Limits on Lorentz Invariance Violation from Coulomb Interactions in Nuclei and Atoms

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Anisotropy in the speed of light that has been constrained by Michelson-Morley-type experiments also generates anisotropy in the Coulomb interactions. This anisotropy can manifest itself as an energy anisotropy in nuclear and atomic experiments. Here the experimental limits on Lorentz violation in ${}^{21}_{10}$Ne are used to improve the limits on Lorentz symmetry violations in the photon sector, namely, the anisotropy of the speed of light and the Coulomb interactions, by 7 orders of magnitude in comparison with previous experiments: the speed of light is isotropic to a part in 10^{28}.

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A special role in the foundation of the theory of relativity was played by the Michelson-Morley experiment searching for the anisotropy in the speed of light. Recent experiments found that the speed of light is isotropic at a level of 10^{-17}–10^{-21} [1–5]. In Ref. [6] it was noted that this limit may be improved to the level of about 10^{-29} using the NMR-type experiment [7]. Similar anisotropy in the maximal attainable speed for massive particles has been constrained for nucleons by NMR experiments (see, e.g., Refs. [7–11]) and for electrons using optical atomic transitions [4,5]. Observation of these or other effects of the Local Lorentz Invariance Violation (LLIV) may pave the way to a new, more general theory (see, e.g., Refs. [12–20]).

Violations of Lorentz symmetry in the photon sector are parametrized in the standard model extension (SME) [20] by the tensor $(k_F)^{\alpha\beta\mu\nu}$. In the presence of Lorentz violation, the Coulomb potential of a point charge becomes anisotropic [13],

$$\Phi(\mathbf{r}) = \frac{q}{r}(1 + (\kappa_{\mathrm{DE}})^{ij} n_i n_j/2), \tag{1}$$

where $\kappa_{\mathrm{DE}}^{ij} = -2(k_F)^{0i0j}$ are the tensor components characterizing the anisotropy in the Coulomb potential and $n_i = x_i/r$ are the unit vectors along the radius vector in the rest frame of the charge. This equation also gives the approximate potential in any frame that is moving slowly with respect to the point charge.

A nucleus that has a finite electric quadrupole moment in the absence of LLIV will exhibit a spatial energy anisotropy due to LLIV caused by the electrostatic interactions of the valence protons with the anisotropic Coulomb potential of the nuclear core. The shift of the electrostatic energy due to the LLIV correction in Eq. (1) can be written as

$$\delta E = (\kappa_{\mathrm{DE}})^{ij} M_{ij}, \tag{2}$$

where M_{ij} is the nuclear tensor which we calculate here. Below we establish a proportionality relation between M_{ij} and the experimental value of the nuclear electric quadrupole moment tensor Q, namely, $M_{ij} = KQ_{ij}$. An estimate of the coefficient K allows us to provide estimates of the LLIV shifts for all nuclei and extract the limits on LLIV constants from corresponding experiments.

We start from a simple analytical estimate of the proportionality coefficient K. To obtain the LLIV correction to the electrostatic potential of a finite size charge distribution one has to integrate Eq. (1) with a charge density distribution, and the result will not be a simple product of the unperturbed nuclear electrostatic potential $\Phi_0(\mathbf{r})$ and the LLIV factor $[1 + (\kappa_{\mathrm{DE}})^{ij} n_i n_j/2]$. However, to clarify the dependence on the parameters of the problem it is instructive to start from a simple estimate assuming $\Phi(\mathbf{r}) \approx \Phi_0(\mathbf{r})[1 + (\kappa_{\mathrm{DE}})^{ij} n_i n_j/2]$. Below we will also perform an accurate integration of Eq. (1) with a realistic charge distribution; this gives a more accurate value of the proportionality factor K. We find that the LLIV shift of the electrostatic energy $\delta E = (\kappa_{\mathrm{DE}})^{ij} M_{ij}$, may be expressed via the nuclear electric quadrupole moment. Indeed, the quadrupole moment tensor is

$$Q_{ij} = \int (3n_i n_j - \delta_{ij})\rho(\mathbf{r}) r^2 d^3 r, \tag{3}$$

where $\rho(\mathbf{r})$ is the proton number density. The correction to energy produced by the LLIV part of the potential $\Phi_0(\mathbf{r})(\kappa_{\mathrm{DE}})^{ij} n_i n_j/2$ may be presented as $\delta E = (\kappa_{\mathrm{DE}})^{ij} M_{ij}$, with

$$M_{ij} \approx \frac{e}{6} \int (3n_i n_j - \delta_{ij})\rho(\mathbf{r})\Phi_0(r) d^3 r. \tag{4}$$

