Quantum State Transfer via Noisy Photonic and Phononic Waveguides

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We describe a quantum state transfer protocol, where a quantum state of photons stored in a first cavity can be faithfully transferred to a second distant cavity via an infinite 1D waveguide, while being immune to arbitrary noise (e.g., thermal noise) injected into the waveguide. We extend the model and protocol to a cavity QED setup, where atomic ensembles, or single atoms representing quantum memory, are coupled to a cavity mode. We present a detailed study of sensitivity to imperfections, and apply a quantum error correction protocol to account for random losses (or additions) of photons in the waveguide. Our numerical analysis is enabled by matrix product state techniques to simulate the complete quantum circuit, which we generalize to include thermal input fields. Our discussion applies both to photonic and phononic quantum networks.

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Introduction.—The ability to transfer quantum states between distant nodes of a quantum network via a quantum channel is a basic task in quantum information processing [1–4]. An outstanding challenge is to achieve quantum state transfer [5,6] (QST) with high fidelity despite the presence of noise and decoherence in the quantum channel. In a quantum optical setup, the quantum channels are realized as 1D waveguides, where quantum information is carried by "flying qubits" implemented either by photons in the optical [7–9] or microwave regime [10–13], or phonons [14,15]. Thus, imperfections in the quantum channel include photon or phonon loss, and, in particular for microwave photons and phonons, a (thermal) noise background [16]. In this Letter, we propose a QST protocol and a corresponding quantum optical setup which allow for state transfer with high fidelity, undeterred by these imperfections. A key feature is that our protocol and setup are *a priori* immune to quantum or classical noise injected into the 1D waveguide, while imperfections such as random generation and loss of photons or phonons during transmission can be naturally corrected with an appropriate quantum error correction (QEC) scheme [17].

The generic setup for QST in a quantum optical network is illustrated in Fig. 1 as transmission of a qubit state from a first to a second distant two-level atom via an infinite 1D bosonic open waveguide. The scheme of Fig. 1(a) assumes a chiral coupling of the two-level atoms to the waveguide [18,19], as demonstrated in recent experiments with atoms [20] and quantum dots [21]. The atomic qubit is transferred in a decay process with a time-varying coupling to a right-moving photonic (or phononic) wave packet propagating in the waveguide, i.e., $(c_g|g\rangle_1 + c_e|e\rangle_1)|0\rangle_p \to |g\rangle_1(c_g|0\rangle_p + c_e|1\rangle_p)$ where $|\rangle_1$ and $|\rangle_p$ denote the atomic and channel states. The transfer of the qubit state is then completed by reabsorbing the photon (or phonon) in the second atom via the inverse operation, essentially mimicking the time-reversed process of the initial decay. Such transfer protocols have been discussed in the theoretical literature [5,6,22–25], and demonstrated in recent experiments [7].

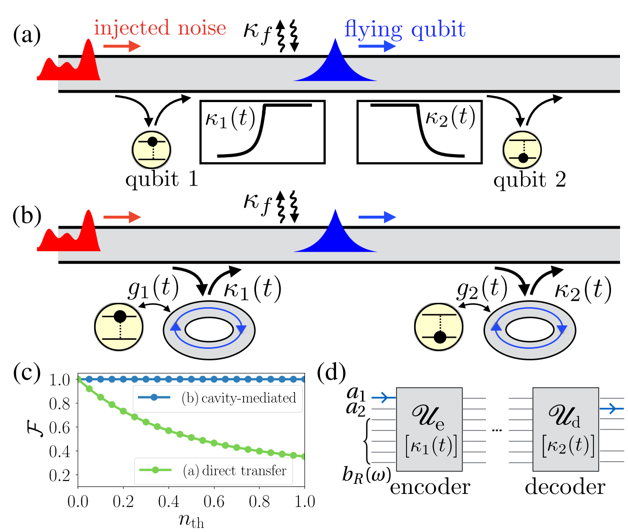

FIG. 1. Quantum state transfer via a noisy waveguide. (a) QST where qubits are coupled directly with chiral coupling to a waveguide representing the quantum channel. (b) QST in a cavity QED setup, where atoms representing qubits are coupled to the waveguide with a cavity as mediator. (c) Fidelity $\mathcal{F}$ for QST of a qubit as a function of photon occupation $n_{\rm th}$ representing a thermal noise injected into the waveguide for setups (a) and (b). For the protocol described in the text, setup (b) is robust to injected noise. (d) "Write" of a quantum state from cavity 1 to a temporal mode in the (noisy) waveguide, and "Read" back to cavity 2 as a linear multimode encoder and decoder with encoding [decoding] functions $\kappa_1(t)$ $[\kappa_2(t)]$ (see text).

