Yangian Symmetry and Integrability of Planar $\mathcal{N}=4$ Supersymmetric Yang-Mills Theory

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In this Letter, we establish Yangian symmetry of planar $\mathcal{N}=4$ supersymmetric Yang-Mills theory. We prove that the classical equations of motion of the model close onto themselves under the action of Yangian generators. Moreover, we propose an off-shell extension of our statement, which is equivalent to the invariance of the action and prove that it is exactly satisfied. We assert that our relationship serves as a criterion for integrability in planar gauge theories by explicitly checking that it applies to the integrable Aharony-Bergman-Jafferis-Maldacena theory but not to the nonintegrable $\mathcal{N}=1$ supersymmetric Yang-Mills theory.

Introduction.—The assumption that planar $\mathcal{N}=4$ supersymmetric Yang-Mills (SYM) theory is integrable (see [1] for a review) has unlocked an enormous body of data—not only perturbatively but also at strong and intermediate coupling—thanks to advanced methods of integrable systems. Two prime examples are the finite-coupling computations of the cusp dimension [2] by means of an integral equation [3] and of the scaling dimension of the Konishi operator [4] by means of the thermodynamic Bethe ansatz [5–7]. Their smooth interpolations between weak and strong coupling are viewed as strong confirmations of the AdS/CFT correspondence. While the feature of integrability in this model is now supported by an overwhelming amount of evidence, it largely remains a conjecture except for certain observables in certain corners of the parameter space. The main obstacle in proving integrability is the lack of a proper definition for this feature within a planar gauge theory.

A key property of integrable systems is the existence of a large amount of hidden symmetries. For this model, the relevant algebra has been identified as the Yangian $Y[\mathfrak{psu}(2,2|4)]$ [8], which is an infinite-dimensional quantum algebra based on the Lie superalgebra $\mathfrak{psu}(2,2|4)$ of superconformal symmetries. The original formulation, however, which merely addresses the spectrum of one-loop anomalous dimensions for which Yangian symmetry, is largely broken due to the pertinent cyclic boundary conditions. Sometime later, Yangian invariance has been established for color-ordered scattering amplitudes at tree level [9]. Unfortunately, at loop level, this symmetry is severely affected by infrared singularities inherent to the

scattering of massless particles. Only recently, the Yangian has been found to be a proper symmetry of the expectation values of certain nonsingular Wilson loops [10]. However, the Yangian has never been shown to be a symmetry of the model itself.

The goal of the present Letter is to fill this gap and to establish Yangian symmetry for planar $\mathcal{N}=4$ SYM. The obvious strategy would be to show that the action of the model is invariant under the Yangian generators. However, there are several difficulties in this approach. First, the large-N_c limit must play a decisive role for integrability is clearly restricted to the planar limit. However, it is not *a priori* evident how to define the planar limit of the Lagrangian or of the Yangian generators. Another difficulty lies in the apparent incompatibility of the Yangian with the cyclicity of the Lagrangian. Finally, one needs to define how exactly the Yangian generators act on the fields. This turns out to be a rather subtle issue within a gauge theory because symmetries necessarily act nonlinearly in the sense that one field is mapped to a single or to several fields. On the one hand, nonlinearity complicates the analysis, especially in combination with the above issues. On the other hand, exact nonlinear invariance is a very strong statement, which typically extends to the quantum theory unless the symmetry is anomalous. The combination of these difficulties has arguably been a show stopper in proving Yangian symmetry of the model in the past. Here, we start with the somewhat more moderate goal to establish Yangian symmetry for the classical equations of motion of $\mathcal{N}=4$ SYM. We first show that the equations of motion close onto themselves under an appropriately defined action of the Yangian generators. This alone is not sufficient to establish the Yangian as a proper symmetry of the model. We therefore propose an off-shell extension of the statement and prove its validity. We claim that our relationship is equivalent to Yangian invariance of the action and thus serves as a suitable criterion for integrability in planar gauge theories. In order to substantiate

these claims, we consider similar superconformal gauge theory models: on the one hand, we show that the evidently nonintegrable $\mathcal{N} = 1$ SYM theory does not satisfy our relationship. On the other hand, we prove Yangian symmetry of Aharony-Bergman-Jafferis-Maldacena (ABJM) theory [11] for which signs of integrability have been found following the work [12].

