Emergence of Supersymmetric Quantum Electrodynamics

Shao-Kai Jian,[1] Chien-Hung Lin,[2] Joseph Maciejko,[2,3,4,†] and Hong Yao[1,*]
[1]Institute for Advanced Study, Tsinghua University, Beijing 100084, China
[2]Department of Physics, University of Alberta, Edmonton, Alberta T6G 2E1, Canada
[3]Theoretical Physics Institute, University of Alberta, Edmonton, Alberta T6G 2E1, Canada
[4]Canadian Institute for Advanced Research, Toronto, Ontario M5G 1Z8, Canada
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Supersymmetric (SUSY) gauge theories such as the minimal supersymmetric standard model play a fundamental role in modern particle physics, but have not been verified so far in nature. Here, we show that a SUSY gauge theory with dynamical gauge bosons and fermionic gauginos emerges naturally at the pair-density-wave (PDW) quantum phase transition on the surface of a correlated topological insulator hosting three Dirac cones, such as the topological Kondo insulator SmB_6. At the quantum tricritical point between the surface Dirac semimetal and nematic PDW phases, three massless bosonic Cooper pair fields emerge as the superpartners of three massless surface Dirac fermions. The resulting low-energy effective theory is the supersymmetric XYZ model, which is dual by mirror symmetry to $\mathcal{N}=2$ supersymmetric quantum electrodynamics in $2+1$ dimensions, providing a first example of emergent supersymmetric gauge theory in condensed matter systems. Supersymmetry allows us to determine certain critical exponents and the optical conductivity of the surface states at the strongly coupled tricritical point exactly, which may be measured in future experiments.

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Spacetime supersymmetry (SUSY) was introduced more than forty years ago as a means to resolve fundamental issues in particle physics such as the hierarchy problem [1–4], but has not been discovered yet. Amazingly, many beautiful theories originating in high-energy physics may be realized and tested in condensed matter systems; for instance, 3D Weyl fermions [5–7] were discovered recently in solid state materials [8–12]. One may wonder whether SUSY can be realized in quantum materials. Indeed, it was proposed that SUSY can emerge at quantum criticality in Bose-Fermi lattice models [13,14] and at the boundary of topological materials [15–18], as well as at multicritical points in low-dimensional systems [19–21]. Further, it was shown in Ref. [22] that SUSY in $3+1$D can emerge at superconducting quantum critical points in ideal Weyl semimetals [23,24].

However, all known examples only realize the simplest type of emergent SUSY: the Wess-Zumino theory [3], which contains a single SUSY multiplet of matter fields (one scalar and one fermion). It is highly desirable to know whether richer types of SUSY can emerge in condensed matter systems, such as theories with dynamical gauge fields. Here, we theoretically show that the nematic pair-density-wave (PDW) tricritical point on the surface of a correlated topological insulator (TI) [25,26] with three Dirac cones can realize a SUSY gauge theory. At this tricritical point, three Dirac fermions and three complex bosons form mutual superpartners and are described by the so-called XYZ model [27], which is dual by mirror symmetry to $\mathcal{N}=2$ supersymmetric quantum electrodynamics (SQED) [28–30].

An ideal candidate correlated TI to possibly realize this new type of SUSY is SmB_6, which is proposed to be a topological Kondo insulator [31,32] with three degenerate Dirac cones on its (111) surface protected by time-reversal and crystal symmetries [33,34]. (Another candidate with similar properties is YbB_6 [35].) Experiments on surface electronic structure [36–38], transport properties [39,40], and quantum oscillations [41,42] in SmB_6 all indicate conducting surface Dirac cones but an insulating bulk. To realize three complex bosons as superpartners of the three surface Dirac fermions, we consider the surface quantum phase transition into a PDW phase with three complex order parameters.

From the effective theory describing the PDW transition on the TI surface hosting three Dirac cones, we analyze the possible phases of the model at the mean-field level and find two PDW phases distinguished by the lattice rotational symmetry. In the nematic PDW phase, which breaks rotational symmetry spontaneously, there is a tricritical point separating first- and second-order PDW phase transitions. A renormalization group (RG) analysis reveals that the SUSY XYZ model, and thus, $\mathcal{N}=2$ SQED, emerges at this tricritical point. To the best of our knowledge, this is the first example of emergent SQED in quantum materials. We calculate certain critical exponents and the optical conductivity of the surface states exactly, which may be tested in future experiments on correlated TIs with PDW transitions.

