Speed of Gravitational Waves from Strongly Lensed Gravitational Waves and Electromagnetic Signals

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We propose a new model-independent measurement strategy for the propagation speed of gravitational waves (GWs) based on strongly lensed GWs and their electromagnetic (EM) counterparts. This can be done in two ways: by comparing arrival times of GWs and their EM counterparts and by comparing the time delays between images seen in GWs and their EM counterparts. The lensed GW-EM event is perhaps the best way to identify an EM counterpart. Conceptually, this method does not rely on any specific theory of massive gravitons or modified gravity. Its differential setting (i.e., measuring the difference between time delays in GW and EM domains) makes it robust against lens modeling details (photons and GWs travel in the same lensing potential) and against internal time delays between GW and EM emission acts. It requires, however, that the theory of gravity is metric and predicts gravitational lensing similar to general relativity. We expect that such a test will become possible in the era of third-generation gravitational-wave detectors, when about 10 lensed GW events would be observed each year. The power of this method is mainly limited by the timing accuracy of the EM counterpart, which for kilonovae is around 10^4 s. This uncertainty can be suppressed by a factor of $\sim 10^{10}$, if strongly lensed transients of much shorter duration associated with the GW event can be identified. Candidates for such short transients include short γ-ray bursts and fast radio bursts.

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Introduction.—Gravitational waves (GWs), which are the transverse waves of spatial strain generated by time variations of the mass quadrupole moment of the source and traveling at the speed of light, were predicted by Albert Einstein in [1]. The first observational evidence for the existence of gravitational waves was made after the discovery of the binary pulsar system PSR 1913 + 16 by Hulse and Taylor [2] and its subsequent follow-up by Taylor and Weisberg [3]. The recent announcement of the first direct detection of gravitational waves (GW150914) by the advanced LIGO detector [4] was a great achievement which opened up a new window on the Universe. Moreover, the first GW signal detected in laboratory came from the merger of two massive black holes, proving the existence of these as-yet speculative binary systems. With GW detectors operating and gathering data, one would also be able to test various aspects of gravitational physics, like the validity of general relativity (GR), in a way unaccessible to other techniques. For example, in alternative theories of gravity, the speed of GWs could be different from the speed of light through the breaking of the weak equivalence principle or the existence of massive gravitons (see the review [5] and references therein). Indeed, the graviton Compton wavelength test has already been performed following the first direct detection of GWs [6] using the dispersion measurement, as well as the Einstein's equivalence principle test [7,8].

Binary neutron stars (NS-NS) are one of the promising sources that can be routinely detected by the ground-based detectors (such as Advanced LIGO and VIRGO) and the third-generation detectors like the Einstein Telescope [9]. What makes them even more interesting is that they are expected to be accompanied by the electromagnetic (EM) counterparts which could be visible as kilonovae or merger-novae (see the review [10] and references therein) with peak r-band magnitude ~22–25 AB magnitude (e.g., [11]). They are transient events of shorter duration (of order of days), similar to supernovae (SNe). Short γ-ray bursts (SGRBs), which have simple and sharp temporal features (of the order of 0.1–1 s), are another very promising EM counterpart of GW from NS-NS and NS–black hole (BH) systems [12]. Because of the jet collimation ~10% of the NS-NS systems will be aligned as to give an observable SGRB. Recently, the fast radio bursts (FRBs) have attracted considerable attention [13]. The origin of FRBs is not known, but they could also be EM counterparts of GW from NS-NS and NS-BH systems [14]. These transients of much shorter duration (of the order of ms) allow us to reach the timing precision ~0.01 ms (e.g., FRB 130628 in [15]). Detailed studies of the EM counterparts of GW signals focused on their properties, rates, and identification strategies are the top research topics in current astrophysics from both the theoretical and observational point of view. Any improvement on the timing accuracy of the EM counterpart in the future will enhance the power of the method proposed in this Letter.

