Trade-Off Between Speed and Cost in Shortcuts to Adiabaticity

Steve Campbell[1,2,3] and Sebastian Deffner[4]
[1]Centre for Theoretical Atomic, Molecular and Optical Physics, Queen's University Belfast, Belfast BT7 1NN, United Kingdom
[2]Dipartimento di Fisica, Università degli Studi di Milano, Via Celoria 16, 20133 Milan, Italy
[3]Istituto Nazionale di Fisica Nucleare, Sezione di Milano, Via Celoria 16, 20133 Milan, Italy
[4]Department of Physics, University of Maryland Baltimore County, Baltimore, Maryland 21250, USA
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Achieving effectively adiabatic dynamics is a ubiquitous goal in almost all areas of quantum physics. Here, we study the speed with which a quantum system can be driven when employing transitionless quantum driving. As a main result, we establish a rigorous link between this speed, the quantum speed limit, and the (energetic) cost of implementing such a shortcut to adiabaticity. Interestingly, this link elucidates a trade-off between speed and cost, namely, that instantaneous manipulation is impossible as it requires an infinite cost. These findings are illustrated for two experimentally relevant systems—the parametric oscillator and the Landau-Zener model—which reveal that the spectral gap governs the quantum speed limit as well as the cost for realizing the shortcut.

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A popular saying states that "there ain't no such thing as a free lunch." Although quite casually formulated, this phrase expresses nothing less but the gist of the second law of thermodynamics, namely, that nonideal processes are always accompanied by the irreversible expense of a thermodynamic resource. Nevertheless, recent research in quantum control and quantum thermodynamics has seen growing popularity of so-called "shortcuts to adiabaticity," i.e., fast processes with the same outcome as an ideal, infinitely slow process [1]. Such shortcuts are fast processes with suppressed nonequilibrium excess energy [2,3], and apparently provide means to circumvent the second law in isolated systems [4–6]. Thus, a variety of techniques have been developed: using dynamical invariants [7], inversion of scaling laws [8], the fast-forward technique for Schrödinger [9–14] and Dirac dynamics [15], transitionless quantum driving [16–19], classical dissipationless driving [20,21], optimal protocols from optimal control theory [22–28], optimal driving from properties of the quantum work statistics [29], "environment" assisted methods [30], using the properties of Lie algebras [31], and approximate methods such as linear response theory [5] and fast quasistatic dynamics [32].

Among this plethora of techniques transitionless quantum driving (TQD) is unique. In its original formulation [16–18] one considers a time-dependent Hamiltonian $H_0(t)$ and constructs an additional counterdiabatic field $H_1(t)$, such that the joint Hamiltonian $H(t) = H_0(t) + H_1(t)$ drives the dynamics precisely through the adiabatic manifold of $H_0(t)$. Moreover, $H_1(t)$ vanishes by construction in the beginning, $t = 0$, and the end, $t = \tau$, of the finite time process. Thus, if only considering the energy balance $\langle H(\tau)\rangle - \langle H(0)\rangle = \langle H_0(\tau)\rangle - \langle H_0(0)\rangle$, implementing such a shortcut to adiabaticity appears to be thermodynamically free [33]. Even more dramatically, it seems that such an energetically free shortcut to adiabaticity could be implemented for any arbitrarily fast process of arbitrarily short duration τ.

That this almost naive interpretation of TQD cannot be entirely sound has been formalized recently in Ref. [34] where a family of cost functionals is introduced. They are given by the time averaged norm of the counterdiabatic field, $C_t^n = \nu_{t,n} \int_0^\tau dt \|H_1(t)\|^n$, where $\nu_{t,n}$ is a setup dependent constant and the index of the norm n depends on the nature of the applied fields (see Ref. [34] for a more detailed discussion). Here, we show that the norm plays the most crucial role in defining the cost of driving. Therefore, we remove any setup dependence by assuming that $\nu_{t,n} = n = 1$. Although insightful, defining a cost *ad hoc* is not entirely satisfactory. In particular, it is not immediately clear how C_t^n corresponds to expended resources. Moreover, the definition of C_t^n also does not address the rather unsettling impression that TQD could be performed in arbitrarily short times τ. In this Letter we resolve both issues by showing that a cost function that depends on the norm of the counterdiabatic term is intimately related to the maximal speed of the evolution.