A central assumption underlying these studies is, however, that the waveguide is initially prepared in the vacuum state; i.e., at zero temperature, and—as shown in Fig. 1(c)—the fidelity for QST (formally defined in the Supplemental Material [26]) will degrade significantly in the presence of noise, e.g., thermal [16]. Below, we show that a simple variant of the setup with a cavity as mediator makes the QST protocol immune against arbitrary injected noise [5,34] [cf. Fig. 1(b)]. Robust QST also provides the basis for distribution of entanglement in a quantum network.

Photonic quantum network model.—We consider the setup illustrated in Fig. 1(b), where each "node" consists of a two-level atom as qubit coupled to a cavity mode. We assume that the cavity QED setup is designed with a chiral light-matter interface with coupling to right-moving modes of the waveguide [35]. In the language of quantum optics, the setup of Fig. 1(b) is a cascaded quantum system [37], where the first node is unidirectionally coupled to the second one. The dynamics is described by a quantum stochastic Schrödinger equation (QSSE) [37] for the composite system of nodes and waveguide as $i(d/dt)|\Psi(t)\rangle = H(t)|\Psi(t)\rangle$ ($\hbar = 1$). The Hamiltonian is $H(t) = \sum_{j=1,2} H_{n_j}(t) + V(t)$ with $H_{n_j}(t) = ig_j(t)(a_j^\dagger \sigma_j^- - \text{H.c.})$ the Jaynes-Cummings Hamiltonian for node $j = 1, 2$ in the rotating wave approximation (RWA). Here, a_j are annihilation operators for the cavity modes and σ_j's are Pauli operators for the two-level atoms with levels $|g_j\rangle$, $|e_j\rangle$. We assume that the cavities are tuned to resonance with the two-level atoms ($\omega_c = \omega_{eg}$), and the Hamiltonian is written in the rotating frame. The coupling of the first and second cavity (located at $x_1 < x_2$) to the right-moving modes of the channel is described by the interaction Hamiltonian

$$V(t) = i \sum_{j=1,2} \sqrt{\frac{\kappa_j(t)}{2\pi}} \int_{\mathcal{B}} d\omega b_R^\dagger(\omega) e^{i(\omega-\omega_c)t - i\omega x_j/c} a_j - \text{H.c.}$$
$$\equiv i(\sqrt{\kappa_1(t)} b_R^\dagger(t) a_1 + \sqrt{\kappa_2(t)} b_R^\dagger(t-\tau) e^{i\phi} a_2 - \text{H.c.}), \tag{1}$$

in the RWA. Here, $b_R(\omega)$ denotes the annihilation operators of the continuum of right-moving modes with frequency ω within a bandwidth $\mathcal{B}$ around the atomic transition frequency, c is the velocity of light, and $\kappa_{1,2}(t)$ is a decay rate to the waveguide. In the second line of Eq. (1), we have rewritten this interaction in terms of quantum noise operators $b_R(t)$ satisfying white noise commutation relations $[b_R(t), b_R^\dagger(s)] = \delta(t-s)$. The parameter $\tau = d/c$, with $d = x_2 - x_1 > 0$, denotes the time delay of the propagation between the two nodes, and $\phi = -\omega_c \tau$ is the propagation phase. For a cascaded quantum system with purely unidirectional couplings, τ and ϕ can always be absorbed in a redefinition of the time and phase of the second node. Noise injected into the waveguide is specified by the hierarchy of normally ordered correlation functions of $b_R(t)$. In particular, the Fourier transform of the correlation function $\langle b_R^\dagger(t) b_R(s)\rangle$ provides the spectrum of the incident noise $S(\omega)$, which for white (thermal) noise corresponds to $\langle b_R^\dagger(t) b_R(s)\rangle = n_{\text{th}} \delta(t-s)$ with occupation number n_{th} and flat spectrum $S(\omega) = n_{\text{th}}$.

