## Rescuing a Quantum Phase Transition with Quantum Noise

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We show that placing a quantum system in contact with an environment can enhance non-Fermi-liquid correlations, rather than destroy quantum effects, as is typical. The system consists of two quantum dots in series with two leads; the highly resistive leads couple charge flow through the dots to the electromagnetic environment, the source of quantum noise. While the charge transport inhibits a quantum phase transition, the quantum noise reduces charge transport and restores the transition. We find a non-Fermi-liquid intermediate fixed point for all strengths of the noise. For strong noise, it is similar to the intermediate fixed point of the two-impurity Kondo model.

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Quantum fluctuations and coherence are key distinguishing ingredients in quantum matter. Two phenomena to which they give rise, for instance, are *quantum phase* transitions [\[1,2\]](#page-4-1), changes in the ground state of a system driven by its quantum fluctuations, and quantum noise [\[3](#page-4-2)–5], the effect on the system of quantum fluctuations in its environment, for no system is truly isolated. Understanding the intersection of these two topics—the effects of quantum noise on quantum phase transitions—is important for understanding quantum matter. It is natural to suppose that decoherence produced by the noise will suppress quantum effects, and in particular, inhibit or destroy a quantum critical state. Indeed, a variety of calculations demonstrate this in both equilibrium [\[3,4,](#page-4-2) 6–[12\]](#page-4-2) and nonequilibrium [\[13](#page-4-3)–17] contexts. There are also a few known cases that do not follow this rule [\[18](#page-4-4)–20]. Here, we present a striking counterexample to the notion that environmental noise necessarily harms quantum manybody effects: in the system we study, the addition of (equilibrium) quantum noise stabilizes a non-Fermi liquid quantum critical state.

We discuss the phase diagram of two quantum dots connected to two leads in the presence of environmental quantum noise. The noiseless model has a quantum phase transition that is transformed into a crossover by charge transport across the double dot. We show that quantum fluctuations of the field associated with the source and drain voltage counteract this charge transport. The competition between these two processes restores the delicate balance of the quantum critical state. The result is that the quantum phase transition is rescued from the undesired crossover for any strength of the noise.

Our double quantum dot setup is shown schematically in Fig. [1](#page-0-0): two small dots are in series between two leads, labeled  $L$  (left) or  $R$  (right). The leads are resistive, thereby coupling the electrons to an Ohmic electromagnetic environment. Experimentally, small double dots have been

studied in several materials [\[21](#page-4-5)–25], and the effect of the environment on transport in simpler systems has been recently studied in detail [\[26](#page-4-6)–29], including transport through a single quantum dot [\[27,28\].](#page-5-0) Thus, all the necessary ingredients for an experimental study of our system are available.

Model for dots and leads.—The model has three parts: leads, dots, and an electromagnetic environment. Following standard procedures, we linearize the spectrum of each lead, notice that a one-dimensional subset of electrons couples to each dot, and represent it using chiral fermions by analytic continuation with open boundary conditions [\[30\]](#page-5-1). The resulting lead Hamiltonian is the sum of four free Dirac fermions,

<span id="page-0-1"></span>
$$
H_{\text{leads}}^{0} = \sum_{\alpha,\sigma} \int_{-\infty}^{\infty} dx \psi_{\alpha,\sigma}^{\dagger}(x) i \partial_{x} \psi_{\alpha,\sigma}(x), \tag{1}
$$

where  $\alpha$  and  $\sigma$  are the lead and spin labels and both the Fermi velocity and  $\hbar$  are set to unity.

For the dots, we consider the Coulomb blockade regime in which charge fluctuations are suppressed and the electron number is odd [\[31\]](#page-5-2). The single-level Anderson model is suitable for each dot as the spacing between levels in the carbon nanotube dots is large [\[27,28\]](#page-5-0). Each dot then has a low-energy spin- $\frac{1}{2}$  degree of freedom  $\vec{S}_{\alpha}$ . Projecting onto this low-energy subspace via a second-order

<span id="page-0-0"></span>

FIG. 1. Schematic of the system: two quantum dots coupled to left and right leads.  $J_{L,R}$  and K refer to the Kondo and exchange coupling strengths, respectively.  $V_{LR}$  is the strength of direct charge transport between the leads. Dissipative modes in the leads are represented by wiggly arrows.