$\mathcal{N} = 4$ *supersymmetric Yang-Mills.*—The $\mathcal{N} = 4$ SYM theory in four dimensions consists of the gauge field A_μ, four Weyl fermions $\Psi_{a\alpha}$, together with their conjugates $\bar{\Psi}_{\dot\alpha}^a$, and six real scalar fields Φ_m. We denote spacetime vector indices by $\mu, \nu, \ldots = 0, \ldots, 3$ and spinor indices of the two chiralities by $\alpha, \beta, \ldots = 1, 2$ and $\dot\alpha, \dot\beta, \ldots = 1, 2$, respectively. Moreover, $m, n, \ldots = 1, \ldots, 6$ and $a, b, \ldots = 1, 2, 3, 4$ denote indices for the vector and spinor representations of SO(6), respectively. All matter fields transform in the adjoint representation of the gauge group. The covariant derivative of a generic covariant field Z is $D_\mu Z := \partial_\mu Z - i[A_\mu, Z]$, and the field strength equals $F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$.

The Lagrangian of the theory takes the form

$$\mathcal{L} = -\frac{1}{4}\mathrm{tr}F_{\mu\nu}F^{\mu\nu} + \mathrm{tr}\,i\bar{\Psi}_{\dot\alpha}^a \sigma_\mu^{\alpha\dot\alpha} D^\mu \Psi_{a\alpha}$$
$$- \frac{i}{2}\mathrm{tr}(\Psi_{a\alpha}\sigma_m^{ab}\epsilon^{\alpha\beta}[\Phi^m, \Psi_{b\beta}] - \bar{\Psi}_{\dot\alpha}^a \sigma_{ab}^m \epsilon^{\dot\alpha\dot\beta}[\Phi_m, \bar{\Psi}_{\dot\beta}^b])$$
$$- \frac{1}{2}\mathrm{tr}D^\mu\Phi^m D_\mu\Phi_m + \frac{1}{4}\mathrm{tr}[\Phi^m, \Phi^n][\Phi_m, \Phi_n]. \quad (1)$$

Here, $\sigma_{\alpha\dot\alpha}^\mu$ and σ_{ab}^m denote the generalizations of the Pauli matrices to $(3+1)D$ and to $6D$, respectively. Furthermore, $\epsilon^{\alpha\beta}$, $\epsilon^{\dot\alpha\dot\beta}$, and ϵ^{abcd} denote the totally antisymmetric tensors. As already alluded to in the introduction, we are mainly going to work with the equations of motion of the theory. To that end, we introduce a short-hand notation for the variation of the action with respect to a generic field Z_A (the indices $A, B, \ldots$ enumerate the various fields) $\check{Z}^A := \delta\mathcal{S}/\delta Z_A$ so that the equations of motion simply read $\check{Z}^A = 0$. We have explicitly that

$$\check{\bar{\Psi}}_a^{\dot\alpha} = i\sigma_\mu^{\alpha\dot\alpha}D^\mu\Psi_{a\alpha} + i\epsilon^{\dot\alpha\dot\gamma}\sigma_{ab}^m[\Phi_m, \bar{\Psi}_{\dot\gamma}^b],$$
$$\check{\Psi}^{a\alpha} = i\sigma_\mu^{\dot\alpha\alpha}D^\mu\bar{\Psi}_{\dot\alpha}^a + i\epsilon^{\alpha\beta}\sigma_m^{ab}[\Phi^m, \Psi_{\beta b}],$$
$$\check{\Phi}_m = D_\mu D^\mu\Phi_m + [\Phi_n, [\Phi^n, \Phi_m]] + \frac{i}{2}\sigma_m^{ab}\epsilon^{\alpha\beta}\{\Psi_{a\alpha}, \Psi_{b\beta}\}$$
$$+ \frac{i}{2}\sigma_{m,ab}\epsilon^{\dot\alpha\dot\gamma}\{\bar{\Psi}_{\dot\alpha}^a, \bar{\Psi}_{\dot\gamma}^b\},$$
$$\check{A}_\mu = D^\nu F_{\nu\mu} + i[\Phi_n, D_\mu\Phi^n] + \sigma_\mu^{\dot\alpha\alpha}\{\bar{\Psi}_{\dot\alpha}^a, \Psi_{a\alpha}\}. \quad (2)$$