It has been established in virtually all areas of quantum physics [35–38] that the Heisenberg uncertainty relation for energy and time sets a quantum speed limit (QSL) [39–43], i.e., a fundamental upper bound on the speed of quantum evolution. These bounds have been extensively studied for isolated [44–50] and open [51–61] systems. It has been shown that the maximal speed of quantum evolution is given by the time averaged norm of the generator of the dynamics [51], which in the case of unitary dynamics and for orthogonal states reduces to the average energy E [48]. Thus, the minimal time during which a quantum system can evolve from the initial to the final state, i.e., the QSL time

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τ_{QSL}, is determined by $\tau_{\mathrm{QSL}} \simeq \hbar/2E$ [48,51]. Since this QSL is a consequence of fundamental properties of quantum dynamics, it also has to apply to quantum processes facilitating shortcuts to adiabaticity. In other words, the QSL prohibits TQD to be performed in arbitrarily short times.

In this Letter we show that the cost of TQD [34] and the QSL [51] are intimately connected. As our main result we rigorously prove a trade-off between the speed and the thermodynamic cost: the faster one wants to implement a shortcut, the higher is the thermodynamic cost of realizing the quantum process. We will further illustrate that this insight is not only of theoretical and conceptual interest, but also of practical relevance. To this end, we will analyze two experimentally important systems, namely, the parametric harmonic oscillator and the Landau-Zener model. Parametric harmonic oscillators have been shown to be ideal test beds for quantum thermodynamic relations [62–65], which can be easily implemented for instance in ion traps [66–69]. The Landau-Zener model, on the other hand, is closely related to the Ising model [70–72] and hence is instrumental for current technological advancements in quantum annealing [73] such as the D-wave machine [72,74,75].

Preliminaries.—Consider a time-dependent Hamiltonian $H_0(t)$ with instantaneous eigenvalues $\{\varepsilon_n(t)\}$ and eigenstates $\{|n_t\rangle\}$. In the limit of infinitely slow variation of $H_0(t)$, i.e., the adiabatic limit, no transitions between eigenstates occur [76]. Now consider a nonadiabatic parametrization of $H_0(t)$. In this case we can construct a corresponding Hamiltonian $H(t) = H_0(t) + H_1(t)$ such that the adiabatic solution of $H_0(t)$ is an exact solution of the dynamics generated by $H(t)$. It can be shown that [16–18,34]

$$H_1(t) = i\hbar[\partial_t|n_t\rangle\langle n_t|, |n_t\rangle\langle n_t|]. \tag{1}$$

Note that computing the counterdiabatic Hamiltonian $H_1(t)$ requires the instantaneous eigenbasis $|n_t\rangle$. Since finding these time-dependent eigenstates can become arbitrarily complicated, hybrid methods have been developed utilizing tools from optimal control theory [25–28].

In Ref. [34] a family of functionals has been proposed to quantify the cost associated with implementing $H_1(t)$. The simplest member of the family is given by the trace norm $\|\cdot\|$ [34,77–79].

$$C_t^1 \equiv C = \int_0^\tau dt \|H_1(t)\| \tag{2}$$

with $\nu_{t,1} = 1$. It is easy to see that for a single two-level spin, $\partial_t C$ is proportional to the average power input [34]; i.e., $H_1(t)$ reduces to an orthogonal, magnetic field. More generally, C can be interpreted as the additional action arising from the counterdiabatic driving. Hence, the relation to the QSL becomes apparent, since loosely speaking the QSL sets a lower bound on the action $E\tau_{\mathrm{QSL}} \simeq \hbar/2$ [48].

