

Anyonic Haldane Insulator in One Dimension

Florian Lange,^{1,2} Satoshi Ejima,² and Holger Fehske²

¹*Computational Condensed Matter Physics Laboratory, RIKEN, Wako, Saitama 351-0198, Japan*

²*Institut für Physik, Ernst-Moritz-Arndt-Universität Greifswald, 17489 Greifswald, Germany*

(Received 30 November 2016; published 23 March 2017)

We demonstrate numerically the existence of a nontrivial topological Haldane phase for the one-dimensional extended (U - V) Hubbard model with a mean density of one particle per site, not only for bosons but also for anyons, despite a broken reflection parity symmetry. The Haldane insulator, surrounded by superfluid, Mott insulator, and density-wave phases in the V - U parameter plane, is protected by combined (modified) spatial-inversion and time-reversal symmetries, which is verified within our matrix-product-state based infinite density-matrix renormalization group scheme by analyzing generalized transfer matrices. With regard to an experimental verification of the anyonic Haldane insulator state the calculated asymmetry of the dynamical density structure factor should be of particular importance.

DOI: 10.1103/PhysRevLett.118.120401

Anyons represent a third fundamental class of particles with fractional exchange statistics that interpolates, to some degree, between those of bosons and fermions having symmetric or antisymmetric wave functions under exchange [1,2]. By contrast, the exchange of two anyons creates a phase factor $e^{i\theta}$ in the many-body wave function, where the statistical parameter θ can be of any value in the interval $(0, \pi)$. In the beginning anyons were thought to be relevant solely for two-dimensional systems. Describing the fractional quantum Hall effect experiments in particular, the quasiparticles could be viewed as anyons with θ fixed by the filling factor [3,4]. With Haldane's generalized Pauli principle and definition of fractional statistics, however, the concept of anyons becomes important in arbitrary dimensions [5].

In one dimension, the physics of anyons might be studied successfully with ultracold atoms in optical lattices [6]. For example, one-dimensional (1D) anyon statistics can be implemented by bosons with occupation-dependent hopping amplitudes generated by assisted Raman tunneling [7,8]. An alternative route to create 1D anyons in an optical lattice exploits lattice-shaking-assisted tunneling against potential offsets generated by a combination of a static potential tilt and strong on-site interactions [9]. Thereby, advantageously, no additional lasers are required, except for those employed on creating optical lattices. However, in spite of the huge experimental efforts, a conclusive detection of 1D anyons in optical lattices has not yet been achieved.

Notwithstanding, from a theoretical point of view, anyons in one dimension have received continuous and legitimate interest on account of their intriguing physical properties. The exact solution of a 1D anyon gas with a delta-function potential has been obtained by a Bethe ansatz technique [10]. Boundary conformal field theory shows that non-Abelian anyons may form topological insulating phases in spin-1/2 $su(2)_k$ chains [11]. For the Abelian 1D anyon-Hubbard

model (AHM), the possibility of a statistically induced quantum phase transition between Mott-insulator (MI) and superfluid phases [7,12] and the asymmetry of the momentum distribution for hard-core [13] and soft-core anyons [14] have been addressed so far. Since the AHM is equivalent to a variant of the Bose-Hubbard model (BHM) with state-dependent bosonic hopping amplitudes [7], the next very interesting question might be whether the symmetry-protected topological (SPT) Haldane state [15,16], observed in the extended BHM (EBHM) with an additional nearest-neighbor particle repulsion [17,18], also shows up in the extended AHM (EAHM). Because of its SPT order, the Haldane phase in the EBHM is separated from the topologically trivial MI phase by a phase transition, as long as the protecting symmetry—being a combination of bond-centered inversion and a local unitary transformation—keeps up [16]. By breaking this symmetry, the two phases can be adiabatically connected without crossing a phase transition. Therefore, a sharp distinction between the two phases is only possible in the presence of the protecting symmetry, even though no spontaneous symmetry breaking occurs. As the hopping phase factor breaks the reflection parity in the system [19], naively one might expect the Haldane state to disappear in the EAHM for any finite fractional phase θ . However, this will not happen if the protecting symmetry is appropriately generalized for finite θ .