$\mathcal{N} = 4$ SYM theory is a superconformal theory; its action is invariant under the four-dimensional extended superconformal algebra $\mathfrak{psu}(2, 2|4)$, and quantum effects do not spoil such invariance. In the following, we will need the action of the generators of dilatations $\mathbb{D}$, translations $\mathbb{P}_\mu$, and Lorentz transformation $\mathbb{M}_{\mu\nu}$ on a generic field Z,

$$\mathbb{D}Z = i(x^\sigma D_\sigma + \Delta_Z)Z,$$
$$\mathbb{P}^\rho Z = iD^\rho Z,$$
$$\mathbb{L}_{\mu\nu}Z = i(x_\mu D_\nu - x_\nu D_\mu + \Sigma_{\mu\nu})Z. \quad (3)$$

In the above equations, Δ_Z is the conformal dimension of the field Z, and $\Sigma_{\mu\nu}$ is a spin-specific part of the transformation. To make transformations consistent for the noncovariant gauge field A_μ, we have to make the peculiar definitions, $\Delta_A := 0$ and $D_\mu A_\nu := F_{\mu\nu}$. Supersymmetry generators $\mathbb{Q}_\alpha^a$ act as

$$\mathbb{Q}_\alpha^a\Phi_m = \sigma_m^{ab}\Psi_{b\alpha},$$
$$\mathbb{Q}_\alpha^a\Psi_{b\beta} = -\frac{1}{2}\sigma_{\beta\dot\epsilon}^\mu\epsilon^{\dot\epsilon\dot\gamma}\sigma_{\alpha\dot\gamma}^\nu F_{\mu\nu}\delta_b^a + \frac{i}{2}\epsilon_{\beta\alpha}\sigma_{bc}^m\sigma_n^{ca}[\Phi_m, \Phi^n],$$
$$\mathbb{Q}_\alpha^a\bar{\Psi}_{\dot\gamma}^b = i\sigma_m^{ab}\sigma_{\alpha\dot\gamma}^\mu D_\mu\Phi^m,$$
$$\mathbb{Q}_\alpha^a A^\mu = i\sigma_{\alpha\dot\gamma}^\mu\epsilon^{\dot\gamma\dot\epsilon}\bar{\Psi}_{\dot\epsilon}^a. \quad (4)$$

Analogous expressions hold for the conjugate generator $\bar{\mathbb{Q}}_{a\dot\alpha}$, with the roles of $\Psi_{a\alpha}$ and $\bar{\Psi}_{\dot\alpha}^a$ interchanged.

Yangian invariance of $\mathcal{N} = 4$ SYM.—We want to study classical Yangian symmetry of planar $\mathcal{N} = 4$ SYM; ordinarily, one would show the invariance of the action $\mathcal{S}$. This works well for the superconformal generators $\mathbb{J}^K$ at level zero of the Yangian [the indices $K, L, \ldots$ enumerate a basis of the level-zero algebra $\mathfrak{psu}(2, 2|4)$], namely $\mathbb{J}^K\mathcal{S} = 0$. As outlined in the introduction, there are several difficulties in formulating invariance of the action to higher-level generators. Gladly, most of them disappear when acting on the equations of motion instead. In the following, we will show how this can be achieved and how to promote their invariance to a powerful off-shell statement.