Effective field theory.—The hexagonal surface Brillouin zone (BZ) of a TI with C_3 symmetry (e.g., Bi_2Se_3 and SmB_6) contains four time-reversal invariant (TRI) points: the $\bar{\Gamma}$ point, and three $\bar{M}$ points related by symmetry [Fig. 1(a)]. There are two different types of surface states: a

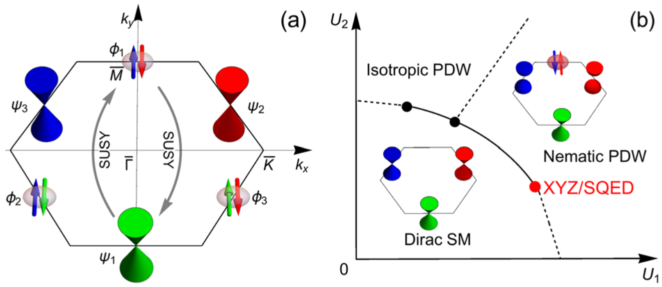

FIG. 1. (a) Hexagonal surface Brillouin zone of a TI with three Dirac cones ψ_1, ψ_2, and ψ_3, as in SmB_6. The three intervalley PDW order parameters are labeled by ϕ_1, ϕ_2, and ϕ_3, which can be rotated into Dirac fermions by a SUSY transformation at the nematic PDW tricritical point. (b) Schematic quantum phase diagram. U_1, U_2 represent combinations of the couplings r, u, u', u'' in the Landau-Ginzburg theory (3). Solid (dashed) lines represent second- (first-) order transitions. The red circle represents the tricritical point between the Dirac SM and nematic PDW phases, where SUSY of the XYZ/SQED type emerges.

single Dirac cone at the $\bar{\Gamma}$ point, as on the (001) surface of Bi_2Te_3 and Bi_2Se_3 [25,26], or three Dirac cones at the $\bar{M}$ points, as on the (111) surface of SmB_6 [33,34] and YbB_6 [35]. Here, we consider the latter case. We further impose the reflection symmetry $\mathcal{M}_x$ ($x \to -x$) which is respected in SmB_6. As a result, the TI surface has C_{3v} symmetry. The three Dirac fermions located at these TRI points are denoted by $\psi_{1,2,3}$ [Fig. 1(a)]. The little group for ψ_1 is generated by time reversal $\mathcal{T}$ and reflection $\mathcal{M}_x$. The low-energy theory of the surface Dirac semimetal (DSM) is dictated by symmetry and given by

$$\mathcal{L}_f = \sum_{i=1}^{3} \psi_i^\dagger (\partial_\tau + h_i^f)\psi_i, \tag{1}$$

where τ is imaginary time and $h_1^f = -i\sigma^y v_x \partial_x + i\sigma^x v_y \partial_y$ is a Dirac-like Hamiltonian with v_i the fermion velocities and σ^i the Pauli spin matrices; h_2^f and h_3^f are obtained from h_1^f by rotations [43]. Though there is no symmetry to enforce $v_x = v_y$, velocity isotropy emerges at the PDW transitions discussed below. We assume that the chemical potential is exactly at the Dirac points, namely, at stoichiometry. By contrast with the (111) surface considered here, on the (001) surface of SmB_6 the three Dirac points are not at equal energy [44].

We consider the system near PDW criticality, where pairing is between different cones and the PDW order parameters are, e.g., $\phi_1 \propto \psi_2\sigma^y\psi_3$ [Fig. 1(a)]. PDW ordering possesses finite momentum but does not spontaneously break time-reversal symmetry [45–49]. The quantum Landau-Ginzburg Lagrangian for the PDW order parameters is constrained by symmetry and reads [43]

$$\mathcal{L}_b = \sum_{i=1}^{3} \phi_i^* (-\partial_\tau^2 + h_i^b)\phi_i + V_b, \tag{2}$$