Next-generation GW detectors like the Einstein Telescope will improve sensitivity by an order of magnitude over the Advanced LIGO. This means that probed volume of the Universe will increase by 3 orders of magnitude. Perspectives for observing strongly lensed GWs from merging double compact objects (NS-NS, NS-BH, BH-BH) has been studied in [16,17] with the prediction that Einstein Telescope should be able to detect several tens up to more than hundred of such events per year. This statistics is, however, dominated by BH-BH systems. Although a pure BH-BH merger is not expected to have an EM counterpart, several papers (e.g., [7,18–20]), motivated by the plausible γ-ray transient associated with GW150914 [21], have proposed the formation channels of the EM counterpart in BH-BH merger systems and discussed their applications.

With new generation of dedicated surveys (e.g., SLACS, CASSOWARY, BELLS, and SL2S), strong gravitational lensing has developed into a important technique in extragalactic astronomy (galactic structure studies) and in cosmology. In this phenomenon, a source (typically a quasar or a distant galaxy) lensed by a foreground massive galaxy or cluster appears in multiple images. Light rays of these images travel along paths differing in length and probe gravitational potential of the lens at different depths experiencing different gravitational time delays. These two effects, the geometrical and the Shapiro effect, combine to produce the time delay between images [22]. If the source is intrinsically variable (and most quasars are), the light curves of its images can be used to extract out the time delay [23]. This technique requires high-quality monitoring with sufficient cadence, season, and campaign lengths so that the microlensing effects caused by the stars can be eliminated. Moreover, quite recently the first detection of the gravitationally lensed supernovae has been reported [24], following the work of Resfdal [25]. In the case of lensed transient sources, like SNe, the measurements of time delays between images can be much more accurate. In addition to its typical use to determine the Hubble constant [26], measurements of strong lensing time delays have also been used to constrain the amplitude of the gravitational wave background [27]. The forthcoming Large Synoptic Survey Telescope (LSST) will find about ~8000 lensed quasars, approximately 3000 of which will be monitored and have the well-measured time delays in six frequency bands within ten years. The estimated number of robust time-delay measurements for these is around 400, each with precision $< 3\%$ and accuracy of 1% [28]. The LSST should also find some 130 lensed supernovae during its survey duration, while the deep, space-based supernovae survey done by Joint Dark Energy Mission is expected to find ~15 lensed SNe [29]. Note that, similar to SNe, the proposed isotropic counterparts to NS-NS mergers (e.g., kilonovae) have signals a few days of duration with a limited timing accuracy.

As already mentioned, one of the most important issues to be studied with GW detectors is the testing of the validity of GR, in particular the question whether GW travels with the speed of light. Similar questions arise within the Lorentz-invariance-violating theories, where the dispersion relation for the photon could be modified, thus making the speed of light energy dependent. The observed time of arrival delay between two events, such as the emission of different energy photon-photon [30], photon-neutrino [31], and GW-EM signals [32,33], has been proposed to constraint the respective propagation speed. The unknown intrinsic time delay in the emission time of two such signals to be compared contributes considerably to the uncertainty of the time of arrival method. Concerning the speed of GWs, it was proposed in [34] that using the phase information of the GWs from inspiralling compact binaries estimated by the matched filtering technique, a bound on the graviton mass (and, hence, on the speed of GWs) could by made using GWs alone. However, the expected bounds depend strongly on other physical effects relevant for the particular inspiralling system detected such as spin-induced precessions, orbital eccentricity, higher waveform harmonics, the merger-ringdown phase, etc.

Here, we propose a method to directly constrain the speed of GWs by using the strong lensing time delays measured with GWs and their EM counterparts. The differential setting of our method makes it free from the intrinsic time delays in the source (i.e., the different emission times of GWs and the EM signal). The general idea to use gravitationally lensed signals registered in GW and EM windows was independently proposed by [35]. Our formulation is slightly different from theirs and is supported with more rigorous calculations. Also worth noting is the paper by Takahashi [36], which claims that even within general relativity it is possible for a lensed GW signal to come earlier than EM one (emitted simultaneously) due to wave effects in gravitational lensing (the breakdown of geometric optics approximation). This result does not apply in our case, where we consider galaxies acting as lenses.