<span id="page-1-1"></span>Schrieffer-Wolff transformation produces two Kondo-like terms with couplings  $J_{L,R}$  and a spin-spin antiferromagnetic interaction with coupling  $K$ ,

$$
H_{\text{dots}} = J_L \vec{s}_L(0) \cdot \vec{S}_L + J_R \vec{s}_R(0) \cdot \vec{S}_R + K \vec{S}_L \cdot \vec{S}_R, \quad (2)
$$

where  $\vec{s}_\alpha = \psi_a^{\dagger}(0)\vec{\sigma}\psi_\alpha(0)$  is the spin density in the lead at the point connected to the dot. Though none of our results depend point connected to the dot. Though none of our results depend on left-right symmetry, we take  $J_L = J_R$  for simplicity.

Charge transfer between the two leads is key to the physics of this system [\[32](#page-5-3)–37]. The effective hopping between the leads that arises from the third-order Schrieffer-Wolff transformation of the original Anderson model must be added [\[37\],](#page-5-4)

<span id="page-1-0"></span>
$$
H_{LR} = V_{LR}[(\psi_{L\uparrow}^{\dagger}\psi_{R\uparrow} + \psi_{R\downarrow}^{\dagger}\psi_{L\downarrow})S_{L}^{-}S_{R}^{+} + (\psi_{L\uparrow}^{\dagger}\psi_{R\uparrow} + \psi_{L\downarrow}^{\dagger}\psi_{R\downarrow})S_{L}^{z}S_{R}^{z} + (\psi_{L\uparrow}^{\dagger}\psi_{R\downarrow} - \psi_{R\uparrow}^{\dagger}\psi_{L\downarrow})(S_{L}^{z}S_{R}^{-} - S_{L}^{-}S_{R}^{z})] + \text{H.c.,}
$$
 (3)

where  $x = 0$  for the lead operators [\[38\]](#page-5-5). This form is obtained because moving an electron across the dots necessarily involves the dot spins. Much of the physics added by [\(3\)](#page-1-0) is obtained from a simpler direct hopping,  $\hat{H}_{LR} = \hat{V}_{LR} \psi_{L\sigma}^{\dagger}(0) \psi_{R\sigma}(0) + \text{H.c.}$  [\[37,42\]](#page-5-4). We therefore simplify the discussion by using  $\hat{H}_{LR}$  rather than  $H_{LR}$ when possible [\[38\]](#page-5-5).

The final ingredient in our system is the "quantum noise." Quantum fluctuations of the source and drain voltage require a quantum description of the tunneling junction [\[5,31\]](#page-4-7). The standard procedure is to introduce junction charge and phase fluctuation operators that are conjugate to each other and (bilinearly) coupled to modes of the Ohmic environment with resistance R. Treating the latter as a collection of harmonic oscillators with the desired impedance, we write the environment as a free bosonic field,  $H_{\varphi}^0 = \int (dx/4\pi)$ <br>(a)  $\varphi$ <sup>2</sup> which is excited in a tunnaling event through the  $(\partial_x \varphi)^2$ , which is excited in a tunneling event through the charge-shift operator  $e^{i\sqrt{2r}\varphi(0)}$  [\[5\].](#page-4-7) Such a shift operator is added to every term in  $H_{LR}$  according to

$$
\psi_{L\sigma}^{\dagger}\psi_{R\sigma} \to e^{i\sqrt{2r}\varphi(0)}\psi_{L\sigma}^{\dagger}\psi_{R\sigma},\tag{4}
$$