To comment on an anyonic topological Haldane state in one dimension, we scrutinize its protecting symmetry in the framework of the EAHM by analyzing the invariance of the density-dependent hopping amplitudes (as for the EBHM in the limit $\theta \rightarrow 0$). Calculating the generalized transfer matrices [20] from the infinite matrix-product state (iMPS) of the infinite density-matrix renormalization-group (iDMRG) [21–23] simulations, we prove the existence of the Haldane insulator (HI) state and derive the complete ground-state phase diagram of this paradigmatic anyonic

model Hamiltonian at unit filling. In order to discriminate the topological HI phase from the other, more conventional Mott and density-wave (DW) insulating phases in possible future experiments, we also determine the dynamical density response of the system, showing a characteristic asymmetry in the Brillouin zone, which can be attributed to the fractional phase factor of the anyons.

The Hamiltonian of the 1D EAHM consists of three terms, $\hat{H}_{\text{EAHM}}^{(a)} \equiv \hat{H}_t + \hat{H}_U + \hat{H}_V$, with

$$\hat{H}_t = -t \sum_j (\hat{a}_j^\dagger \hat{a}_{j+1} + \text{H.c.}), \quad (1)$$

$\hat{H}_U = U \sum_j \hat{n}_j (\hat{n}_j - 1)/2$, and $\hat{H}_V = V \sum_j \hat{n}_j \hat{n}_{j+1}$, describing the nearest-neighbor anyon transfer ($\propto t$), as well as the repulsive on-site ($\propto U$) and nearest-neighbor ($\propto V$) particle interaction, respectively. The anyon creation (\hat{a}_j^\dagger), annihilation (\hat{a}_j), and particle number ($\hat{n}_j = \hat{a}_j^\dagger \hat{a}_j$) operators at lattice site j are defined by the generalized commutation relations [7,10]:

$$\hat{a}_j \hat{a}_\ell^\dagger - e^{-i\theta \text{sgn}(j-\ell)} \hat{a}_\ell^\dagger \hat{a}_j = \delta_{j\ell}, \quad (2)$$

$$\hat{a}_j \hat{a}_\ell - e^{i\theta \text{sgn}(j-\ell)} \hat{a}_\ell \hat{a}_j = 0, \quad (3)$$

where the sign function $\text{sgn}(j-\ell) = 0$ for $j = \ell$ is mandatory, since two anyons on the same site behave as ordinary bosons. Anyons with $\theta = \pi$ represent so-called ‘‘pseudofermions,’’ namely, they are fermions off site, while being bosons on site.

Performing a fractional Jordan–Wigner transformation [7],

$$\hat{a}_j = \hat{b}_j e^{i\theta \sum_{\ell=1}^{j-1} \hat{n}_\ell}, \quad (4)$$

where \hat{b}_j^\dagger (\hat{b}_j) is a boson creation (annihilation) operator, $\hat{H}_{\text{EAHM}}^{(a)}$ becomes $\hat{H}_{\text{EAHM}}^{(b)}$ with density-dependent hopping amplitudes,

$$\hat{H}_t = -t \sum_j (\hat{b}_j^\dagger \hat{b}_{j+1} e^{i\theta \hat{n}_j} + e^{-i\theta \hat{n}_j} \hat{b}_{j+1}^\dagger \hat{b}_j). \quad (5)$$

That is, when a boson hops to the left from site $j+1$ to site j it acquires an occupation dependent phase $e^{i\theta \hat{n}_j}$. Of course, $\hat{n}_j = \hat{a}_j^\dagger \hat{a}_j = \hat{b}_j^\dagger \hat{b}_j$, which means that \hat{H}_U and \hat{H}_V are form invariant under the anyon-boson mapping (4).