The QSL is a fundamental upper bound on the rate with which a quantum state can evolve. For our present purposes we are interested in the evolution of pure states under the time-dependent Schrödinger equation $i\hbar\partial_t|\psi_t\rangle = H(t)|\psi_t\rangle$. It has been shown that in this case the maximal rate of change of the angle between the initial and time-evolved state $\mathcal{L}_t = \arccos|\langle\psi_0|\psi_t\rangle|$ is given by

$$\partial_t\mathcal{L}_t \leq v_{\mathrm{QSL}} \equiv \frac{|\epsilon_t|}{\hbar\cos(\mathcal{L}_t)\sin(\mathcal{L}_t)}, \tag{3}$$

where $\epsilon_t = \|H(t)|\psi_t><\psi_t|\|$ [51]. From this maximal quantum speed one easily obtains the QSL time [51]

$$t \geq \tau_{\mathrm{QSL}} \equiv \frac{\hbar}{2E_\tau}[\sin(\mathcal{L}_\tau)]^2, \tag{4}$$

where E_τ is the time-averaged norm of the energy, $E_\tau = 1/\tau\int_0^\tau dt|\epsilon_t|$, and τ is the length of the driving protocol. Note that Eq. (4) is an expression of the Heisenberg uncertainty principle of energy and time for time-dependent, driven quantum systems [48].

Trade-off between speed and cost.—It is easy to see that, in the case of TQD, the instantaneous cost, i.e., the trace norm of the counterdiabatic Hamiltonian $H_1(t)$, reduces to

$$\partial_t C = \|H_1(t)\| = \sqrt{\langle\partial_t n_t|\partial_t n_t\rangle}, \tag{5}$$

where we used that $\langle\partial_t n_t|n_t\rangle = 0$, which is true for all Hamiltonians with an entirely discrete eigenvalue spectrum. By further noting that $|\psi_t\rangle = |n_t\rangle$ and $H(t) = H_0(t) + H_1(t)$ with $H_1(t)$ as given in Eq. (1) ϵ_t simply becomes

$$\epsilon_t = \sqrt{\varepsilon_n^2(t) + \langle\partial_t n_t|\partial_t n_t\rangle}, \tag{6}$$

where we employed again $\langle\partial_t n_t|n_t\rangle = 0$.

Substituting Eqs. (5) and (6) into Eq. (3) we obtain the maximal speed with which a quantum state can undergo transitionless quantum driving

$$v_{\mathrm{QSL}} = \frac{\sqrt{\varepsilon_n^2(t) + (\partial_t C)^2}}{\hbar\cos(\mathcal{L}_t)\sin(\mathcal{L}_t)}, \tag{7}$$

and the QSL time becomes

$$\tau_{\mathrm{QSL}} = \frac{\hbar\tau[\sin(\mathcal{L}_\tau)]^2}{2\int_0^\tau dt\sqrt{\varepsilon_n^2(t) + (\partial_t C)^2}}. \tag{8}$$

The latter two equations constitute our main results. First, we have related the cost for TQD introduced in Ref. [34] to one of the most fundamental results in modern quantum physics—the Heisenberg uncertainty principle for energy and time. Second, Eqs. (7) and (8) express, in a transparent and immediate way, the trade-off between speed and cost of a shortcut to adiabaticity. In particular, Eq. (7) shows that the faster a quantum system evolves along its adiabatic manifold, the higher is the cost of implementing the shortcut [80]. Equation (8) states that the shorter the time is during which a quantum system is driven from the initial to the final energy eigenstate, the more thermodynamic resources have to be expended [81]. The remainder of this analysis is dedicated to two experimentally relevant case studies, which illustrate this trade-off for practical applications.