We subtracted δ_{ij} in the expression above since it does not produce any anisotropic LLIV effect. We see that the angular factors in the integrals Eqs. (3), (4) are the same, we only need to evaluate the radial parts. It is convenient to separate the spherically symmetrical part and the angular-dependent part of the density:

$$\rho(\mathbf{r}) = \rho_0(r) + \rho_2(\mathbf{r}). \tag{5}$$

For a deformed nucleus of a constant density ρ the nonspherical (mainly, quadrupole) part of the density $\rho_2(\mathbf{r})$ is responsible for the deformation and may be viewed as a correction to the density located near the nuclear surface. Indeed, for a spherical nucleus of a constant density ρ the correction $\rho_2 = \rho$ outside the spherical nuclear edge (on the z axis, for the angle $\theta = 0$) makes one axis longer and $\rho_2 = -\rho$ in the perpendicular direction (for the angle $\theta = \pi/2$) makes another axis shorter; i.e., $\rho_2(\mathbf{r})$ transforms the spherical nucleus into the spheroidal one. The electrostatic potential near the nuclear surface is equal to $\Phi_0(\mathbf{r}) \approx Ze/R$ and is continuous across the surface. So, for small quadrupolar deformations we obtain from Eqs. (3), (4) the relation between $M = M_{zz}$ and $Q = Q_{zz}$:

$$M_{\text{estimate}} \sim \frac{1}{6}\frac{(Z-1)e^2 Q}{R^3}. \tag{6}$$

Note that we replaced the nuclear charge factor Z by $Z-1$ to account for the charge quantization. Indeed, to exclude the interaction of a proton with itself the result must vanish for $Z = 1$. Transformation from the frozen body frame to the laboratory reference frame does not change the ratio M/Q.

We have also performed numerical calculations of the correction to the energy due to the LLIV part of the electrostatic potential Eq. (1) for the interaction of the point charges using the accurate numerical integration

$$\delta E = \frac{1}{2}\int \Phi(\mathbf{r}_1 - \mathbf{r}_2)\rho(\mathbf{r}_1)\rho(\mathbf{r}_2) d^3\mathbf{r}_1 d^3\mathbf{r}_2. \tag{7}$$

For the density $\rho(\mathbf{r})$ we assume the model of the spheroidal nucleus with a constant charge density and the nuclear radius given by $R = R_0[1 + \beta Y_{20}(\theta, \phi)]$. We have found that the numerical results in this model for nuclei with different R, Z, and Q may be approximated by the analytical formula

$$M = 0.06\frac{(Z-1)e^2 Q}{R^3} = 0.055\frac{Z-1}{A}\frac{Q}{\text{fm}^2}\text{MeV}. \tag{8}$$

We assume the nuclear radius $R = 1.15A^{1/3}$ fm, where A is the number of nucleons. The ratio M/Q changes by about $\pm 10\%$ as β changes from 0 to 0.4, as shown in Fig. 1, so the result can be applied to all nuclei. The experimental values of the nuclear quadrupole moments Q may be found in the tables [21]. For the $^{21}_{10}$Ne nucleus $Q = 10.3$ fm^2, and we obtain $M = 0.25$ MeV.

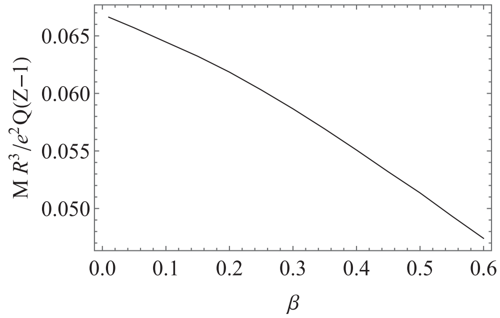

FIG. 1. Dependence of the ratio M/Q on the nuclear quadrupole deformation size β.