If we limit the maximum number of particles per site as $n_p = 2$, the EBHM, resulting in the limit $\theta \rightarrow 0$ from $\hat{H}_{\text{EAHM}}^{(b)}$, maps to an effective XXZ spin-1 chain [18]:

$$\begin{aligned} \hat{H}_{\text{eff}} = & -t \sum_j (\hat{S}_j^+ \hat{S}_{j+1}^- + \text{H.c.}) + \frac{U}{2} \sum_j (\hat{S}_j^z)^2 \\ & + V \sum_j \hat{S}_j^z \hat{S}_{j+1}^z \end{aligned} \quad (6)$$

with the pseudospin operator $\hat{S}_j^z = \hat{n}_j - 1$. Here, we have neglected terms that break the particle-hole symmetry. We note the negative sign of the first term compared to the usual XXZ spin-chain Hamiltonian. This leads to a protecting modified inversion symmetry \mathcal{I}' for the Haldane state of the EBHM [16]:

$$\mathcal{I}' = e^{i\pi \sum_j \hat{S}_j^z} \mathcal{I} = e^{i\pi \sum_j (\hat{n}_j - 1)} \mathcal{I}. \quad (7)$$

Owing to the occupation-dependent hopping in Eq. (5) the HI phase in the EAHM seems not be protected by the modified inversion symmetry \mathcal{I}' .

To clarify whether \hat{H}_t is invariant under certain symmetry operations, let us first consider the inversion symmetry operator \mathcal{I} , acting on $\hat{H}_t \rightarrow \hat{H}_t' = \mathcal{I} \hat{H}_t \mathcal{I}^\dagger$ with

$$\hat{H}_t' = -t \sum_j (\hat{b}_{j+1}^\dagger \hat{b}_j e^{i\theta \hat{n}_{j+1}} + e^{-i\theta \hat{n}_{j+1}} \hat{b}_j^\dagger \hat{b}_{j+1}). \quad (8)$$

Applying next a time-reversal transformation \mathcal{T} , $\hat{H}_t' \rightarrow \hat{H}_t'' = \mathcal{T} \hat{H}_t' \mathcal{T}^{-1}$, we obtain

$$\hat{H}_t'' = -t \sum_j (\hat{b}_{j+1}^\dagger \hat{b}_j e^{-i\theta \hat{n}_{j+1}} + e^{i\theta \hat{n}_{j+1}} \hat{b}_j^\dagger \hat{b}_{j+1}). \quad (9)$$

To see that \hat{H}_t stays invariant under the combined symmetry operations, we make the following transformation:

$$\hat{b}_j^\dagger \rightarrow e^{i\theta \hat{n}_j (\hat{n}_j - 1)/2} \hat{b}_j^\dagger e^{-i\theta \hat{n}_j (\hat{n}_j - 1)/2} = \hat{b}_j^\dagger e^{i\theta \hat{n}_j}, \quad (10)$$

$$\hat{b}_j \rightarrow e^{i\theta \hat{n}_j (\hat{n}_j - 1)/2} \hat{b}_j e^{-i\theta \hat{n}_j (\hat{n}_j - 1)/2} = e^{-i\theta \hat{n}_j} \hat{b}_j. \quad (11)$$

Since the second term of Eq. (9) transforms as $e^{i\theta \hat{n}_{j+1}} \hat{b}_{j+1}^\dagger \hat{b}_j \rightarrow \hat{b}_{j+1}^\dagger \hat{b}_j e^{i\theta \hat{n}_j}$, it is equal to the first term of \hat{H}_t . Therefore, the Hamiltonian $\hat{H}_{\text{EAHM}}^{(b)}$ is invariant under the transformation

$$\mathcal{K} = e^{i\theta \sum_j \hat{n}_j (\hat{n}_j - 1)/2} \mathcal{I} \mathcal{T}. \quad (12)$$

We now show that the combination of $\mathcal{R}^z = e^{i\pi \sum_j \hat{S}_j^z} = e^{i\pi \sum_j (\hat{n}_j - 1)}$ and \mathcal{K} is related to an SPT phase in the EAHM, and define a corresponding topological order parameter. Following Ref. [24], we use the iMPS representation formed by complex $\chi \times \chi$ matrices Γ_σ and a positive, real, diagonal matrix Λ :

$$|\psi\rangle = \sum_{\dots \sigma_j, \sigma_{j+1} \dots} \dots \Lambda \Gamma_{\sigma_j} \Lambda \Gamma_{\sigma_{j+1}} \dots |\dots, \sigma_j, \sigma_{j+1}, \dots\rangle, \quad (13)$$