Method.—Our method is an extension of the idea proposed by [37] in the context of testing the Lorentz invariance violation by using the energy dependence of time delays in gravitationally lensed systems. Let us assume that we observed a strongly lensed GW signal and identified its electromagnetic counterpart in the optical or radio waves or in γ rays. Then we would be able to measure time delays between the images independently in GW detectors, Δt_{GW}, and in the electromagnetic window, Δt_γ. They will be different if the speed of gravity v_{GW} is different from c. The difference $(\Delta t_\gamma - \Delta t_{\mathrm{GW}})$ will bear information about the speed of GWs. The bound on the v_{GW} will have the following general form valid for a broad set of analytical lens models:

$$1 - \left(\frac{v_{\mathrm{GW}}}{c}\right)^2 \leq \frac{\delta T}{\Delta t_\gamma F_{\mathrm{lens}}(z_l, z_s)}, \qquad (1)$$

where δT is the timing accuracy with which time delays are determined and $F_{\mathrm{lens}}(z_l, z_s) \sim O(1)$ is some factor (weakly) dependent on the lens model and background cosmology (see below). Let us stress that our method is purely empirical: we do not assume any model of massive gravitons (a consistent theory of which is so far nonexistent). We refer to differences in time delays in GW and EM windows, assuming, however, that the theory of gravity is metric and predicts gravitational lensing similar as general relativity. It means we refer to a purely classical, rather than quantum, regime. However, in order to be more specific in calculations we will assume below that gravitons are massive and travel along timelike geodesics. In this sense, the term "graviton" should be perceived as a useful jargon rather than reference to the quantum nature of GWs. There is a wide diversity of possible alternative theories of gravity, not all of which will be well constrained by the method we propose. For some more recent reviews see, e.g., [38] or [39], where the constraining power of observed GW signals has been demonstrated and discussed.

Propagation of massive gravitons on the cosmological background.—The hypothesis that the speed v_{GW} of GWs could be different from c means that gravitons should be treated as massive particles (having the rest mass m_{GW}) moving along timelike geodesics. Therefore their dispersion relation would be

$$E_{\mathrm{GW}}^2 - p_{\mathrm{GW}}^2 c^2 = m_{\mathrm{GW}}^2 c^4 \tag{2}$$

instead of

$$E_\gamma^2 - p_\gamma^2 c^2 = 0, \tag{3}$$

as for the photons. Let us moreover assume that GWs travel along radial geodesics in the flat Friedman-Robertson-Walker (FRW) model with the metric

$$ds^2 = c^2 dt^2 - a(t)^2 [dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2]. \tag{4}$$

Generalization to nonflat FRW would be straightforward. The covariant and contravariant radial components of GW four-momentum are related as

$$p_r = a^2 p^r \tag{5}$$

and, obviously,

$$\frac{dr}{dt} = \frac{p^r c^2}{E} = \frac{p_r c^2}{a^2 E}. \tag{6}$$

Then, it is easy to see that velocity of gravitons is

$$v_{\mathrm{GW}} = \frac{dr}{dt} = \frac{c}{a}\left[1 - \frac{1}{2}\frac{m_{\mathrm{GW}}^2 c^2 a^2}{p_r^2}\right]. \tag{7}$$

