were calculated from the program ENTING using the same configuration as the RTAB database [22] in our energy range of interest, where the contribution of 10 inner shell electrons is calculated using the relativistic second-order S matrix and of 40 outer shell electrons is calculated using form factors [21]. The T amplitudes were obtained from the routines in the code ENTING used to generate the RTAB database [22].

Figure 2 shows results of the calculations of the differential cross section $d\sigma_{\|_{R+T+\mathrm{GDR}}}/d\Omega$, for $R + T + \mathrm{GDR}$ for γ rays in the range 1.10 ± 0.05 MeV and scattering angles $40° \leq \theta \leq 120°$ in the scattering plane. In the energy range we have chosen $d\sigma_{\|_{R+T+\mathrm{GDR}}}/d\Omega$ has a minimum around 70°. Figure 3 shows the real and imaginary parts of the scattering amplitudes $A_{\|}$ for D and $R + T + \mathrm{GDR}$ at 1.1 MeV. The real part of the amplitude, $R + T + \mathrm{GDR}$, goes from negative to positive, crossing zero around 70°. Although T and GDR go to zero at 90°, the contribution from R shifts the crossing point to lower angles. In addition, the imaginary parts of the amplitude are small compared to the real parts. Since T and GDR do not have imaginary components, this part comes only from R.

The event rate dR/dt for a given cross section σ_p is calculated from

$$\frac{dR}{dt} = \mathcal{L}\sigma_p, \quad (16)$$

where $\mathcal{L}$ is the luminosity for a fixed target given by

$$\mathcal{L} = \Phi\rho_t l, \quad (17)$$

where Φ is the flux, ρ_t is the target density, and l is the target length.

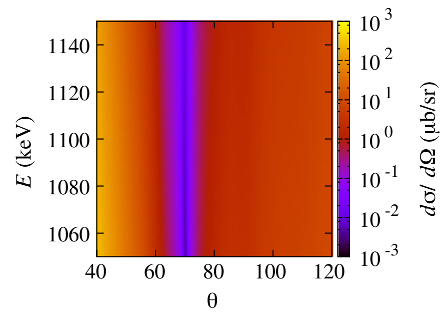

FIG. 2. The differential cross section $d\sigma_{\|_{R+T+\mathrm{GDR}}}/d\Omega$ for tin including $R + T + \mathrm{GDR}$ for scattering angles from $40° \leq \theta \leq 120°$ in the scattering plane. The angular resolution is $\Delta\theta = 0.15°$, and the energy resolution is 1 eV.

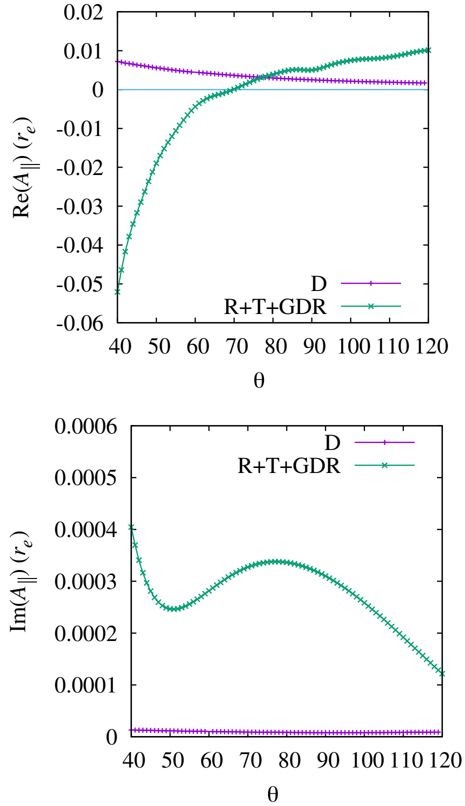

FIG. 3. The real and imaginary parts of the scattering amplitudes $A_\parallel$ for D and $R+T+\mathrm{GDR}$ as indicated in the figures at 1100 keV of tin in units of r_e. The angular resolution is $\Delta\theta = 1°$ (excluding 60°) with a relative error of 1% for D.