Now we can extract the limits on the coefficients $(\kappa_{\text{DE}})^{ij}$ responsible for the asymmetry in the speed of light from the measurements of LLIV in the $^{21}_{10}$Ne nucleus [7]. We are interested in two contributions,

$$\delta E = (\kappa_{\text{DE}})^{ij} M_{ij} - c_{ij} P_{ij}. \tag{9}$$

The coefficients c_{ij} characterize anisotropy in the maximal attainable speed for massive particles or the anisotropy in the kinetic energy term, which in the nonrelativistic limit may be presented as $-c_{ij}P_{ij}$, as derived in Ref. [17]. Here the tensor $P_{ij} = \langle 3p_i p_j - p^2\delta_{ij}\rangle/6m$, p, and m are the particle momentum and mass. In Ref. [7] the limits on c_{ij} for neutrons were obtained assuming $(\kappa_{\text{DE}})^{ij} = 0$:

$$\begin{aligned}
c_n^X &= c_n^{YZ} + c_n^{ZY} = (4.8 \pm 4.4) \times 10^{-29},\\
c_n^Y &= c_n^{XZ} + c_n^{ZX} = -(2.8 \pm 3.4) \times 10^{-29},\\
c_n^Z &= c_n^{XY} + c_n^{YX} = -(1.2 \pm 1.4) \times 10^{-29},\\
c_n^- &= c_n^{XX} - c_n^{YY} = (1.4 \pm 1.7) \times 10^{-29}.
\end{aligned} \tag{10}$$

The limits on the c_{ij} above have been obtained assuming the Schmidt (single valence neutron) value $P^n = P^n_{zz} = -0.66$ MeV [7]. More accurate calculations of P [6,22] have shown that both neutrons and protons contribute to c_{ij}. According to Ref. [22] $P^p = 0.54$ MeV, $P^n = 0.57$ MeV and the experimental limits on c_{ij} actually contain the linear combination $c = -(0.8c^p + 0.9c^n)$. In Ref. [6], the recommended values for the matrix elements are $P^p = 0.47$ and $P^n = 0.7$ MeV, so the results of the two approaches are quite consistent. We need to add the photon contribution $(\kappa_{\text{DE}})^{ij}$ calculated in the present work with $M = 0.25$ MeV. This gives the linear combination of the coefficients that are constrained by the experimental measurements in ^{21}Ne presented above:

$$\begin{aligned}
c_n^X + 0.9c_p^X - 0.4\kappa_{\rm DE}^X &= -(5.3 \pm 4.9) \times 10^{-29},\\
c_n^Y + 0.9c_p^Y - 0.4\kappa_{\rm DE}^Y &= (3.1 \pm 3.8) \times 10^{-29},\\
c_n^Z + 0.9c_p^Z - 0.4\kappa_{\rm DE}^Z &= (1.3 \pm 1.6) \times 10^{-29},\\
c_n^- + 0.9c_p^- - 0.4\kappa_{\rm DE}^- &= -(1.6 \pm 1.9) \times 10^{-29}. \qquad (11)
\end{aligned}$$

It is instructive to compare these calculations to the case of Coulomb interaction in atoms. For example, for an electron in a one-electron atom, one can write the main LLIV energy contributions as

$$\delta E = -\frac{Ze^2}{r}\frac{(\kappa_{\rm DE})^{ij}n_i n_j}{2} - c_{ij}p_i p_j/m. \qquad (12)$$

The two terms can be related to each other using a generalization of the virial theorem. In particular, for an atomic eigenstate

$$\frac{d\langle p_i r_j\rangle}{dt} = 0 = \frac{i}{\hbar}\langle[H, p_i r_j]\rangle, \qquad (13)$$

where $H = -Ze^2/r + p_i p_i/2m$. It follows that

$$\left\langle\frac{p_i p_j}{m}\right\rangle = \left\langle\frac{Ze^2 n_i n_j}{r}\right\rangle. \qquad (14)$$

The virial theorem is valid in many-electron atoms too. We should include double sum over interaction energies of all particles into the Hamiltonian and make corresponding changes in the equations above. Therefore, for any atom with the electron energy dominated by the nonrelativistic Hamiltonian, the two LLIV terms always enter as a combination $(\kappa_{\rm DE})^{ij}/2 + c_{i,j}$.