where the index σ labels the basis states of the local Hilbert spaces. The iMPS is assumed to be in the canonical form: $\sum_\sigma \Gamma_\sigma \Lambda^2 \Gamma_\sigma^\dagger = \sum_\sigma \Gamma_\sigma^\dagger \Lambda^2 \Gamma_\sigma = \mathbb{1}$. If a state $|\psi\rangle$ is invariant under an internal symmetry that is represented by a unitary matrix $\Sigma_{\sigma\sigma'}$, then the transformed Γ_σ matrices satisfy [16,25]

$$\sum_{\sigma'} \Sigma_{\sigma\sigma'} \Gamma_{\sigma'} = e^{i\varphi} U^\dagger \Gamma_\sigma U, \quad (14)$$

where U is a unitary matrix that commutes with Λ matrices, and $e^{i\varphi}$ is a phase factor. Similar relations hold for time reversal symmetry, inversion symmetry, and a combination of both. In those cases Γ_σ on the left-hand side is replaced by its complex conjugate Γ_σ^* , its transpose Γ_σ^T , and its Hermitian transpose Γ_σ^\dagger , respectively. The properties of the matrices U can be used to classify SPT phases [16,26]. For instance, in the case of time reversal or (modified) inversion symmetry the matrices satisfy $U_T U_T^* = \pm \mathbb{1}$ or $U_{\mathcal{I}^{(v)}} U_{\mathcal{I}^{(v)}}^* = \pm \mathbb{1}$, and the sign distinguishes between two symmetric phases. In the EAHM, the situation is slightly different because time reversal and inversion are not symmetries of the system; only a combination \mathcal{K} of them is. For \mathcal{R}^z and \mathcal{K} we have $U_{\mathcal{R}^z}^2 = e^{i\alpha_{\mathcal{R}^z}} \mathbb{1}$ and $U_{\mathcal{K}}^2 = e^{i\alpha_{\mathcal{K}}} \mathbb{1}$. From this we can derive an SPT order parameter similar to the case of the $Z_2 \times Z_2$ spin rotation symmetry of \mathcal{R}^z and \mathcal{R}^x in the spin-1 XXZ chain [16]. Since the phase factors $e^{i\alpha_{\mathcal{R}^z}}$ and $e^{i\alpha_{\mathcal{K}}}$ can be removed by absorbing them into the corresponding matrices $U_{\mathcal{R}^z}$ and $U_{\mathcal{K}}$ they have no physical meaning. However, if both \mathcal{R}^z and \mathcal{K} are preserved, the combination $\mathcal{R}^z \mathcal{K}$ is a symmetry as well and its phase factor is not arbitrary if $U_{\mathcal{R}^z}$ and $U_{\mathcal{K}}$ have been fixed. Indeed, one can show that $U_{\mathcal{R}^z} U_{\mathcal{K}} = \pm U_{\mathcal{K}} U_{\mathcal{R}^z}$, which defines two different phases. To verify that the EAHM has a nontrivial topological phase protected by \mathcal{R}^z and \mathcal{K} , we calculate the order parameter [20]

$$O = \frac{1}{\chi} \text{tr}(U_{\mathcal{K}} U_{\mathcal{R}^z} U_{\mathcal{K}}^\dagger U_{\mathcal{R}^z}^\dagger), \quad (15)$$

if the state is symmetric under both \mathcal{K} and \mathcal{R}^z . Otherwise, if one of the symmetries is broken, the order parameter is zero.

The iDMRG results for the order parameter are shown in Fig. 1. If $U_{\mathcal{K}}$ and $U_{\mathcal{R}^z}$ commute ($O = 1$), the system is in a trivial phase, i.e., a site-factorizable MI state, whereas if they anticommute ($O = -1$), the system realizes a nontrivial HI state. Since the order parameter O changes its sign only if a phase transition takes place, the HI is a well-defined phase of the EAHM. Increasing the number of particles per site n_p at fixed $U/t = 5$, the HI phase ($O = -1$) slightly shifts to larger value of V/t but, most notably, the Haldane phase still occupies a solid parameter region, see the data for $n_p = 3$ and 5 in Fig. 1(a). Increasing the fractional angle θ for $n_p = 2$, the Haldane state region narrows [see Fig. 1(b) for $\theta = \pi/2$] and disappears (at least) for $\theta = \pi$ [33]. We would like to emphasize that the HI sector marked in Fig. 1 by the gray area agrees with that extracted from the correlation length, the entanglement spectrum, and the numerically obtained central charge [27].