Choosing parameters of the proposed ELI-NP-GBS as a γ-ray source, the energy range is between 1 and 20 MeV with a root mean square (rms) bandwidth less than 0.5% and a time average spectral density greater than 5×10^3 ph/ sec /eV [17]. Taking the spectral density to be 5000 ph/ sec /eV at a photon energy of 1.1 MeV with the above bandwidth, the photon flux Φ becomes 2.75×10^7 ph/ sec. Assuming that the target is tin, we get $\rho_t = 7.31\ \mathrm{g/cm^3}/(118.71)(1.660\,538\,92 \times 10^{-24}\ \mathrm{g}) = 3.7 \times 10^{22}\ \mathrm{cm^{-3}}$ with $l = 10$ cm and then $\mathcal{L}$ becomes $1.02 \times 10^{31}\ \mathrm{cm^{-2}\,s^{-1}}$. Around 70°, Fig. 4 shows that $d\sigma_\parallel/d\Omega$ is approximately 1 μb/sr. Using a typical angular acceptance of $\Delta\theta = 0.8°$ (for example, Ref. [26]), we get $\Delta\Omega \approx 1.5 \times 10^{-4}$ giving $\sigma_\parallel \approx \Delta\Omega d\sigma_\parallel/d\Omega \approx 1.5 \times 10^{-34}\ \mathrm{cm^2}$. The resulting event rate is $dR/dt = 1.5 \times 10^{-3}/$ sec. The total number of scattering events is $N = \tau dR/dt$, where τ is the measurement time with an error of $\delta = 1/\sqrt{N}$. To get an error of $\delta = 1/\sqrt{\tau dR/dt} = 1\%$, the measurement time would be $\tau = (\delta^2 dR/dt)^{-1} = 6.5 \times 10^6$ sec or ≈76 days. Previously, measurements of the elastic scattering cross section obtained 2% accuracy for tin at 75° [26]. However, since these measurements were performed with unpolarized photons and at a higher photon energy of 2.754 MeV, all four processes contributed to the scattering, and higher-order corrections to the Delbrück scattering needed to be incorporated [26]. So an error of $\delta = 1\%$ would not only give us measurements of the Delbrück scattering with accuracies approximately a factor of 2 higher in accuracy than before but be nearly isolated. From Fig. 4, we can see that the overall effect of Delbrück scattering is to shift the angle of the minimum scattering cross section from $\theta = 70°$ down to $\theta = 60°$.

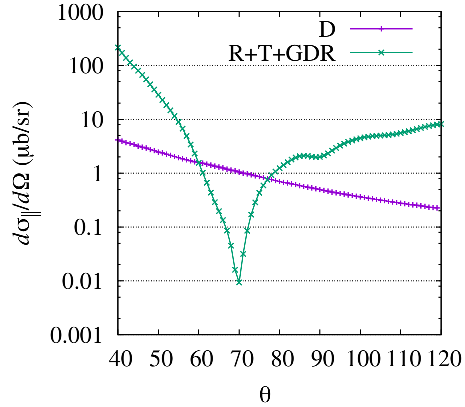

FIG. 4. The differential scattering amplitude $d\sigma_\parallel/d\Omega$ for D and $R+T+\mathrm{GDR}$ as indicated in the figure for tin at 1100 keV. The angular resolution is $\Delta\theta = 1°$ (excluding 60°) with a relative error of 1% for D.

We show in Fig. 5 the differential scattering amplitudes $d\sigma_\perp/d\Omega$ for D and $R+T+\mathrm{GDR}$ as indicated in the figure for tin at 1.1 MeV for scattering photons with polarizations perpendicular to the polarization plane. At scattering angles near $\theta = 102°$, the Delbrück scattering contribution is nearly zero, because the real part of D crosses zero and its imaginary part is small. With $d\sigma/d\Omega_\perp \approx 20\ \mu\mathrm{b/sr}$, this cross section could be measured with a very high accuracy of $\delta = 0.2\%$ in the same amount of time for the nearly isolated Delbrück scattering measurement. As a result, the accuracy of the calculation of $R+T+\mathrm{GDR}$ could be experimentally verified simultaneously with the nearly isolated Delbrück component using two detectors.

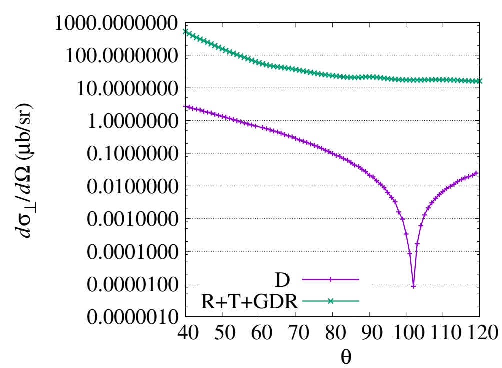

FIG. 5. The differential scattering amplitude $d\sigma_\perp/d\Omega$ for D and $R+T+\mathrm{GDR}$ as indicated in the figure for tin at 1.1 MeV. The angular resolution is $\Delta\theta = 1°$ (excluding 60°) with a relative error of 1% for D.

The polarization of the incoming photons can be altered by the scattering process if the photons are not completely polarized [32]. In this study, we have ignored the change in polarization of the photons. Because it has been shown that with future sources such as ELI-NP-GBS a very high degree of polarization can be maintained [33], we consider this to be an adequate assumption. However, future studies could take into account unexpected changes in polarizations as indications of the possibility of new physics.

In conclusion, we have shown that, using linearly polarized photons with polarizations parallel to the scattering plane at a sufficiently low energy impinging on targets having a sufficiently small atomic charge Z to avoid higher-order effects, the vacuum contribution to the elastic scattering, Delbrück scattering, of these photons can be nearly isolated and precisely measured. With the proposed ELI-NP-GBS [17] measurements, within 1% accuracy could be made in 76 days. Such precise measurements would allow for the search for possible deviations from theoretical predictions indicating the need for new physics.

The calculations in this Letter were performed on the "SGI ICE X" supercomputer at the Tokai Research and Development Center in the Japan Atomic Energy Agency. We thank L. Kissel for providing the code ENTING, which calculates the Rayleigh scattering matrix amplitudes, and help in getting it running. This work was supported by JSPS KAKENHI Grant No. 15H03665. We also acknowledge fruitful discussions with S. V. Bulanov, C. Angell, M. Omer, and M. Kando.

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