Quantum state transfer protocol.—To illustrate immunity to injected noise in QST, first, we consider a minimal model of a pair of cavities coupled to the waveguide. The quantum Langevin equations (QLEs) for the annihilation operators of the two cavity modes $a_{1,2}(t)$ in the Heisenberg picture read [26]

$$\frac{da_1}{dt} = -\frac{1}{2}\kappa_1(t) a_1(t) - \sqrt{\kappa_1(t)} b_R(t),$$
$$\frac{da_2}{dt} = -\frac{1}{2}\kappa_2(t) a_2(t) - \sqrt{\kappa_2(t)} [b_R(t) + \sqrt{\kappa_1(t)} a_1(t)]. \tag{2}$$

These equations describe the driving of the first cavity by an input noise field $b_R(t)$ [38], while the second cavity is driven by both $b_R(t)$ and the first cavity. We can always find a family of coupling functions $\kappa_{1,2}(t)$, satisfying the time-reversal condition $\kappa_2(t) = \kappa_1(-t)$ [see inset of Fig. 1(a)], which achieves a mapping

$$a_1(t_i) \to -a_2(t_f); \tag{3}$$

i.e., the operator of the first cavity mode at initial time t_i is mapped to the second cavity mode at final time t_f, with no admixture from $b_R(t)$ [26]. In other words, an arbitrary photon superposition state prepared initially in the first cavity can be faithfully transferred to the second distant cavity without being contaminated by incident noise. This result holds without any assumption on the noise statistics. It is intrinsically related to the linearity of the above QLEs, which allows the effect of noise acting equally on both cavities to drop out by quantum interference. Thus, the setup can be combined with other elements of linear optics, such as beam splitters [26].

Robustness of QST to injected noise generalizes immediately to more complex systems representing effective "coupled harmonic oscillators." We can then add atomic ensembles of N two-level atoms represented by atomic hyperfine states [3,39,40] to the first and second cavities ($j = 1$, 2). Spin excitations in atomic ensembles [41,42], generated by the collective spin operator $S_j^+ = (1/\sqrt{N}) \sum_{i=1}^N \sigma_{i,j}^+$ with i the sum over atomic spin-operators of node j, are again harmonic for low densities. Moreover, they can be coupled in a Raman process to the cavity mode, $H_{n_j} = \tilde{g}_j(t)(S_j^+ a_j + \text{H.c.})$, as familiar from the read and write of photonic quantum states to atomic ensembles as quantum memory [43]. This provides a way of getting an effective time-dependent coupling to the waveguide in a setup with constant cavity decay. Thus, our protocol generalizes to the transfer of quantum states stored as a long-lived spin excitation in a first atomic ensemble to a second remote ensemble [26].

Returning to the setup of Fig. 1(b), with a single atom as qubit coupled to a cavity mode, we achieve—in contrast to the setup of Fig. 1(a)—QST immune to injected noise in a three step process. (i) We first map the atomic qubit state $c_g|g\rangle_1 + c_e|e\rangle_1$ to the cavity mode $c_g|0\rangle_1 + c_e|1\rangle_1$ with the cavity decoupled from the waveguide [44]. (ii) With atomic qubits decoupled from cavities, we transfer the photon superposition state to the second cavity as above [45]. (iii) We perform the time-inverse of step (i) in the second node. This QST protocol generalizes to several atoms as a quantum register representing an entangled state of qubits, which can either be transferred sequentially or mapped collectively to a multiphoton superposition state in the cavity, to be transferred to the second node [46]. As depicted in Fig. 1(d), we can understand our QST protocol in the chiral cavity setup [Fig. 1(b)], consisting of a write operation of the qubit in the first cavity to the waveguide as a quantum data bus, followed by a read into the second cavity. This write and read are both linear operations on the set of operators consisting of cavity and waveguide modes, or as an encoder and decoder into temporal modes specified by $\kappa_{1,2}(t)$, and physically implemented by the chiral cavity-waveguide interface.