Case study 1: harmonic oscillator.—The "unperturbed" Hamiltonian of the parametric harmonic oscillator reads

$$H_0(t) = \frac{p}{2m} + \frac{1}{2} m\omega_t^2 x^2. \tag{9}$$

For the sake of simplicity we only consider situations in which the system is initially prepared in its ground state

$$\psi_0(x) = \left(\frac{m\omega_0}{\pi\hbar}\right)^{1/4} \exp\left(-\frac{m\omega_0 x^2}{2\hbar}\right), \tag{10}$$

where the corresponding energy eigenvalue is $\varepsilon_0(0) = \hbar\omega_0/2$. A straightforward calculation then reveals that the cost of keeping the oscillator in its instantaneous ground state at all times is

$$\partial_t C = \left|\frac{\partial_t \omega_t}{\sqrt{8}\omega_t}\right|, \tag{11}$$

and the maximal quantum speed reads

$$v_{\mathrm{QSL}} = \frac{\sqrt{(\hbar\omega_t/2)^2 + (\partial_t\omega_t/\sqrt{8}\omega_t)^2}}{\hbar\cos(\mathcal{L}_t)\sin(\mathcal{L}_t)}. \tag{12}$$

Finally, the instantaneous angle can be written as

$$\mathcal{L}_t = \arccos\left(\sqrt{2\sqrt{\omega_0\omega_t}/(\omega_0+\omega_t)}\right). \tag{13}$$

Note that the maximal speed of quantum evolution v_{QSL} is fully determined by the parametrization of the angular frequency ω_t. Therefore, v_{QSL} fully characterizes the dynamics, and we can analyze the quantum process without having to solve the time-dependent Schrödinger equation.

In Figs. 1 we examine the case of a compression [Fig. 1(a)] and an expansion [Fig. 1(b)] using the simple linear ramp $\omega_t = \omega_0 + \omega_d(t/\tau)$. Interestingly, we see that generally the QSL time is significantly larger for the expansion. Nevertheless, we also observe that using the shortcut can bring the QSL time to arbitrarily small values. However, as evidenced in the insets, a smaller τ corresponds to a diverging instantaneous cost. Remarkably, v_{QSL} and $\partial_t C$ exhibit qualitatively opposite behaviors for the two protocols. For the compression, we see that v_{QSL} tends to increase as we decrease the total evolution time τ. However, all curves collapse on top of one another toward the end of the protocol. Conversely, in the case of an expansion the speeds diverge as we evolve. The instantaneous cost qualitatively behaves in the same way.

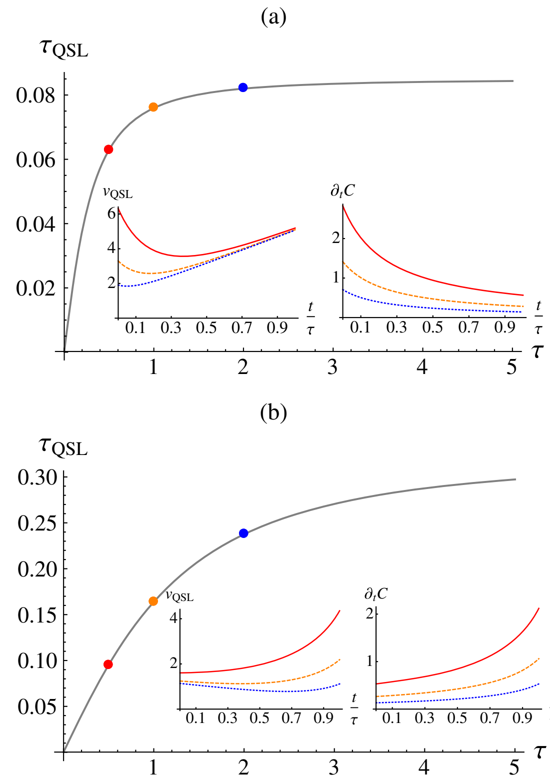

FIG. 1. We consider the harmonic oscillator with time-dependent frequency $\omega_t = \omega_0 + \omega_d(t/\tau)$. Main panels: QSL time. The points correspond to the QSL times considered in the insets. Insets: maximal speed v_{QSL}, and instantaneous cost $\partial_t C$, for $\tau = 0.5$ (solid), 1 (dashed), and 2 (dotted). (a) A compression with $\omega_0 = 1$ and $\omega_d = 4$. (b) An expansion with $\omega_0 = 1$ and $\omega_d = -0.75$.