Figure 2 represents the ground-state phase diagram of the 1D EAHM in the V - U plane, as obtained from large-scale

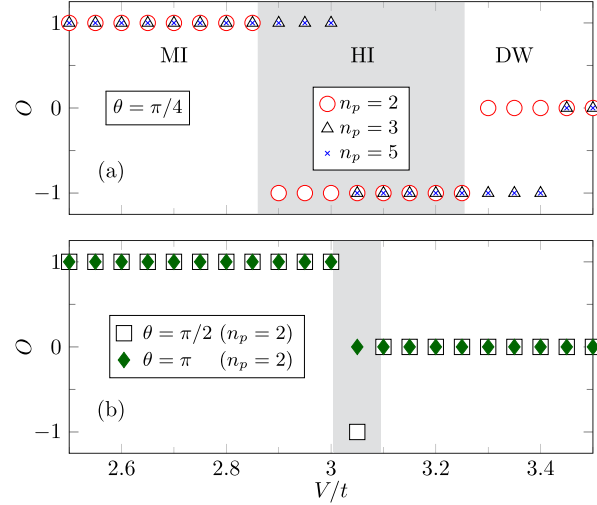


FIG. 1. Order parameter O , defined by Eq. (15), selecting the topological state in the EAHM at fixed $U/t = 5$ and $\theta = \pi/4$ for different n_p (a), and at fixed $\theta = \pi/2$ and π for $n_p = 2$ (b). Data obtained by iDMRG calculations with a (relatively small) bond dimension $\chi = 100$.

iDMRG calculations for $\theta = \pi/4$ and $n_p = 2$. The phase boundaries are determined simulating the order parameter O , as well as the correlation length and the entanglement spectrum [27]. The EAHM exhibits three different

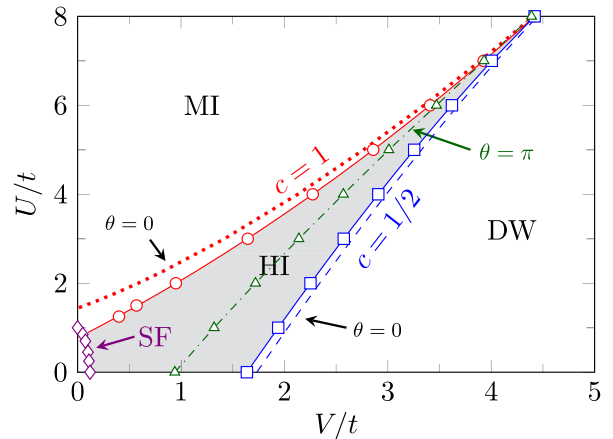


FIG. 2. Ground-state phase diagram of the extended anyon-Hubbard model in one dimension, where the particle density $\rho = 1$, $n_p = 2$, and $\theta = \pi/4$. Most notably the Haldane insulator (HI), located between Mott insulator (MI) and density wave (DW) insulating phases in the EBHM, survives for any $\theta > 0$, i.e., in the anyonic case. Likewise, the superfluid (SF) appears in the very weak-coupling regime. The MI-HI (squares) and HI-DW (circles) transition points can be determined by a divergent correlation length ξ_χ as χ increases; i.e., the model becomes critical with the central charge $c = 1$ and $c = 1/2$, respectively (see Ref. [27]). For comparison, the dotted (dashed) line marks the MI-HI (HI-DW) transition in the EBHM ($\theta = 0$) [34]. The dash-dotted line with triangles up denotes the first-order MI-DW phase transition for $\theta = \pi$.