Numerical techniques.—We now study the sensitivity of the above protocol to errors. Imperfections may arise from inexact external control parameters including timing and deviations from perfect chirality. Moreover, loss or addition of photons can occur during propagation. Below, we describe a QEC scheme which corrects for such single photon errors.

A study of imperfections in QST will necessarily be numerical in nature, as it requires solution of the QSSE with injected noise accounting for nonlinearities in atom-light coupling. Beyond Eq. (1), the Hamiltonian must include coupling to both right- and left-propagating modes in the waveguide, and should account for possible couplings of waveguide and cavities to additional reservoirs representing decoherence [26]. We have developed and employed three techniques to simulate the complete dynamics of the quantum circuits as depicted in Figs. 1(a) and 1(b). First, we use matrix product states techniques to integrate the QSSE discretized in time steps, as developed in Ref. [48], which we generalize to include injected quantum noise. Our method allows a general input field to be simulated using purification techniques, by entangling time-bins of the photonic field with ancilla copies in the initial state (for related techniques developed in condensed matter physics, see Ref. [49]). This method also allows the study of non-Markovian effects (i.e., for finite retardation $\tau > 0$) in the case of imperfect chiral couplings, and is well suited to represent various kinds of noise. Second, we solve the master equation describing the nodes, which allows for efficient simulations valid in the Markovian limit. Finally, to simulate the QST in nonchiral setups as described at the end of this Letter, we solve the dynamics of the nodes and of a discrete set of waveguide modes, following Ref. [23]. For a detailed description of the complete model and numerical methods, we refer to Ref. [26], and present, below, our main results assuming thermal injected noise $n_{\rm th}$.

Sensitivity to coupling functions $\kappa_{1,2}(t)$.—In Figs. 2(a) and 2(b), we study the sensitivity of QST to the functions $\kappa_{1,2}(t)$ for the minimal model of nodes represented by cavities. Figure 2(a) shows the effect of the protocol duration $T = t_f - t_i$ which, in the ideal case, is required to fulfill $T \gg 1/\kappa_{\rm max}$, with $\kappa_{\rm max}$ the maximum value of $\kappa_{1,2}(t)$. For finite durations, the effect on the fidelity scales linearly with the noise intensity but quickly vanishes for $\kappa_{\rm max} T \gtrsim 10$, above which $\mathcal{F} \geq 0.99$. In all other figures of this work, we use $\kappa_{\rm max} T = 20$. In Fig. 2(b), we show the effect of an imperfection δ_τ in the timing of the coupling functions, namely, $\kappa_2(t) = \kappa_1(\delta_\tau - t)$. The digression from unity is quadratic in δ_τ but linear in noise intensity. This result illustrates that only the proper decoding function allows one to unravel the quantum state emitted by the first cavity on top of the injected noise. Note that, in addition to errors in the coupling functions, the fidelity is also sensitive to the frequency matching of the cavities [50], which we discuss in [26].