This behavior is due to the effect that these protocols have on the energy spectrum. In the case of a compression, the energy levels become more spaced and therefore the first excited state becomes progressively harder to reach. The larger gap then means that we can drive the system comparatively faster without exciting it and the associated cost of achieving this dynamics decreases. In the case of an expansion the energy spacing decreases. Therefore, to avoid excitations, the system must be driven more slowly. Achieving this dynamics using a shortcut is then necessarily accompanied by an increasingly larger cost.

The latter interpretation is further supported by considering Fermi's "golden rule" for time-dependent perturbation theory [76]. This rule states that the rate of quantum transitions is determined by the time-integrated magnitude of the perturbation. In TQD we seek to suppress these transitions. This means that larger gaps have a lower probability of observing a transition in the unperturbed dynamics compared to smaller gaps. Hence, it is also "cheaper" to suppress transitions in processes with larger gaps than in denser energy spectra.

Clearly, the gap between the driven state and the rest of the spectrum plays the most crucial role in determining both the QSL and the cost of achieving finite time adiabatic dynamics. Such an observation is of particular relevance in critical many-body systems, where quantum phase transitions often occur at avoided crossings in the spectrum. In the following, we examine the avoided crossing (AC) in the Landau-Zener (LZ) model, which serves to elucidate all the relevant features of driving the many-body Ising model through its critical point [71] and is also relevant to the Lipkin-Meshkov-Glick model [25].

Case study 2: Landau-Zener model.—Consider the Hamiltonian

$$\mathcal{H}_{\mathrm{LZ}} = \Delta\sigma_x + g(t)\sigma_z, \tag{14}$$

where Δ is the energy splitting and $g(t)$ is the time-dependent field. As shown in Ref. [71] the Ising model can be expressed as a series of independent LZ crossings, and therefore the following results extrapolate to driving a critical many-body system. For the sake of clarity we further rescale $\mathcal{H}_{\mathrm{LZ}}$ by Δ, $H_0 = \mathcal{H}_{\mathrm{LZ}}/\Delta$, and hence set the minimal energy gap to 1. The corresponding correction term (1) is readily determined to be [18]

$$H_1 = -\frac{g'(t)\Delta}{2(\Delta^2 + [g(t)]^2)}\sigma_y, \tag{15}$$

which allows us to evaluate Eqs. (5) and (7).

In Fig. 2 we examine the role the energy splitting and the total time plays in setting the maximal speed at which the system can be driven through the AC using the simple linear ramp $g(t) = g_0 + g_d(t/\tau)$ [83]. In Fig. 2(a) we set $\Delta = 0.001$ and consider $\tau = 10^3$ (bottom curve), which is close to the adiabatic limit and therefore $\partial_t C \simeq 0$. We observe that the speed steadily decreases as we approach the AC and cusps at $t = 0.5\tau$. Comparing to the same evolution time for a larger splitting, $\Delta = 0.01$, while the same qualitative behavior is observed we see that the cusp is smoothed out. This has a clear physical interpretation: close to the adiabatic limit we can drive the system at a finite speed far from the AC; however, the vanishingly small gap means as we approach the AC we must drive the system extremely slowly, approaching a speed of zero, in order to avoid the excitations that are more likely to occur. Increasing the splitting allows for an increase in the speed at which we can still evolve the system effectively adiabatically. Physically, this is the same behavior that we found for the harmonic oscillator in Fig. 1.