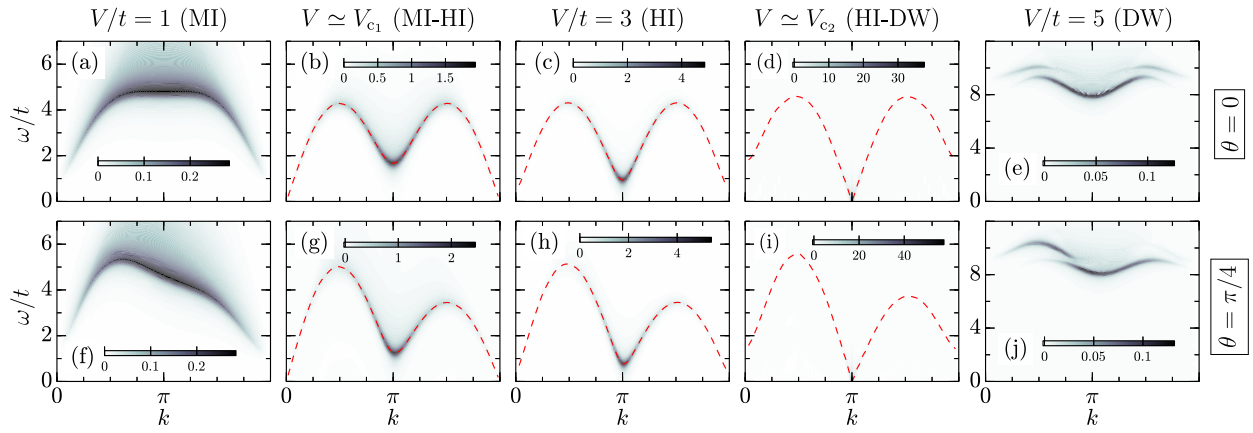


FIG. 3. Intensity plots of the dynamical structure factor $S(k, \omega)$ in the EBHM ($\theta = 0$, upper panels) and in the EAHM ($\theta = \pi/4$, lower panels) for characteristic values of V/t at fixed $U/t = 5$. Again, the maximum number of particles per site is limited to $n_p = 2$. Dashed lines in panels (b)–(d) and (g)–(i) mark the highest intensity of $S(k, \omega)$ in the k - ω plane.

insulating phases (MI, DW, and HI) and a superfluid state in the weak interaction regime, just as for the EBHM [34] but with the addition that the region of the intervening anyonic HI phase at $\theta = \pi/4$ is slightly reduced. The HI vanishes in the pseudofermionic case ($\theta = \pi$). According to field theory for the EBHM [18,35], which is based on the bosonization procedure developed for integer-spin chains [36,37], the MI-HI and HI-DW quantum phase transitions belong to the universality class of Tomonaga-Luttinger liquid and Ising model, with central charge $c = 1$ and $1/2$, respectively, see Fig. S1(c) in Ref. [27]. That is, the universality classes are not modified by the fractional angle.

Perhaps the most striking feature of the AHM is the asymmetry of the momentum distribution function in k space [13,14]. The position of the maximum strongly depends on the fractional phase θ [recall that the momentum distribution diverges at $k = 0$ in the BHM ($\theta = 0$)]. We expect that this asymmetry can also be observed in dynamical quantities such as the dynamical structure factor $S(k, \omega)$. Hence, if an anyonic system will be realized in optical lattices, $S(k, \omega)$ might be one of the best physical quantities to look at, comparing theoretical predictions with real experiments, like for 1D Bose–Hubbard type models [38]. $S(k, \omega)$ should be easily accessible by momentum resolved Bragg spectroscopy [39]. Furthermore, it has been recently demonstrated that $S(k, \omega)$ can also be used to distinguish the topological HI from the conventional MI and DW states [34,40], in analogy to exploiting the dynamical spin-spin structure factor in the spin-1 XXZ chain [41].

The dynamical density structure factor is defined as

$$S(k, \omega) = \sum_n |\langle \psi_n | \hat{n}_k | \psi_0 \rangle|^2 \delta(\omega - \omega_n), \quad (16)$$

where $|\psi_0\rangle$ ($|\psi_n\rangle$) denotes the ground (n th excited) state, and $\omega_n = E_n - E_0$. To compute this quantity, we follow Ref. [42] and first determine the two-point correlation

function $\langle \psi_0 | \hat{n}_j(\tau) \hat{n}_0(0) | \psi_0 \rangle$ by real-time evolution of the iMPS $|\psi_0\rangle$. Fourier transformation then provides us with accurate numerical results of the dynamical structure factor in the EAHM.