A level-one generator $\hat{\mathbb{J}}^K$ has a bilocal contribution determined completely by the level-zero generators

$$\hat{\mathbb{J}}_{\mathrm{biloc}}^K = \frac{1}{2}f_{MN}^K\mathbb{J}^M \wedge \mathbb{J}^N; \quad (5)$$

here, f_{MN}^K denotes the structure constants of $\mathfrak{psu}(2, 2|4)$. Explicitly, the bilocal term acts in the following way on a sequence $Z_1 Z_2 \ldots Z_n$ of fields:

$$f_{MN}^K \sum_{1=i<j}^n Z_1 \ldots (\mathbb{J}^M Z_i) \ldots (\mathbb{J}^N Z_j) \ldots Z_n. \quad (6)$$

Notice that this is where the planar limit is relevant for Yangian symmetry: only in the planar limit, the ordering of fields within a matrix product is universally and unambiguously defined because there are no identities between matrix polynomials when N_c is sufficiently large.

Let us apply the simplest level-one generator $\hat{\mathbb{P}}^\rho$, also known as the dual superconformal generator [9], to the equations of motion (2). We will work with the easiest of them, the Dirac equation. The bilocal part of the level-one momentum $\hat{\mathbb{P}}^\rho$ takes the form

$$\hat{\mathbb{P}}_{\mathrm{biloc}}^\rho = \mathbb{D} \wedge \mathbb{P}^\rho - \mathbb{L}_\mu^\rho \wedge \mathbb{P}^\mu - \frac{i}{4}\sigma^{\rho,\dot\alpha\beta}\bar{\mathbb{Q}}_{a\dot\alpha} \wedge \mathbb{Q}_\beta^a. \quad (7)$$

Applying it to the Dirac equation, we get

$$\hat{\mathbb{P}}^{\mu}_{\text{biloc}} \check{\Psi}^{\dot{\alpha}}_a = -i\epsilon^{\dot{\alpha}\dot{\gamma}}\sigma^m_{ab}\{\Phi_m, D^{\mu}\bar{\Psi}^b_{\dot{\gamma}}\} - \frac{i}{2}\epsilon^{\dot{\alpha}\dot{\gamma}}\sigma^m_{ab}\{D^{\mu}\Phi_m, \bar{\Psi}^b_{\dot{\gamma}}\}$$

$$-\frac{i}{2}\epsilon^{\dot{\alpha}\dot{\gamma}}\sigma^m_{ab}\sigma^{\mu}_{\alpha\dot{\gamma}}\sigma^{\alpha\dot{\epsilon}}_{\nu}\{D^{\nu}\Phi_m, \bar{\Psi}^b_{\dot{\epsilon}}\}$$

$$-\frac{i}{2}\sigma^{\mu,\rho\dot{\alpha}}\sigma^m_{ac}\sigma^{cb}_n\{\Psi_{\rho b}, [\Phi_m, \Phi^n]\}. \tag{8}$$

It is useful to observe that the explicit x dependence due to some of the bosonic generators in (3) drops out completely. This is related to the triviality of $[\mathbb{P}^{\mu}, \hat{\mathbb{P}}^{\rho}]$.

A level-one generator can also have some local contributions, which act on a single field at a time just like the level-zero generators. These contributions are not determined by level-zero symmetry, and we may adjust them according to our needs. We now ask whether there is a single-field action of $\hat{\mathbb{P}}^{\rho}$, ensuring the invariance of the equations of motion. With the following choice for the single-field action of $\hat{\mathbb{P}}^{\mu}$,

$$\hat{\mathbb{P}}^{\mu}\Phi_m = 0,$$

$$\hat{\mathbb{P}}^{\mu}\Psi_{a\alpha} = \frac{1}{2}\sigma^{\mu}_{\alpha\dot{\gamma}}\epsilon^{\dot{\gamma}\dot{\epsilon}}\sigma^m_{ab}\{\Phi_m, \bar{\Psi}^b_{\dot{\epsilon}}\},$$

$$\hat{\mathbb{P}}^{\mu}\bar{\Psi}^a_{\dot{\alpha}} = \frac{1}{2}\sigma^{\mu}_{\beta\dot{\alpha}}\epsilon^{\beta\gamma}\sigma^{ab}_m\{\Phi^m, \Psi_{\gamma b}\},$$

$$\hat{\mathbb{P}}^{\mu}A^{\rho} = \frac{i}{2}\eta^{\mu\rho}\{\Phi_m, \Phi^m\}, \tag{9}$$

the combination of local and bilocal terms in the action of $\hat{\mathbb{P}}$ gives

$$\hat{\mathbb{P}}^{\mu}\check{\Psi}^{\dot{\alpha}}_a = -\frac{1}{2}\sigma^{\mu}_{\alpha\dot{\gamma}}\sigma^m_{ba}\epsilon^{\dot{\gamma}\dot{\alpha}}\{\Phi_m, \check{\Psi}^{\alpha b}\}. \tag{10}$$