Achieving the same evolution in shorter times requires the use of the counterdiabatic field (15). In Fig. 2 we see as the system approaches the AC the speed using the counterdiabatic field increases. This behavior can again be understood with the help of Fermi's golden rule for time-dependent perturbations [76]. The transition probabilities for the unperturbed problem are proportional to the time-integrated perturbation. Hence, if the relative magnitude of the perturbation is large, i.e., if the gap is small, transitions

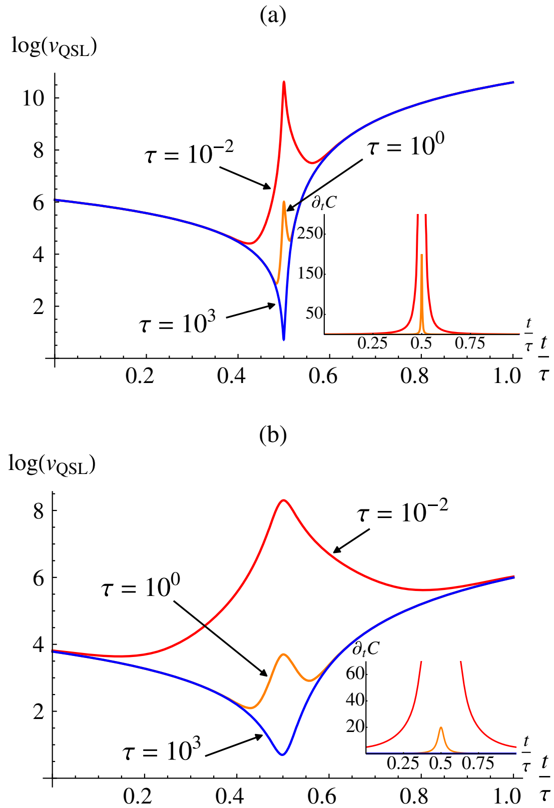

FIG. 2. Maximal quantum speed $\log(v_{\mathrm{QSL}})$ for the LZ model evolved through the AC using the linear ramp $g(t) = 0.2–0.4(t/\tau)$. (a) $\Delta = 0.001$ and (b) $\Delta = 0.01$. Both insets show the corresponding instantaneous cost $\partial_t C$.

can be suppressed if the quantum system is prohibited from lingering at the AC.

A further interesting feature is the clear emergence of a "critical" region, which is delicately dependent on both the splitting and the evolution time. It is clear in both panels that when sufficiently far from the AC, v_{QSL} is largely independent of evolution time and in these regions the instantaneous cost is close to zero. Approaching the AC requires that either the evolution is accordingly slowed down or a counterdiabatic field is used. This behavior is typical for critical systems, and this is also what is at the core of the Kibble-Zurek mechanism [84]. Far away from the critical point the dynamics is essentially adiabatic. However, close to the phase transition the response of the system "freezes out," and the so-called impulse regime emerges. The longer a system lingers in the impulse regime the higher the chances for a transition to occur. Suppressing excitations in the impulse regime, however, is costly, and therefore TQD seeks to rapidly drive the system back into the adiabatic regime.

Concluding remarks.—We have achieved three important results. (i) We have rigorously proven the relationship between the cost of TQD and the QSL. (ii) We have elucidated the trade-off between speed and the cost of a shortcut to adiabaticity. (iii) Finally, by illustrating our general findings with two experimentally relevant systems, we have highlighted the crucial role of the gap for the cost and the speed with which a shortcut can be facilitated. In particular, we have found that effectively instantaneous yet adiabatic dynamics can be achieved at the expense of an infinite cost.

Interestingly, in its original formulation, TQD does not provide any physical intuition as to why it can achieve such fast dynamics. Furthermore, it is reasonable to assume that even using TQD, a small energy gap would require slower driving. Our results show that the shortcut comes from the increased allowed speed of evolution and, quite counterintuitively, TQD encourages faster driving when the energy gap closes. Such an insight could be highly relevant for experimental implementations of TQD [85,86].

Our analysis of the LZ model extrapolates to many critical spin systems, such as the Ising and the LMG model. In any system, for which the gap vanishes as $N \to \infty$, our findings for small Δ qualitatively apply (see also Ref. [87]). Hence, our analysis is particularly important for current efforts in building and improving quantum computing hardware [72,74,75]. Finally, there are two immediate directions for generalizations of our results: open systems [88] and non-Schrödinger dynamics [15]. While we expect such an intuitive trade-off to persist, a reasonable notion of a thermodynamic cost will have to be found first.

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