Figure 3 compares the intensity of the dynamical wave-vector–resolved density response in the EBHM ($\theta = 0$) with those in the EAHM for $\theta = \pi/4$, for $U/t = 5$, at five characteristic V/t values. One point worthy of remark is that each of the phases and phase transitions can be distinguished by looking at $S(k, \omega)$. In the MI, at $V = t$ [Figs. 3(a) and 3(f)], the excitation gap appears at $k \approx 0$. With increasing V/t , the MI-HI transition occurs at $V \approx V_{c1}$, where the excitation gap closes at $k = 0$, as shown in Figs. 3(b) and 3(g). Deep in the HI phase, $V = 3t$ [Figs. 3(c) and 3(h)], the spectral weight exclusively concentrates at $k \approx \pi$, and there are finite excitation gaps at $k = 0$ and π . It is of particular interest to see whether the gap $S(k, \omega)$ closes at the HI-DW transition point. Indeed, the excitation gap at $V = V_{c2}$ closes, but at momentum $k = \pi$, reflecting the lattice-period doubling in the DW phase. Moreover, in the DW phase [Figs. 3(e) and 3(j)], we find a large excitation gap at $k = \pi$ and two dispersive branches, where a changeover of the intensity maximum occurs at $k = \pi/2$ ($k = 3\pi/4$) for $\theta = 0$ ($\theta = \pi/4$). Interestingly, the influence of the occupation-dependent phase of \hat{H}_i in Eq. (5) shows up in $S(k, \omega)$ as well, which helps to differentiate the results from those of the EBHM. $S(k, \omega)$ of the EAHM is asymmetric for any $0 < \theta < \pi$, while $S(k, \omega)$ in the EBHM is always symmetric about $k = \pi$.

To summarize, we carried out an unbiased numerical investigation of the extended anyon-Hubbard model in one dimension and determined its ground-state phase diagram with high precision exploiting the behavior of correlation lengths and entanglement spectra. Defining an order parameter that distinguishes trivial and nontrivial topological phases, we were able to show that the EAHM possesses

an anyonic Haldane insulator state sandwiched between superfluid, Mott insulator, and density-wave phases. Both the HI-MI and HI-DW quantum phase transitions are critical with central charge 1 and $1/2$, respectively. While the HI state survives the EBHM limit ($\theta = 0$), it vanishes when the system is composed of pseudofermions ($\theta = \pi$). If a 1D interacting anyonic system could be realized experimentally in the future, maybe in an optical-lattice setup with ultracold atoms, we suggest performing momentum-resolved Bragg spectroscopy to look for the pronounced asymmetry of the density response spectra in momentum space that we have demonstrated in our model calculation theoretically.

The iDMRG simulations were performed using the ITensor library [43]. This work was supported by Deutsche Forschungsgemeinschaft (Germany), SFB 652, Project No. B5.