If the GW signal was emitted at the moment t_e and detected (observed) at t_0, then the travel distance of the GWs is

$$r_{\mathrm{GW}} = r_\gamma - \Delta r_{\mathrm{GW}}, \tag{8}$$

where

$$r_\gamma = \int_{t_e}^{t_0} \frac{c}{a(t)} dt = c\int_0^z \frac{dz}{H(z)} \tag{9}$$

is the usual comoving distance to the GW source, and

$$\Delta r_{\mathrm{GW}} = \frac{1}{2}\frac{m_{\mathrm{GW}}^2 c^3}{p_r^2}\int_{t_e}^{t_0} a(t) dt. \tag{10}$$

In addition to the expansion rate $H(z)$, we will also use its dimensionless form $h(z)$ defined as $H(z) = H_0 h(z)$, where H_0 is the Hubble constant. Using

$$p_r = a(t_e)\frac{E}{c}, \tag{11}$$

with the notation

$$I_n(z_1, z_2) \coloneqq \int_{z_1}^{z_2} \frac{dz'}{(1+z')^n h(z')}, \tag{12}$$

one has

$$\Delta r_{\mathrm{GW}} = \frac{1}{2}\frac{c}{H_0}\frac{m_{\mathrm{GW}}^2 c^4}{E^2}(1+z)^2 I_2(0, z). \tag{13}$$

The above formulas should be understood in the following way: if the emission time t_e and detection time t_0 are fixed, i.e., the same for the GW and electromagnetic sources, then the GW source is by Δr_{GW} closer than electromagnetic source. On the other hand, if they are emitting from the same location, the GW signal would come by $\Delta t_{\mathrm{GW}} = \Delta r_{\mathrm{GW}}/c$ later than electromagnetic counterpart. In other words, travel time for GWs would be by Δt_{GW} longer, as if the source was located by Δr_{GW} farther away.

Strong lensing time delays.—For the purpose of illustrating our ideas we shall restrict our attention to the singular isothermal sphere (SIS) model, which has been proved to be a useful and reliable phenomenological model of early-type galaxies that dominate the population of lenses. The generalization to singular isothermal ellipsoids and general power-law spherically symmetric mass distribution is rather straightforward and would not change our conclusions.

The Einstein ring radius for the SIS model is

$$\vartheta_E = 4\pi \frac{D_{ls}}{D_s}\frac{\sigma_v^2}{c^2}, \tag{14}$$

where σ_v denotes the one-dimensional velocity dispersion of stars in a lensing galaxy. If the lensing is strong, i.e., the misalignment angle β between the directions to the lens and to the source is $\beta < \vartheta_E$, then two colinear images A and B form on the opposite side of the lens, at radial distances $\vartheta_A = \beta + \vartheta_E$ and $\vartheta_B = \vartheta_E - \beta$. These have time delays between the images

$$\Delta t_{\mathrm{SIS}} = \frac{1+z_l}{2c}\frac{D_l D_s}{D_{ls}}(\vartheta_A^2 - \vartheta_B^2), \tag{15}$$

which, according to Eqs. (9) and (14), can also be written as

$$\Delta t_{\mathrm{SIS}} = \frac{32\pi^2}{H_0}\left(\frac{\sigma}{c}\right)^4 y \frac{\tilde{r}(z_l)\tilde{r}(z_l, z_s)}{\tilde{r}(z_s)}, \tag{16}$$

where $\tilde{r}(z_l)$ denotes the dimensionless (i.e., with c/H_0 factored out) comoving distance to the lens and $y = \beta/\vartheta_E$.

In the context of massive photons, it was shown by [40] that the bending angle is modified by a factor

$1 + (m^2c^4/2E^2)$. These considerations are valid in our case, which means that impact parameters of photons and GWs from the same image are different and the Einstein angle gets modified to $\vartheta_{E,\mathrm{GW}} = \vartheta_E[1 + (m_{\mathrm{GW}}^2 c^4/2E^2)]$. Therefore, while calculating the time delay between images seen in GWs, one has to consider this effect, which affects the Shapiro time delay together with geometrical terms using corrections Eq. (10) in the distances D_{ls} and D_l.