$$V_b = r\sum_{i=1}^{3}|\phi_i|^2 + u(|\phi_1|^2|\phi_2|^2 + \text{c.p.}) + u'[(\phi_1^{*2}\phi_2^2 + \text{H.c.}) + \text{c.p.}] + u''\sum_{i=1}^{3}|\phi_i|^4, \tag{3}$$

where r, u, u', u'' are phenomenological constants, c.p. denotes cyclic permutations, and h_2^b, h_3^b can be obtained from $h_1^b = -c_x^2\partial_x^2 - c_y^2\partial_y^2$ by rotations. Terms linear in spatial derivatives are forbidden due to time-reversal symmetry (we also implicitly assumed particle-hole symmetry to rule out terms linear in time derivative), and higher order terms omitted in $\mathcal{L}_b$ are irrelevant in the RG sense. Boson and fermion velocities are initially different, but flow to a common value in the infrared as discussed later in the text. Hereafter, we assume $u' < 0$ since the Josephson coupling between different condensates normally minimizes their superconducting phase difference. Moreover, the Dirac fermions and PDW order parameter fluctuations are coupled

$$\mathcal{L}_{bf} = g(\phi_1\psi_2\sigma^y\psi_3 + \text{c.p.}) + \text{H.c.}, \tag{4}$$

where g is a coupling constant.

Mean-field analysis.—To facilitate the analysis of the possible PDW phases at the mean-field level, we rewrite the boson potential as

$$V_b = u''\left(\sum_{i=1}^{3}|\phi_i|^2\right)^2 + (u - 2u'')(|\phi_1|^2|\phi_2|^2 + \text{c.p.}), \tag{5}$$

where we have implicitly absorbed the u' term into the u term (i.e., $u + 2u' \to u$) because the phase differences between different condensates (Leggett modes) are gapped in the ordered phase [50]. For now, we neglect the Dirac fermions in the lowest order approximation. The sign of the second (anisotropic) term in Eq. (5) is crucial for determining which PDW ground state is preferred. In the PDW ordered phases, the mass term is negative $r < 0$. When $u - 2u'' < 0$ and $u + u'' > 0$, the anisotropic term in the potential favors the ordering $|\phi_i| = \sqrt{|r|/2(u + u'')}$ and $\phi_1 = \phi_2 = \phi_3$, which we denote the isotropic PDW (IPDW) phase because it preserves the crystalline C_{3v} symmetry. In the IPDW phase, all three surface Dirac fermions are gapped by pairing. On the other hand, $u - 2u'' > 0$ and $u'' > 0$ favors a qualitatively different type of PDW ordering: only one component condenses with $|\phi_i| = \sqrt{|r|/2u''}$ while the other two $\phi_{j\neq i}$ vanish. We call this phase the nematic PDW (NPDW) because it breaks C_{3v} spontaneously. There is no secondary charge-density-wave order formed in the NPDW phase. In the NPDW phase, only two Dirac points are gapped and one remains massless. For the special case $u = 2u''$, the theory describes a bicritical point where the DSM-IPDW and DSM-NPDW phase boundaries meet [Fig. 1(b)].

In the analysis above, we have implicitly assumed that the transition between the DSM and PDW phases is continuous. However, it is always possible for a transition to be discontinuous. Thus, we also consider the possibility of first-order PDW transitions as well as tricritical points between the first- and second-order transitions. The transition into the nematic PDW phase (i.e., $u - 2u'' > 0$) should be first-order when $u'' < 0$, in which case a sixth-order term like $w(\sum_{i=1}^{3}|\phi_i|^2)^3$ with $w > 0$ should be added to V_b to stabilize the free energy. $u'' = 0$ is, thus, a tricritical point between the continuous ($u'' > 0$) and first-order ($u'' < 0$) transitions into the nematic PDW phase [Fig. 1(b)]. Similarly, $u + u'' = 0$ describes a tricritical point between the continuous transition ($u + u'' > 0$) and the first-order transition ($u'' + u < 0$) into the isotropic PDW phase.

We have identified three multicritical points through the mean-field analysis above: one bicritical, and two tricritical. In the remainder of the Letter, we analyze the emergent low-energy, long-wavelength properties at these multicritical points. Remarkably, the tricritical point into the nematic PDW phase features an emergent SUSY of the XYZ/SQED type, as discussed below.