<span id="page-1-3"></span>where  $r = Re^2/h$  is the dimensionless resistance. The environment does not modify the second-order exchange couplings, Eq. [\(2\),](#page-1-1) because those virtual processes occur on the very short time scale of the inverse charging energy [\[43\]](#page-5-6), typically smaller than the time scale of the environment. This model of noisy tunneling has been used previously in work on a resonant level [\[44,45\]](#page-5-7), including in our own work [\[27,28,46,47\]](#page-5-0), and for a quantum dot in the Kondo regime [\[43\].](#page-5-6) In summary, the starting point of our discussion is the Hamiltonian

$$
H = H_{\text{leads}}^{0} + H_{\varphi}^{0} + H_{\text{dots}} + H_{LR}(r). \tag{5}
$$

Quantum phase transition or crossover?—First, we bosonize the chiral fermions describing the leads, Eq. [\(1\)](#page-0-1), thereby introducing chiral bosonic fields  $\phi_{\alpha,\sigma}$ [\[30,36,38\]](#page-5-1). One can then see that the ultraviolet fixed point, described by  $H_{\text{leads}}^0 + H_{\varphi}^0$ , is unstable. There are two<br>important energy scales connected to this instability; the important energy scales connected to this instability: the Kondo temperature  $T_K$ , associated with the screening of each dot by its own lead, and the "crossover temperature,"  $T^* < T_K$  [\[36,42\]](#page-5-8).

To explain  $T^*$ , we start by considering  $V_{LR} = 0$ , yielding the two-impurity Kondo model. For  $T < T_K$ , there are two Fermi-liquid phases with a critical coupling that separates them [\[32,48\],](#page-5-3) denoted by  $K_c$ . (i) For  $K > K_c$ , the two dots become maximally entangled in a singlet state—the localsinglet phase controlled by a fixed point, denoted LSFP, with a scattering phase shift of 0. (ii) For  $K < K_c$ , each dot becomes maximally entangled with its respective lead, forming two decoupled Kondo singlets—the Kondo phase controlled by a fixed point, denoted KFP, phase shift of  $\pi/2$ . The fact that the phase shifts are different implies the existence of an intermediate (unstable) fixed point [\[48,49\]](#page-5-9), which we call  $\text{IFP}_1$  (see Fig. [2\)](#page-1-2).

Interlead tunneling,  $V_{LR} \neq 0$ , changes the behavior dramatically. In the absence of dissipation,  $r = 0$ , it is known that  $H_{LR}$  destabilizes IFP<sub>1</sub> [34–[37,50\]](#page-5-10), becoming effective below a scale  $T^*$ . The low-energy physics is described by Fermiliquid Hamiltonians, with scattering phase shift varying from  $\delta = 0$  to  $\pi/2$ , depending on the initial values of the couplings [\[33,35,36,42\]](#page-5-11). The finite temperature conductance is  $G = G_0 \sin(2\delta) [1 - \kappa (T/T^*)^2]$ , where  $G_0 = 2e^2/h$  and  $\kappa$ <br>is nonuniversal. Therefore, for  $T < T^*$  the quantum phase is nonuniversal. Therefore, for  $T < T^*$ , the quantum phase transition of the two-impurity Kondo model is transformed into a crossover between the Kondo and local-singlet regimes.

Quantum noise effects.—Close to the KFP and LSFP, the tunneling Hamiltonian in the absence of noise  $H_{LR}$  is a marginal operator [\[35,36,42\]:](#page-5-12) without noise, any bilinear

<span id="page-1-2"></span>

FIG. 2. Stability diagram for different noise strengths  $r$ , and the temperature dependence of the conductance at each fixed point. The KFP and LSFP are stable for any nonzero  $r$ , while the nature of the IFP changes as a function of r. For  $r < 1/2$ , the intermediate state is controlled by  $IFP_2$ —the fixed point that evolves from one of the  $r = 0$  Fermi-liquid fixed points. In contrast, for  $r > 1/2$ , IFP<sub>1</sub>—which evolves from the twoimpurity-Kondo IFP—is relevant. For  $r = 1/2$ , a line of fixed points connects  $\text{IFP}_1$  to  $\text{IFP}_2$ .