Now, we can see that the difference between image time delays observed in GW detectors and in the electromagnetic domain is

$$\Delta t_{\mathrm{SIS,GW}} - \Delta t_{\mathrm{SIS},\gamma} = \Delta t_{\mathrm{SIS},\gamma}\frac{m_{\mathrm{GW}}^2 c^4}{E^2}F_{\mathrm{lens}}(z_l, z_s), \quad (17)$$

where

$$F_{\mathrm{lens}}(z_l,z_s)=1+\frac{(1+z_s)I_2(0,z_s)}{2\tilde{r}(z_l,z_s)} - \frac{(1+z_l)I_2(0,z_l)}{2\tilde{r}(z_l)} - \frac{(1+z_l)I_2(0,z_l)}{\tilde{r}(z_l,r_s)}. \quad (18)$$

Therefore, we could specify the speed of GWs through Eq. (1) using the information from the lensed GW-EM system in terms of a “graviton,”

$$\frac{m_{\mathrm{GW}}^2 c^4}{E^2} = 1 - \left(\frac{v_{\mathrm{GW}}}{c}\right)^2. \quad (19)$$

If one would be able to measure such a difference in time delays, this would also be a proof that gravitons are massive (i.e., that GR needs to be modified).

The accuracy δT of time-delay measurements sets constraints on the v_{GW}. Assuming the galaxy-galaxy strong lensing system with $z_l = 1$ and $z_s = 2$, one has the following bound coming from the GW-EM difference in lensing time delays:

$$1 - \left(\frac{v_{\mathrm{GW}}}{c}\right)^2 \leq 4.26\times 10^{-10}\left(\frac{\delta T}{1\ \mathrm{ms}}\right)\left(\frac{\sigma}{250\ \mathrm{km/s}}\right)^{-4} \times \left(\frac{y}{0.1}\right)^{-1}, \quad (20)$$

where we also assumed ΛCDM cosmology with the Hubble constant $H_0 = 68\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, $\Omega_m = 0.3$. Numerical factor setting the scale corresponds to lens velocity dispersion $\sigma = 250$ km/s, a timing accuracy of $\delta T = 1$ ms, and source-lens misalignment $y = 0.1$. Recent discovery of the “Refsdal supernova” [24], and especially its reappearance [41], demonstrated that we are starting to discover transient events lensed by a cluster. The cluster-scale images have much bigger time delays than in the case of galaxy-scale lenses. This means that in such a case our method of differences in time delays would be more restrictive. For example, taking the value of the time delay for the Refsdal supernova image SX, which reappeared as predicted, one would get a bound $1 - (v_{\mathrm{GW}}/c)^2 \leq 3.2\times 10^{-11}$ assuming 1-ms timing accuracy.

Apart from the limitation due to the accuracy with which the EM lensing time delay can be measured, the method described above is less restrictive than travel-time techniques because it cannot take advantage of the cumulative effect along the whole path. However, the strong lensing system seen both in EM and GW windows offers the additional possibility to compare the moments of arrival of the same image seen in the EM and GW events, respectively. This would be possible only for transient EM sources like kilonovae or, better yet, SGRBs associated with a GW signal. Then, according to Eq. (10), the expected time delay (in each image) would be

$$\Delta t_{\gamma,\mathrm{GW}} = \frac{1}{2H_0}(1+z_s)^2 I_2(0, z_s). \quad (21)$$

For the source at the redshift $z_s = 2$, one has the following bound: $1 - (v_{\mathrm{GW}}/c)^2 \leq 9.92\times 10^{-22}$.