Effective theory of the bicritical point.—First, we explore universal properties of the continuous DSM-PDW transition (i.e., $r = 0$) via a one-loop RG analysis in $4-\epsilon$ spacetime dimensions (the physical dimension corresponds to $\epsilon = 1$) [43]. At this transition, we find that the anisotropy in fermion and boson velocities vanishes, i.e., $c_x = c_y \equiv c$ and $v_x = v_y \equiv v$ at low energies and long distances. Moreover, they flow to a common value $c = v$ in the infrared such that Lorentz symmetry emerges at the continuous PDW transition, no matter whether the PDW phase is isotropic or nematic. Even though emergent Lorentz symmetry was previously observed at various quantum critical points involving Dirac fermions [22,51–54], it is more exotic here because it involves an odd number of Dirac cones on the surface of a correlated TI. This emergent Lorentz symmetry allows us to set $c = v = 1$ in discussing the PDW quantum critical points.

From the RG equations for the coupling constants g, u, u', u'',

$$\frac{dg^2}{dl} = \epsilon g^2 - \frac{3\pi}{2}g^4,$$
$$\frac{du}{dl} = \epsilon u - \pi g^2 u + \pi g^4 - \frac{\pi}{2}(3u^2 + 16u'^2 + 8uu''),$$
$$\frac{du'}{dl} = \epsilon u' - \pi g^2 u' - \pi(u'^2 + 2uu' + 2u'u''),$$
$$\frac{du''}{dl} = \epsilon u'' - \pi g^2 u'' + \frac{\pi}{2}g^4 - \frac{\pi}{2}(u^2 + 4u'^2 + 10u''^2),$$

we find a unique stable fixed point at $g^2_{\rm st} = (2/3\pi)\epsilon$, $u'_{\rm st} = 0$, and $u_{\rm st} = 2u''_{\rm st} = (1 + \sqrt{57}/21\pi)\epsilon$, where the subscript "st" means "stable." At this stable fixed point, the boson potential becomes $V_b = u''_{\rm st}(\sum_{i=1}^{3}|\phi_i|^2)^2$ and has an emergent SO(6) symmetry. However, the full theory only has the reduced $U(1) \times C_{3v}$ symmetry due to the finite fermion-boson coupling $g_{\rm st}$.

Based on our earlier analysis, the fixed point with $u_{\rm st} - 2u''_{\rm st} = 0$ corresponds to a bicritical point where three phases (DSM, IPDW, and NPDW) meet. However, this multicritical point is a novel one as it has only one relevant direction (the mass term r). The term proportional to $u - 2u''$ in Eq. (5) is dangerously irrelevant, and ground states on the ordered side crucially depend on its sign.

Emergent XYZ/SQED at the NPDW tricritical point.—Besides the stable fixed point discussed above, the RG equations also support another (unstable) fixed point at $g^2_{\rm susy} = u_{\rm susy} = \frac{2}{3\pi}\epsilon$ and $u'_{\rm susy} = u''_{\rm susy} = 0$. The fixed point action is invariant under the SUSY transformations $\delta\phi_i = \sqrt{2}\xi\psi_i$, $\delta\psi_1 = i\sqrt{2}\sigma^\mu\bar{\xi}\partial_\mu\phi_1 + g\sqrt{2}\xi\phi_2\phi_3$, and $\delta\psi_{2,3}$ are obtained by permutations of $\delta\psi_1$, where the infinitesimal transformation parameters ξ, $\bar{\xi}$ are Grassmann-valued two-component spinors, and $\sigma^0 = -I$ with I the identity matrix.