The rhs. is proportional to $\check{\Psi}$ and vanishes on shell. Hence, $\hat{\mathbb{P}}^{\rho}$ is an on-shell symmetry of the $\mathcal{N} = 4$ SYM Dirac equation.

The invariance of the action $\mathcal{S}$ under a superconformal generator $\mathbb{J}^K$ implies a stronger, off-shell relationship for the equations of motion. To that end, consider the invariance of the action, $\mathbb{J}^K\mathcal{S} = \check{Z}^A(\mathbb{J}^K Z_A) = 0$. Now, vary this equation with respect to a generic field Z_C to obtain an equivalent relation, which holds *off-shell*

$$\mathbb{J}^K\check{Z}^C = -\check{Z}^A\frac{\delta(\mathbb{J}^K Z_A)}{\delta Z_C}. \tag{11}$$

The r.h.s. is now a specific combination of the equations of motion $\check{Z}^A$, given by the action of the generators $\mathbb{J}^K$ on the fields Z_A of the theory. Getting inspiration from the structure of the bilocal term (5), as well as from the level-zero formula (11), we propose an analogous formula for the level-one Yangian generators $\hat{\mathbb{J}}^K$,

$$\hat{\mathbb{J}}^K\check{Z}^C = -\check{Z}^A\frac{\delta(\hat{\mathbb{J}}^K Z_A)}{\delta Z_C} + f^K_{MN}\check{Z}^A\left(\mathbb{J}^M \wedge \frac{\delta}{\delta Z_C}\right)(\mathbb{J}^N Z_A). \tag{12}$$

Let us explicitly demonstrate how the two terms on the r.h.s. are to be understood. The former is the direct counterpart of the r.h.s. of (11). Assuming that, e.g., $\hat{\mathbb{J}}^K Z_A = Z_1 Z_2$, it evaluates to

$$-[\delta^C_1\check{Z}^A Z_2 + \delta^C_2 Z_1\check{Z}^A]. \tag{13}$$

Concerning the second term, we first observe that it vanishes for the linear contribution of $\mathbb{J}^N$ acting on Z_A. Assuming that $\mathbb{J}^N Z_A = Z_1 Z_2$, it evaluates to

$$+f^K_{MN}[\delta^C_2(\mathbb{J}^M Z_1)\check{Z}^A - \delta^C_1\check{Z}^A(\mathbb{J}^M Z_2)]. \tag{14}$$

It is now a straightforward exercise to verify that the formula (12) indeed reproduces (10) correctly. Similar checks can also be made for all the other equations of motion. As the supersymmetry generators $\mathbb{Q}^a_{\alpha}$ and $\bar{\mathbb{Q}}_{a\dot{\alpha}}$ map the Dirac equation to the remaining equations of motion, already the algebraic relations guarantee their invariance under $\hat{\mathbb{P}}^{\rho}$. Most importantly, they also imply invariance under the remaining infinitely many Yangian generators.

We conclude that the relationship (12) holds in classical planar $\mathcal{N} = 4$ SYM theory. Furthermore, we will demonstrate in [13], that it can be lifted to an invariance statement for the action of $\mathcal{N} = 4$ SYM. In that sense, classical planar $\mathcal{N} = 4$ SYM is invariant under Yangian symmetry.