It would be appropriate to compare the above bounds with the results published in [4,6] concerning the constraints on violations of general relativity leading to massive gravitons. The bound obtained from the GW150914 event was formulated in terms of graviton Compton wavelength $\lambda_{\mathrm{GW}} > 10^{13}$ km, which turned out to be the strongest dynamical bound probing the propagation of gravitational interactions. Translating this into a bound on the speed of gravity, one obtains $1 - (v_{\mathrm{GW}}/c)^2 < 10^{-19}$. This constraint is much stronger than one can get from differences in time delays. Let us remind the reader, however, that the aforementioned bound was obtained as a result of sophisticated analysis using waveform models that allow for parameterized general-relativity violations during the inspiral and merger phases and using the dispersion measurement. On the other hand, the second method discussed by us—using the GW vs EM arrival times in lensed images—is more restrictive by 3 orders of magnitude. This means that even a single instant of gravitationally lensed GW signal accompanied by an EM transient counterpart would be valuable.

Perspectives.—One can expect that observations over the next decades carried out together in the GW and EM windows will be sufficient to give a strong constraint on the GW speed and graviton mass. The main limitation of our method is the accuracy of the EM time delay, while the timing in the GW detectors is very precise ($< 10^{-4}$ ms).

The planned third-generation gravitational-wave detectors, such as the Einstein Telescope, could observe the strongly lensed GW. The rate of yearly detections of strongly lensed GWs from NS-NS and NS-BH sources are in the range ~2–10/yr [16,17], depending on different configurations of the Einstein Telescope and stellar population synthesis models [42]. Cadenced wide-field EM imaging surveys in the next decade will increase the catalog of strongly lensed systems by 2 orders of magnitude. In addition, one can imagine a dedicated follow-up project based on the observed GW events. Short-duration EM counterparts transients (such as kilonovae) have a strong and pronounced feature on the light curve (i.e., the maximum point), which creates a unique opportunity for time-delay extracting algorithms that result in a accurate estimate. For these objects, we expect to obtain the time-delay precision $\sim 10^4$ s through the dedicated photometry related to maximum point. EM counterparts of much shorter duration, like SGRBs, FRBs, or any new signal discovered in the future, will be measured with much better time-delay accuracy. The constraint on GW speed can be enhanced by increasing the number N of lensed systems observed in GW and EM windows. In such a case, the statistical uncertainty would be reduced by a factor of $\sqrt{N}$. Such a population of much-shorter-duration EM counterparts, such as ~10 FRBs with 0.01-ms time-delay accuracy, could suppress the uncertainty of time delays by a factor of $\sim 10^{10}$ comparing with that measured by a kilonovae. The issue of expected rates of joint EM-GW strongly lensed events is interesting on its own and merits further studies. However, even a single such event—discovered either serendipitously or as a result of dedicated surveys—would be very important.

Our approach has a number of advantages. The first is its differential setting, which makes it robust, as already mentioned. However, the price paid for this is that it is much less restrictive. Second, the lensed EM-GW event is perhaps the best way to identify an EM counterpart: even with poor resolution of GW detectors, if we see a lensed GW (i.e., two strains of similar temporal structure) coincident with a lensed EM source we can be almost sure about the source location. An extra bonus, then, is that we would be able to measure time-of-flight differences (GW vs EM) in each image, in addition to differential time delays. If there are more than two images (e.g., quads, which are typical in the strong lensing systems discovered so far), we could have several measurements from a single lensed source. One has to remark, however, that as discussed in [43] the peak amplitude of GW emission associated with the time of the merger could be registered long before the EM prompt SGRB signal. The two signals would be separated by the lifetime of the supramassive NS, which can easily exceed 10^3 s. The intrinsic time delay between the EM and GW signals is very hard to disentangle from the possible time delay due to hypothetical difference between the speed of light and the speed of gravity. Therefore, it is a serious obstacle to the method of EM-GW time-of-flight differences in each image.

We can conclude that, according to the anticipated development of GW astrophysics, massive EM surveys and the synergy between them will create the possibility to use strongly lensed GW-EM events as complementary tests of fundamental physics and astrophysics. When this Letter was under review we became aware of the paper [44], in which the authors independently discussed the idea and applications of multi-messenger time delays from lensed GWs.

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