Remarkably, this fixed point is described by a new type of SUSY qualitatively different from all previously predicted in condensed matter. The bosonic PDW fields ϕ_i and Dirac fermions ψ_i combine into three chiral superfields $\Phi_i = \phi_i + \sqrt{2}\theta^\alpha\sigma^y_{\alpha\beta}\psi_i^\beta + \cdots$, $i = 1, 2, 3$, where θ is a Grassmann-valued two-component spinor and α, β are (pesudo-)spin indices. Intravalley pairing would have resulted in three decoupled copies of the $\mathcal{N} = 2$ Wess-Zumino theory with superpotential Φ_i^3 studied previously [13–17,22]. By contrast, in the intervalley pairing scenario considered here, the three valleys are strongly coupled via the superpotential $\Phi_1\Phi_2\Phi_3$ [43], and the resulting theory is known as the XYZ model. It flows in the infrared to a strongly coupled fixed point, which is dual via mirror symmetry—a SUSY version of the Peskin-Dasgupta-Halperin or particle-vortex duality [55–57]—to the infrared fixed point of $\mathcal{N} = 2$ SQED [28–30]. The latter is a theory of a vector superfield V and two chiral superfields Q, $\tilde{Q}$, playing the role of gauge field and matter field in the "vortex" theory, respectively. In addition to an emergent bosonic gauge field A_μ, the vector superfield V also contains a fermionic gaugino λ.

We now show that the XYZ/SQED fixed point with $u' = u'' = 0$ and $u > 0$ corresponds to the tricritical point that separates the continuous and first-order transitions into the nematic PDW phase. Linearizing the RG equations for g, u, u', u'' near the SUSY fixed point, we can determine the eigenoperators at this fixed point and their eigenvalues, which are $(-\frac{7}{3}, -1, -1, 1)$. The positive eigenvalue indicates that there is one relevant direction (besides the relevant direction of r). Consequently, the XYZ/SQED fixed point is unstable. This is consistent with our mean-field analysis of the tricritical point on the transition boundary between the DSM and NPDW phases, which is reached by tuning two parameters.

The emergence at the NPDW tricritical point of the XYZ SUSY, which is dual to SQED, may be intuitively understood as follows. Heuristically, for a fermionic quantum critical point with the same number of Dirac fermions and complex order parameters to be possibly supersymmetric, one necessary condition is that the coupling among different bosonic order parameters should avoid flowing to infinity (namely, it should not be relevant), otherwise, the number of remaining effective gapless bosonic modes would be less than the number of fermionic ones. The nematic PDW breaks the $U(1)$ gauge symmetry, as well as the lattice C_3 symmetry which is effectively a $U(1)$ symmetry at criticality due to the irrelevance of anisotropic terms. Thus, from a symmetry point of view, it is natural to expect two gapless complex bosonic modes at a generic NPDW quantum critical point. However, the low-energy theory has three gapless Dirac fermions. To have a chance of being supersymmetric, the quantum phase transition must be tuned to a multicritical point such that a third complex bosonic mode remains gapless. Here, this multicritical point is the NPDW tricritical point.

Like the Wess-Zumino model, the XYZ model enjoys an R symmetry [1,28]. The R charge of the superpotential $\Phi_1\Phi_2\Phi_3$ should be 2, i.e., $\sum_{i=1}^{3}\mathcal{R}(\Phi_i)=2$, where $\mathcal{R}(\Phi_i)$ denotes the R charge of the superfield Φ_i. The assignment of R charge for the superfield Φ_i is simple owing to the rotational symmetry in our case: they should be equal, $\mathcal{R}(\Phi_i)=\frac{2}{3}$. For a chiral superfield, the scaling dimension is exactly equal to the R charge [28,58] in $2+1$ dimensions. Thus, we obtain the exact scaling dimensions of the bosonic order parameter fluctuations and Dirac fermions as $\Delta_\phi=\frac{2}{3}$ and $\Delta_\psi=\Delta_\phi+\frac{1}{2}=\frac{7}{6}$, respectively. Setting $\epsilon=1$, our one-loop RG result $\Delta_\phi=\frac{1}{2}+\frac{\epsilon}{6}=\frac{2}{3}$ for the boson scaling dimension is consistent with the exact result. Accordingly, the order parameter anomalous dimension or critical exponent η is $\frac{1}{3}$. On the other hand, the correlation length exponent ν is related to the scaling dimension of nonchiral fields $|\phi_i|^2$ and cannot be simply related to the R charge. We obtain $\nu=\frac{1}{2}+(\epsilon/4)+\mathcal{O}(\epsilon^2)$ at the one-loop level [43].