A similar result can be derived using a coordinate transformation $x^\mu \to x^\mu - (k_F)^{\alpha\mu}_{\alpha\nu}x^\nu/2$, which removes Lorentz violation in the photon sector and modifies the electron LLIV coefficients $c_{\mu\nu} \to c_{\mu\nu} + (k_F)^\alpha_{\mu\alpha\nu}/2$, as discussed in Ref. [23]. Using Riemann tensor symmetries of the $(k_F)^\alpha_{\mu\alpha\nu}$ tensor one can show that $(k_F)^\alpha_{j\alpha k} = (\tilde{\kappa}_{e^-})^{jk}$ for the traceless components of the tensor. Furthermore, by imposing stringent constraints on birefringence components of the photon LLIV tensor k_{e^+} at a level of 10^{-38} [24], one obtains $(\kappa_{\rm DE})^{ij} = (\tilde{\kappa}_{e^-})^{ij}$ for the traceless components. Hence in an atom held together by Coulomb interactions only one linear combination of the $(\tilde{\kappa}_{e^-})^{ij}$ and c_{ij} tensors can be measured. This relationship based on a coordinate transformation was used to relate the photon sector coefficients to the electron c_{ij} coefficients in Ref. [5].

The situation is different in the nucleus, where the relationship (14) does not hold since the total potential energy is dominated by strong interactions. Note that the linear combination in Eqs. (11) has the opposite sign between the nucleon and photon coefficients because the Coulomb potential energy is positive in the nucleus, unlike an atom. Our calculations implicitly assume that there is no LLIV in the strong interaction. Existing limits on Lorentz violation in strong interactions come from ultrahigh energy particles of astrophysical origin, in particular limits on the difference between the maximum attainable velocity for quarks and gluons, which are on the order of 10^{-21} to 10^{-24} [25,26]. In general, as discussed in Ref. [23], Lorentz violation can be only defined in the differences in the properties of two types of particles. It is convenient therefore to use the freedom of coordinate transformation to remove LLIV for strong interactions. In this coordinate system, the relationships (11) can be interpreted as providing a constraint on the sum of Lorentz violation for nucleons and photons.

Modern analogues of the Michelson-Morley experiments [1–3] measure the anisotropy in the speed of light relative to the length of a material cavity. Lorentz violation due to changes in the cavity length have been considered in Ref. [27]. Such analyses use a coordinate system where the centers of mass of the nuclei in the material lattice are at rest on average and have a kinetic energy on the order of the Debye temperature, much less than 1 eV. Furthermore, the internal motion of the nucleons inside each nucleus has no preferential direction since the nuclei are not spin polarized as in the ^{21}Ne experiment [7] and the expectation value of the P_{ij} tensor is nearly zero [28]. This largely suppresses the Lorentz-violating effects from the anisotropy of the kinetic energy of protons and neutrons. Therefore, Michelson-Morley style experiments place constraints primarily on the combination of the electron and the photon coefficients. To compare our limits to previous measurements of LLIV in the photon sector it is natural therefore to assume $c_{ij}^p = c_{ij}^n = c_{ij}^e = 0$. With these assumptions we obtain the following:

$$\begin{aligned}
\tilde{\kappa}_{e^-}^X &= \tilde{\kappa}_{e^-}^{YZ} + \tilde{\kappa}_{e^-}^{ZY} = -(13 \pm 12) \times 10^{-29},\\
\tilde{\kappa}_{e^-}^Y &= \tilde{\kappa}_{e^-}^{XZ} + \tilde{\kappa}_{e^-}^{ZX} = (7 \pm 9) \times 10^{-29},\\
\tilde{\kappa}_{e^-}^Z &= \tilde{\kappa}_{e^-}^{XY} + \tilde{\kappa}_{e^-}^{YX} = -(3.2 \pm 3.7) \times 10^{-29},\\
\tilde{\kappa}_{e^-}^- &= \tilde{\kappa}_{e^-}^{XX} - \tilde{\kappa}_{e^-}^{YY} = -(3.7 \pm 4.5) \times 10^{-29}. \qquad (15)
\end{aligned}$$

These limits are 7–11 orders of magnitude better than that obtained in the previous laboratory measurements [1–5].

To summarize, in this work we performed calculations which give a new interpretation of existing and future experiments searching for the anisotropy in the speed of light and the anisotropy in the maximal attainable speed for massive particles. New interpretation of the experiment with ^{21}Ne [7] has already allowed us to improve the limits on some LLIV interaction parameters for photons describing asymmetry in the speed of light (studied in the Michelson-Morley experiment) and anisotropy in the Coulomb interaction by 7 orders of magnitude.

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