Correlation functions.—The novel symmetry relationship (12) should have implications for observables. A fundamental class of observables in a quantum field theory is given by correlators of fields. Although not gauge invariant on their own, they are building blocks for observables like Wilson loops, scattering amplitudes (via the Lehmann-Symanzik-Zimmermann (LSZ) reduction formula), correlators of local operators, or form factors. An ordinary symmetry results in Ward-Takahashi identities for correlators. The goal is thus to derive Ward-Takahashi identities for level-one Yangian symmetries, and subsequently use them to prove symmetries of the aforementioned gauge-invariant observables. In the following, we shall briefly sketch the level-one symmetry for planar correlators of the fields; a detailed treatment can be found in the follow-up paper [13].

In order to set up the quantum theory, we should first fix the gauge and thus make correlation functions of the fields well-defined observables. This step can potentially break symmetries, but gladly, it leaves the level-one Yangian symmetry intact: we show in [13] that there exists a (trivial) extension of the symmetry representation on the Faddeev-Popov ghosts such that the invariance statement (12) continues to hold. Note that it must be supplemented by suitable terms exact under Becchi-Rouet-Stora-Tyutin (BRST) symmetry, which are irrelevant for physical observables.

Ward-Takahashi identities for Yangian symmetry are obtained by acting with the nonlinear level-one momentum $\hat{\mathbb{P}}$ on some collection of fields. Symmetry implies that the planar quantum expectation value of the resulting expression vanishes. This amounts to some extra differential

constraints on planar correlation functions. Note, however, the following few caveats: in a gauge-fixed theory, one should expect some residual terms due to the unphysical modes which may violate the symmetry. At leading order, these should take the form of total derivatives of the external fields. Furthermore, the resulting identity can receive divergences at loop level. If the level-one symmetry can be renormalized appropriately to make the identity hold at higher orders in perturbation theory, it is a proper quantum symmetry. Otherwise, the level-one Yangian symmetry would be anomalous.

In [13], we demonstrate that (discarding issues related to gauge fixing) the level-one relationship (12), combined with superconformal symmetry, results in additional Ward-Takahashi identities at tree level at least up to four external points. Therefore, Yangian symmetry indeed has concrete implications for planar correlators.

Other superconformal theories.—It is tempting to consider the relationship of Eq. (12) as a criterion for integrability in other planar gauge theories. The criterion can be applied to all models whose global symmetries form a semisimple Lie (super)algebra. In the following, we support our proposal by showing that the criterion agrees with our expectations for two sample gauge theories with classical superconformal symmetry.

First, we consider $\mathcal{N} = 1$ pure SYM theory in four dimensions, which is nonintegrable. This theory is classically superconformal, its global symmetry algebra being $\mathfrak{su}(2,2|1)$; it contains a single vector field and a single Weyl fermion. The Lagrangian of this theory can be obtained from $\mathcal{N} = 4$ SYM by dropping all the scalar fields and retaining one single Weyl fermion; it reads

$$\mathcal{L} = -\frac{1}{4}\text{tr} F_{\mu\nu}F^{\mu\nu} + \text{tr}\, i\bar{\Psi}_{\dot{\alpha}}\sigma_{\mu}^{\dot{\alpha}\beta}D^{\mu}\Psi_{\beta}. \quad (15)$$

Similarly, the action of the superconformal generators can be obtained from the corresponding expressions for $\mathcal{N} = 4$ SYM, [Eqs. (3) and (4)].

As before, we can check our criterion for Yangian symmetry using the level-one generator $\hat{\mathbb{P}}^{\rho}$ of $Y[\mathfrak{su}(2,2|1)]$, acting on the Dirac equation of this theory. Considering the quantum numbers, one finds no admissible terms for the single-field action of $\hat{\mathbb{P}}^{\rho}$,

$$\hat{\mathbb{P}}^{\rho}A_{\mu} = \hat{\mathbb{P}}^{\rho}\Psi_{\alpha} = \hat{\mathbb{P}}^{\rho}\bar{\Psi}_{\dot{\beta}} = 0. \quad (16)$$

The computation of the lhs. of Eq. (12) in this case yields a term proportional to $\{\bar{\Psi}_{\dot{\alpha}}, F_{\mu\nu}\}$, whereas the r.h.s. vanishes identically; in $\mathcal{N} = 4$ SYM, the latter is nontrivial and cancels the former as well as all other arising terms. Our criterion therefore states that $\mathcal{N} = 1$ SYM does not possess Yangian symmetry in accordance with the fact that this theory, as a whole, is not integrable. Importantly, this demonstrates that it is not merely an artefact due to other properties, such as gauge or superconformal symmetry.