Experimental signatures of SUSY at the NPDW tricritical point.—Owing to the strong constraints imposed by $\mathcal{N}=2$ superconformal symmetry at the XYZ/SQED fixed point, several dynamical properties can be obtained exactly [59,60] despite the presence of strong interactions at this fixed point. According to linear response theory, the optical conductivity at frequency ω is given by the current-current correlation function

$$\sigma(\omega)=\frac{e^2}{\hbar}\frac{1}{i\omega}\langle J_x(\omega)J_x(-\omega)\rangle, \tag{6}$$

where $e^2/\hbar$ is the quantum of conductance. The current-current correlation function is highly constrained by conformal symmetry through the conformal Ward identity [59,61].

We now compute the exact optical conductivity at the NPDW tricritical point. Utilizing the R symmetry of $\mathcal{N}=2$ superconformal field theories in $2+1$ dimensions, one can find [60] that $\sigma_0(\omega)=\frac{5}{4}\tau_{RR}(e^2/\hbar)$, where $\sigma_0(\omega)$ is the optical conductivity at zero temperature and τ_{RR} is the dimensionless coefficient of the two-point correlation function of the R current [43,62–64]. Assuming $\omega\ll\Lambda$ where Λ represents microscopic energy scales above which quantum critical behavior ceases to exist, the zero-temperature optical conductivity $\sigma_0(\omega)=\sigma_0$ is a universal constant independent of frequency that characterizes the universality class of the transition, just like critical exponents [65]. In an $\mathcal{N}=2$ SUSY field theory with only chiral superfields, τ_{RR} is given by an integral that depends only on the R charge of the chiral superfields [59]. As mentioned before, owing to the C_3 rotational symmetry relating the three chiral superfields Φ_i, we obtain $\mathcal{R}(\Phi_i)=\frac{2}{3}$, $i=1,2,3$. This is the same as the R charge of the chiral superfield in the Wess-Zumino model [28]. As a result, τ_{RR} at the XYZ/SQED fixed point is simply three times that in the Wess-Zumino model, which was evaluated analytically in Ref. [60]. Thus, the exact zero-temperature optical conductivity at the nematic PDW tricritical point is given by

$$\sigma_0(\omega)=\frac{15}{243}\left(16-\frac{9\sqrt{3}}{\pi}\right)\frac{e^2}{\hbar}\approx 0.681\,\frac{e^2}{\hbar}, \tag{7}$$

which may be tested in future experiments.

Other experimental signatures include various critical exponents already mentioned, such as the fermion or boson anomalous dimension $\eta=\frac{1}{3}$, which is exact owing to SUSY, and the correlation length exponent $\nu\approx 0.75$. The exact value of η implies that the local electronic density of states $\rho(\omega)$ scales as $|\omega|^{4/3}$ at low energies [16], which can be measured by scanning tunneling microscopy (STM).

Concluding remarks.—We have shown that a novel type of SUSY emerges at the nematic PDW tricritical point on the surface of a correlated TI that hosts three Dirac cones, like SmB_6. At this tricritical point, the three surface Dirac fermions and three complex bosons (corresponding to PDW order parameter fluctuations) are described by the so-called XYZ model, which is dual in the low-energy and long-wavelength limit to a SUSY gauge theory, $\mathcal{N}=2$ SQED. As such, our result also provides a direct physical setting for the investigation of mirror symmetry in condensed matter systems [66–68]. This is an area of increased recent activity [69,70] owing to its connection with a series of recently proposed dualities in $2+1$ dimensions [71–78], with applications to a wide range of problems in contemporary condensed matter physics, such as quantum spin liquids, topological phases, and the half-filled Landau level.

We have also predicted various critical exponents and the zero-temperature optical conductivity at the nematic PDW tricritical point, which could be tested in future experiments. If the SUSY proposed in the present Letter is realized in condensed matter systems, it would help to determine various quantities nonperturbatively, such as the critical exponent ν of SQED in $2+1$ dimensions, which are theoretically known only perturbatively (or numerically by bootstrap calculations [79,80]). We hope the present results will stimulate the theoretical and experimental search for various types of emergent SUSY and, more generally, emergent phenomena in condensed matter systems [81].

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[*]yaohong@tsinghua.edu.cn
[†]maciejko@ualberta.ca

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