operator that transfers charge between the leads is marginal at these two fixed points. A key effect of the noise is that the scaling dimension of such an operator increases, making it irrelevant. In the tunneling operator  $H_{LR}$ , the increase is caused by the exponential charge-shift operator introduced in Eq. [\(4\)](#page-1-3). The line of Fermi-liquid fixed points existing at  $r = 0$  is then destroyed. The conductance around the KFP and LSFP follows from perturbation theory in the tunneling, leading to  $G(T) \sim T^{2r}$  [\[5,51\].](#page-4-7)

The newfound stability of the Kondo and local-singlet fixed points with respect to tunneling demands once again the existence of an intermediate fixed point. We denote this "dissipative intermediate fixed point" by  $\text{IFP}_2$ , as shown in Fig. [2](#page-1-2).

 $IFP<sub>2</sub>$  occurs for the same value of K as  $IFP<sub>1</sub>$ , namely  $K = K_c$ , as we now show. It is known that the effect of a resistive environment on a bilinear tunneling operator is connected to the partition noise produced by the tunneling [\[52,53\]](#page-5-13): when there is no partition noise, the environment is not excited by the current and so has no effect. This is the case at  $K = K_c$ : the phase shift is  $\delta = \pi/4$ , so the zerotemperature conductance is  $G = 2e^2/h$ , and the transmission is unity. Thus, there is no partition noise: from the line of  $r = 0$  Fermi-liquid fixed points, this fixed point survives at nonzero  $r$  and is, in fact, IFP<sub>2</sub>. We now turn to characterizing both IFP's in detail.

Effective Hamiltonian at the intermediate fixed points.— In order to derive an effective Hamiltonian at the critical coupling,  $K = K_c$ , we follow the dissipationless discussion of J. Gan in Ref. [\[54\]](#page-5-14). First, we define new bosonic fields,

$$
\phi_{c/s} = (\phi_{L\uparrow} \pm \phi_{L\downarrow} + \phi_{R\uparrow} \pm \phi_{R\downarrow})/2,
$$
  

$$
\phi_{cf/sf} = (\phi_{L\uparrow} \pm \phi_{L\downarrow} - \phi_{R\uparrow} \mp \phi_{R\downarrow})/2.
$$
 (6)

Physically,  $\phi_c$  ( $\phi_s$ ) represents the total charge (spin) in the leads, and  $\phi_{cf}$  ( $\phi_{sf}$ ) represents the corresponding difference between the left and right leads. Next, one applies the unitary rotation  $U = e^{-i(S_1^z + S_2^z)\phi_s(0)}$ , thus dressing the spin states and making the exchange couplings anisotropic [38] states and making the exchange couplings anisotropic [\[38\]](#page-5-5). A key aspect of the physics at the IFP's is the degeneracy in the dots between the two dressed spin states,  $|0\rangle \equiv (|\uparrow \uparrow \rangle +$  $|\downarrow \downarrow\rangle$ )/ $\sqrt{2}$  and  $|1\rangle \equiv (|\uparrow \downarrow \rangle - |\downarrow \uparrow \rangle)/\sqrt{2}$ , that leads to an affective Kondo problem with Kondo temperature  $\tilde{T}$ , [54] effective Kondo problem with Kondo temperature  $\tilde{T}_K$  [\[54\]](#page-5-14). It is convenient to introduce Majorana operators  $a$  and  $b$  for this two-dimensional Hilbert space (the string from the Jordan-Wigner transformation is incorporated into the lead operators). The end result [\[54\]](#page-5-14) is an effective Hamiltonian for  $K = K_c$  [\[38\]](#page-5-5),

$$
H_{K=K_c} = \sum_{\beta=\{c,s,sf,cf,\varphi\}} H_{\beta}^0 + 2i\tilde{J} \frac{F_{sf}}{\sqrt{\pi \alpha}} \sin \left[\phi_{sf}(0)\right] a + 2\tilde{V}_{LR} \frac{F_{cf}}{\sqrt{\pi \alpha}} \cos \left[\phi_{cf}(0) + \sqrt{2r}\phi(0)\right] b, \tag{7}
$$

where  $F_{sf}$ ,  $F_{cf}$  are Klein factors,  $\alpha$  is of order the inverse cutoff, J is the renormalized Kondo coupling, and  $V_{LR}$  is the renormalized charge tunneling strength. Explicit expressions for  $\tilde{J}$  and  $\tilde{V}_{LR}$  are given in the Supplemental Material [\[38\].](#page-5-5)