The second example we study is the so-called ABJM theory, a Chern-Simons matter theory with gauge group, $U(N_c) \times U(N_c)$. The matter sector consists of four chiral multiplets Φ^a, $\Psi_{a\alpha}$, transforming in the bifundamental of the gauge groups, together with their conjugate multiplets $\bar{\Phi}_a$, $\bar{\Psi}_{\alpha}^a$, transforming in the conjugate bifundamental; the gauge fields associated with the two components of the gauge group are A_{μ} and $\tilde{A}_{\mu}$. ABJM is a superconformal theory with $\mathcal{N} = 6$ supersymmetry, its global symmetry algebra being $\mathfrak{osp}(6|4)$. Similarly to $\mathcal{N} = 4$ SYM, this theory appears to be exactly integrable in the planar limit, and also in this case, its integrability is encoded in the existence of a Yangian symmetry [14] based on its superconformal algebra.

In order to study the Yangian symmetry of planar ABJM, we start from the Lagrangian,

$$\mathcal{L} = \mathcal{L}_{\text{CS}} - \mathcal{L}_{\tilde{\text{CS}}} + \bar{\Psi}^{a\alpha}D_{\mu}\gamma_{\alpha\beta}^{\mu}\Psi_a^{\beta} + D_{\mu}\Phi^a D^{\mu}\bar{\Phi}_a + \dots. \quad (17)$$

Here, $\mathcal{L}_{\text{CS}}$ and $\mathcal{L}_{\tilde{\text{CS}}}$ are the Chern-Simons terms, and we omitted the Yukawa couplings and scalar potential.

We want to check whether Eq. (12) holds for the Dirac equation of ABJM,

$$\breve{\Psi}_{a\alpha} = -\gamma_{\alpha\gamma}^{\mu}\epsilon^{\gamma\beta}D_{\mu}\Psi_{a\beta} + \Psi_{a\alpha}\bar{\Phi}_b\Phi^b - \Phi_b\bar{\Phi}^b\Psi_{a\alpha}$$
$$+ 2\Phi^b\bar{\Phi}_a\Psi_{b\alpha} - 2\Psi_{b\alpha}\bar{\Phi}_a\Phi^b + 2\epsilon_{abcd}\Phi^b\bar{\Psi}_{\alpha}^c\Phi^d. \quad (18)$$

As before, we consider the level-one generator $\hat{\mathbb{P}}^{\rho}$ of $Y[\mathfrak{osp}(6|4)]$, whose bilocal part is of the same form described in Eq. (7),

$$\hat{\mathbb{P}}_{\text{biloc}}^{\rho} = \mathbb{D} \wedge \mathbb{P}^{\rho} - \mathbb{L}_{\mu}^{\rho} \wedge \mathbb{P}^{\mu} + \frac{1}{16}\epsilon^{abcd}\gamma^{\rho,\alpha\beta}\mathbb{Q}_{ab\alpha} \wedge \mathbb{Q}_{cd\beta}. \quad (19)$$

In order to compute the action of $\hat{\mathbb{P}}^{\rho}$ on $\breve{\Psi}_{a\alpha}$, we need the action of the supercharges on the fields

$$\mathbb{Q}_{ab\alpha}\Phi^c = \delta_a^c\Psi_{b\alpha} - \delta_b^c\Psi_{a\alpha},$$
$$\mathbb{Q}_{ab\alpha}\Psi_{c\beta} = -\epsilon_{abcd}\gamma_{\alpha\beta}^{\mu}D_{\mu}\Phi^d - 2\epsilon_{\alpha\beta}\epsilon_{abde}\Phi^d\bar{\Phi}_c\Phi^e$$
$$- \epsilon_{\alpha\beta}\epsilon_{abcd}(\Phi^d\bar{\Phi}_e\Phi^e - \Phi^e\bar{\Phi}_e\Phi^d),$$
$$\mathbb{Q}_{ab\alpha}A^{\mu} = 2\gamma_{\alpha\beta}^{\mu}\epsilon^{\beta\gamma}(\Psi_{b\gamma}\bar{\Phi}_a - \Psi_{a\gamma}\bar{\Phi}_b + \epsilon_{abcd}\Phi^c\bar{\Psi}_{\gamma}^d), \quad (20)$$