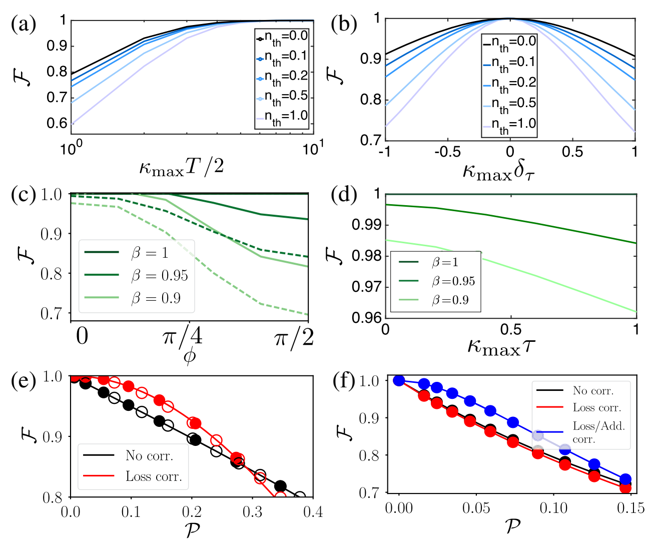

FIG. 2. Role of imperfections. (a) Effect on the fidelity of a finite transfer time $T = t_f - t_i$, and (b) of an imperfect timing of $\kappa_2(t)$. (c) Fidelity as a function of ϕ for different β factors and $\kappa_{\rm max}\tau \approx 0$ (see text). Solid lines: $n_{\rm th} = 0$. Dashed lines: $n_{\rm th} = 0.25$. The fidelity is maximal when ϕ is a multiple of π. (d) By increasing $\kappa_{\rm max}\tau$ for $\phi = 0$ the fidelity decreases. (e) QEC with the setup subject to waveguide and cavity losses. (f) QEC with the setup coupled to a reservoir with photon occupation $n'_{\rm th} = 1$. Black: no error correction. Red: correction against single photon losses. Blue: correction against single photon losses or additions. Solid lines: $n_{\rm th} = 0$, $\kappa' = 0$. Full circles: $n_{\rm th} = 0.5$, $\kappa' = 0$. Empty circles: $n_{\rm th} = 0$, $\kappa_f = 0$.

Imperfect chirality.—For an optical fiber with chirally coupled resonators [36], the nodes emit only a fraction $\beta < 1$ of their excitations in the right direction. The dynamics, then, also depends on the propagation phase ϕ [18] and on the time delay τ. As illustrated in Fig. 2(c), the effect of imperfect chirality in the Markovian regime ($\kappa_{\max}\tau \approx 0$) crucially depends on ϕ, as a consequence of interferences between the photon emissions of the two cavities in the left direction. In particular, for $\phi = 0$, they interfere destructively, leading to a higher fidelity. This interference decreases for finite values of $\kappa_{\max}\tau$, as shown in Fig. 2(d).

Quantum error correction.—In contrast to "injected" noise, loss and injection of photons occurring during propagation between the two cavities represent decoherence mechanisms, which affect the fidelity of the protocol [2]. Such errors can be corrected in the framework of QEC. Instead of encoding the qubits in the Fock states $|0\rangle$ and $|1\rangle$, we use multiphoton states, with the requirement that the loss or addition of a photon projects them onto a new subspace where the error can be detected and corrected. A possibility is to use a basis of cat states, i.e., superposition of coherent states [51,52], where a photon loss only induces a change of parity of the photon number [53]. While we present the efficiency of QST with cat states in Ref. [26], here, we use a basis of orthogonal photonic states for the qubit encoding [17].

We first consider a protocol protecting against single photon losses. Here, the state of the first qubit is mapped to the first cavity as $c_g|g\rangle_1 + c_e|e\rangle_1 \to c_g|+\rangle_1 + c_e|-\rangle_1$, where the cavity logical basis $|\pm\rangle_j = (|0\rangle_j \pm \sqrt{2}|2\rangle_j + |4\rangle_j)/2$ has even photon parity. This unitary transformation can be realized with optimal control pulses driving the qubit and the cavity while using the dispersive shift between the qubit and the cavity mode as nonlinear element [53]. Waveguide losses, with rates κ_f, can be modeled with a beam splitter with transmission probability $\exp(-\kappa_f\tau)$, whereas the rate of cavity losses is denoted κ'. The single photon loss probability is, then, $\mathcal{P} = 1 - \exp(-\kappa_f\tau - \kappa' T)$. The density matrix ρ_f of the second cavity at the end of the protocol reads

$$\rho_f = |\Psi_0\rangle\langle\Psi_0| + \mathcal{P}|\Psi_{-1}\rangle\langle\Psi_{-1}| + \mathcal{O}(\mathcal{P}^2), \qquad (4)$$

where the unnormalized states $|\Psi_0\rangle$ and $|\Psi_{-1}\rangle$, written explicitly in Ref. [26], have even or odd parity, respectively, and satisfy $|\Psi_{-1}\rangle = a_2|\Psi_0\rangle$. The state $|\Psi_0\rangle$ corresponds to the case where no stochastic photon loss occurred, whereas the state $|\Psi_{-1}\rangle$ is obtained if one photon was lost in the process. The last step of the protocol consists in measuring the photon number parity in the second cavity, and—conditional on the outcome—apply unitary operations transferring the photon state to qubit 2. As shown in Fig. 2(e), this encoding significantly improves the fidelity for small losses $\mathcal{P} \ll 1$, up to a threshold value $\mathcal{P} \approx 0.29$. Note that both protocols are insensitive to injected noise.