-
- [1] J. M. Leinaas and J. Myrheim, *Nuovo Cimento B* **37**, 1 (1977).
- [2] F. Wilczek, *Phys. Rev. Lett.* **49**, 957 (1982).
- [3] D. C. Tsui, H. L. Stormer, and A. C. Gossard, *Phys. Rev. Lett.* **48**, 1559 (1982).
- [4] R. B. Laughlin, *Phys. Rev. Lett.* **50**, 1395 (1983).
- [5] F. D. M. Haldane, *Phys. Rev. Lett.* **67**, 937 (1991).
- [6] I. Bloch, J. Dalibard, and W. Zwerger, *Rev. Mod. Phys.* **80**, 885 (2008).
- [7] T. Keilmann, S. Lanzmich, I. McCulloch, and M. Roncaglia, *Nat. Commun.* **2**, 361 (2011).
- [8] S. Greschner and L. Santos, *Phys. Rev. Lett.* **115**, 053002 (2015).
- [9] C. Sträter, S. C. L. Srivastava, and A. Eckardt, *Phys. Rev. Lett.* **117**, 205303 (2016).
- [10] A. Kundu, *Phys. Rev. Lett.* **83**, 1275 (1999).
- [11] W. DeGottardi, *Phys. Rev. B* **90**, 075129 (2014).
- [12] J. Arcila-Forero, R. Franco, and J. Silva-Valencia, *Phys. Rev. A* **94**, 013611 (2016).
- [13] Y. Hao, Y. Zhang, and S. Chen, *Phys. Rev. A* **79**, 043633 (2009).
- [14] G. Tang, S. Eggert, and A. Pelster, *New J. Phys.* **17**, 123016 (2015).
- [15] Z.-C. Gu and X.-G. Wen, *Phys. Rev. B* **80**, 155131 (2009).
- [16] F. Pollmann, A. M. Turner, E. Berg, and M. Oshikawa, *Phys. Rev. B* **81**, 064439 (2010).
- [17] E. G. D. Torre, E. Berg, and E. Altman, *Phys. Rev. Lett.* **97**, 260401 (2006).
- [18] E. Berg, E. G. D. Torre, T. Giamarchi, and E. Altman, *Phys. Rev. B* **77**, 245119 (2008).
- [19] F. Wilczek, *Fractional Statistics and Anyon Superconductivity* (World Scientific, Singapore, 1990).
- [20] F. Pollmann and A. M. Turner, *Phys. Rev. B* **86**, 125441 (2012).
- [21] S. R. White, *Phys. Rev. Lett.* **69**, 2863 (1992).
- [22] U. Schollwöck, *Ann. Phys. (Berlin)* **326**, 96 (2011).
- [23] I. P. McCulloch, [arXiv:0804.2509](https://arxiv.org/abs/0804.2509).
- [24] G. Vidal, *Phys. Rev. Lett.* **98**, 070201 (2007).
- [25] D. Pérez-García, M. M. Wolf, M. Sanz, F. Verstraete, and J. I. Cirac, *Phys. Rev. Lett.* **100**, 167202 (2008).
- [26] X. Chen, Z.-C. Gu, and X.-G. Wen, *Phys. Rev. B* **84**, 235128 (2011).
- [27] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevLett.118.120401>, which includes Refs. [28–32], for further quantities to determine and characterize the phase boundaries and quantum phase transitions.
- [28] H. Li and F. D. M. Haldane, *Phys. Rev. Lett.* **101**, 010504 (2008).
- [29] I. Affleck, T. Kennedy, E. H. Lieb, and H. Tasaki, *Phys. Rev. Lett.* **59**, 799 (1987).
- [30] F. Verstraete, M. A. Martín-Delgado, and J. I. Cirac, *Phys. Rev. Lett.* **92**, 087201 (2004).
- [31] S. Nishimoto, *Phys. Rev. B* **84**, 195108 (2011).
- [32] S. Ejima, H. Fehske, F. Gebhard, K. zu Münster, M. Knap, E. Arrigoni, and W. von der Linden, *Phys. Rev. A* **85**, 053644 (2012).
- [33] Note that our iDMRG scheme holds for arbitrary θ . Additional iDMRG calculations confirm that the HI phase exists not only for the specific fractional angles θ used in Fig. 1 but also for other values of $\theta \in (0, \pi)$ (not shown). The critical θ_c , where the HI phase disappears, depends on the model parameters U , V , and n_p .
- [34] S. Ejima, F. Lange, and H. Fehske, *Phys. Rev. Lett.* **113**, 020401 (2014).
- [35] Y.-W. Lee, Y.-L. Lee, and M.-F. Yang, *Phys. Rev. B* **76**, 075117 (2007).
- [36] H. J. Schulz, *Phys. Rev. B* **34**, 6372 (1986).
- [37] A. M. Tsvelik, *Phys. Rev. B* **42**, 10499 (1990).
- [38] D. Clément, N. Fabbri, L. Fallani, C. Fort, and M. Inguscio, *Phys. Rev. Lett.* **102**, 155301 (2009).
- [39] P. T. Ernst, S. Götze, J. S. Krauser, K. Pyka, D.-S. Lühmann, D. Pfannkuche, and K. Sengstock, *Nat. Phys.* **6**, 56 (2010).
- [40] S. Ejima and H. Fehske, *J. Phys. Conf. Ser.* **592**, 012134 (2015).
- [41] S. Ejima and H. Fehske, *Phys. Rev. B* **91**, 045121 (2015).
- [42] H. N. Phien, G. Vidal, and I. P. McCulloch, *Phys. Rev. B* **86**, 245107 (2012).
- [43] <http://itensor.org/>.