and similarly for $\bar{\Phi}_a$, $\bar{\Psi}_{\alpha}^a$, and $\tilde{A}_{\mu}$. We can now show that Eq. (12) holds for $\hat{\mathbb{P}}^{\rho}$ acting on $\breve{\Psi}_{a\alpha}$, provided that the single-field action of $\hat{\mathbb{P}}^{\rho}$ is

$$\hat{\mathbb{P}}^{\rho}\Phi^a = 0,$$
$$\hat{\mathbb{P}}^{\rho}\Psi_{a\alpha} = \gamma_{\alpha\beta}^{\rho}\epsilon^{\beta\gamma}[\Psi_{a\gamma}\bar{\Phi}_b\Phi^b + \Phi^b\bar{\Phi}_b\Psi_{a\gamma}$$
$$- 2\Psi_{b\gamma}\bar{\Phi}_a\Phi^b - 2\Phi^b\bar{\Phi}_a\Psi_{b\gamma}],$$
$$\hat{\mathbb{P}}^{\rho}A^{\mu} = \frac{1}{2}\epsilon^{\rho\mu\nu}(D_{\nu}\Phi^a\bar{\Phi}_a + \Phi^a D_{\nu}\bar{\Phi}_a)$$
$$+ 2\eta^{\mu\rho}(\Phi^a\bar{\Phi}_b\Phi^b\bar{\Phi}_a - \epsilon^{\alpha\beta}\Psi_{a\alpha}\bar{\Psi}_{\beta}^a), \quad (21)$$

and similarly for $\bar{\Phi}_a$, $\bar{\Psi}_\gamma^a$, and $\tilde{A}_\mu$. It is similarly possible to show that the off-shell invariance condition in Eq. (12) holds for all the equations of motion of ABJM, and that therefore planar ABJM is classically Yangian invariant. However, the compatibility with gauge fixing, cf. the comments in the previous section, is subtler than for $\mathcal{N} = 4$ SYM. At the moment, we do not understand how to compensate terms due to the nonlinear action in Eq. (20) of the supercharges on the gauge fields. We will return to this issue in [13].

Comments and conclusions.—conclusions In this Letter, we showed that the equations of motion of both $\mathcal{N} = 4$ supersymmetric Yang-Mills and ABJM—two superconformal field theories which are apparently integrable in the planar limit—are invariant under the Yangian of the relevant superconformal algebra. Moreover, we derived an *off-shell* relationship (12), which serves as a formal statement of invariance of the classical theory. It can therefore be seen as a criterion for integrability in planar gauge theories.

In a companion paper [13], we will provide a more detailed account of our claims and elaborate on some more advanced aspects, which we can merely touch upon in the present Letter: starting from (12), we will derive a definition for Yangian invariance of the action of a planar gauge theory. Furthermore, we will show how Yangian symmetry survives gauge fixing. Finally, we will establish novel Ward-Takahashi identities for planar correlators of fields.

An important next goal is to analyze the validity of Yangian symmetry at the quantum level. In other words, do the Ward-Takahashi identities continue to hold, at least at the level of loop integrands? And does renormalization and performing the loop integrations render the symmetry anomalous? Our strong expectation is that the symmetry will hold exactly at all values of the coupling constant because we know that integrability leads to consistent results at weak, intermediate, and strong coupling, see [1–7]. Note that, likewise, the AdS/CFT dual string theory model is integrable [15]. However, it is to be expected that particular observables, e.g., Wilson loops with cusps, will break Yangian symmetry to some extent. Therefore, it will be most important to apply our definition of Yangian symmetry to observables and to understand where and how it is broken concretely.

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