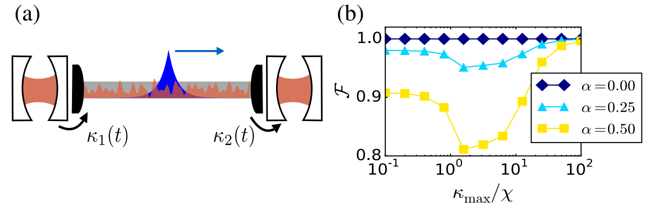

FIG. 3. QST in noncascaded systems. (a) QST in a closed system with two cavities coupled to a finite waveguide. (b) Fidelity as a function of the cavity nonlinearities χ and for different initial occupation of the waveguide. The fidelity approaches unity in the linear limit $\chi \to 0$.

Now, we consider a situation where the waveguide is coupled to a finite temperature reservoir with $n'_{\rm th} = 1$ thermal occupation number which stochastically adds and absorbs photons. Here, the qubit state is encoded as $c_g|+\rangle_1 + c_e|-\rangle_1$, where $|\pm\rangle_1 = (|0\rangle_1 \pm \sqrt{2}|3\rangle_1 + |6\rangle_1)/2$ have photon number 0 modulo 3. The state ρ_f after the transfer is a mixture of $|\Psi_k\rangle\langle\Psi_k|$ with $k = -1, 0, +1$ corresponding to the cases of a single photon loss, of no photon loss or addition, and of a single photon addition. These states satisfy $|\Psi_{-1}\rangle = a_2|\Psi_0\rangle$ and $|\Psi_{+1}\rangle = a_2^\dagger|\Psi_0\rangle$ and are distinguishable by measurement of the photon number modulo 3. In the limit of small error probabilities, one retrieves the original qubit state by applying a unitary operation conditioned on the measurement outcome. In Fig. 2(f), we show that this protocol corrects the errors for $\mathcal{P} \ll 1$ independently of injected noise intensity. This approach extends to an arbitrary number of photon losses and additions, although at the cost of a lower range of achievable $\mathcal{P}$ [17].

Closed systems.—Our results can also be observed in closed systems [cf. Fig. 3(a)], where two cavities are coupled, for instance, via a finite optical fiber or a microwave transmission line [54]. Note that, in circuit QED setups, time-dependent couplings $\kappa_j(t)$ can be realized via tunable couplers [12,50,55]. This system is not chiral, as the dynamics of the first cavity can be perturbed by reflections from the second one. We numerically demonstrate robustness against noise, which, here, is represented as initial occupation of the waveguide. In addition, we consider the effect of Kerr nonlinearities; i.e., we add terms $-\chi a_j^\dagger a_j^\dagger a_j a_j$ which are relevant for circuit QED setups [53] to the Hamiltonian [26]. The results are presented in Fig. 3(b) with each (discrete) waveguide mode initially in a coherent state $|\alpha\rangle$. QST becomes robust against noise in the transition from the cavity as an effective two-level system ($\chi \to \infty$) to perfect harmonic oscillator ($\chi \to 0$).

Conclusion.—Robustness to arbitrary injected noise in transferring a quantum state between two cavities relies on the linearity of the write and read into temporal modes [cf. Fig. 1(d)], with quantum noise canceled by quantum interference. While we have focused on QST between two distant cavity modes here, our approach generalizes to a setup involving many nonlocal bosonic resonator modes [26], which can be realized with various physical platforms, and as hybrid systems.

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B. V. and P. O. G. contributed equally to this work.

Note added.—A related setup and protocol have been proposed in an independent work by Z. L. Xiang *et al.* [57].

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