The bosonic fields can be further untangled by performing a rotation that combines the field representing charge transfer between the leads  $\phi_{cf}$  with the environmental noise  $\varphi$ :  $\tilde{\phi}_{cf} \equiv (\phi_{cf} + \sqrt{2r}\varphi)/\sqrt{1+2r}$  and  $\tilde{\varphi} \equiv$  $(\sqrt{2r}\phi_{cf} - \varphi)/\sqrt{1+2r}$ . The symmetries of the model ore explicitly shown by defining six Majorana fermionic are explicitly shown by defining six Majorana fermionic fields [\[35,36,49\]](#page-5-12) with Ramond boundary conditions,  $\chi^{1,2}_{\beta}(0^+) = \chi^{1,2}_{\beta}(0^-): \; \chi^{(1)}_{\beta=\{c,s,sf\}}(x) = (F_{\beta}/\sqrt{\pi\alpha}) \, \sin[\phi_{\beta}(x)]$ and  $\chi_{\beta=\{c,s,sf\}}^{(2)}(x) = (F_{\beta}/\sqrt{\pi\alpha}) \cos[\phi_{\beta}(x)]$ . Because the boundary interaction  $2i\tilde{J}\chi_{sf}^{(1)}(0)a$  has scaling dimension  $1/2$  (*a* is an impurity operator),  $\tilde{J}$  flows to strong coupling [\[35,36\]](#page-5-12).  $\chi_{sf}^{(1)}$  then incorporates a and can be expressed as a simple change of boundary condition from Ramond to Neveu-Schwarz:  $\chi_{sf}^{(1)}(0^+) = -\chi_{sf}^{(1)}(0^-)$  [\[49\].](#page-5-15)<br>The offective LEB Hemiltonian can thus be v

<span id="page-2-0"></span>The effective IFP Hamiltonian can thus be written in terms of six free Majorana fields—five with Ramond and one with Neveu-Schwarz boundary condition—one free bosonic field  $(\tilde{\varphi})$ , and a boundary sine-Gordon model for  $\phi_{cf}$ :

$$
H_{\text{IFP}} = \sum_{j=1}^{5} \int \frac{dx}{2} \chi_j(x) i \partial_x \chi_j(x) + \int \frac{dx}{2} \chi_{sf}^{(1)}(x) i \partial_x \chi_{sf}^{(1)}(x) + \int \frac{dx}{4\pi} [\partial_x \tilde{\varphi}(x)]^2 + \int \frac{dx}{4\pi} [\partial_x \tilde{\varphi}_{cf}(x)]^2 + 2i \tilde{V}_{LR} \frac{F_{cf}}{\sqrt{\pi \alpha}} \cos \left[\sqrt{1 + 2r} \tilde{\varphi}_{cf}(0)\right] b.
$$
 (8)

This Hamiltonian has an inherent  $SO(5) \times U(1)$  symmetry from the five Majorana fields and the dressed dissipation field  $\tilde{\varphi}$ . With regard to the dot degrees of freedom, while Majorana mode  $a$  is effectively incorporated into the leads, mode b is coupled to the charge transport. For the two-impurity Kondo model,  $\tilde{V}_{LR} = 0$  and b is a decoupled Majorana zero mode.

Dependence of IFP on dissipation.—The boundary sine-Gordon model, which is the last element in Eq. [\(8\)](#page-2-0), is well known to have a quantum phase transition [\[30,55,56\]](#page-5-1) as the parameter in the boundary term varies, in our case r. The simplest description of this transition is via the scaling equation,  $(d\tilde{V}_{LR}/d\ell)=(\frac{1}{2}-r)\tilde{V}_{LR}$ , which results from noticing that the scaling dimension of the operator  $\cos[\sqrt{1+2r\phi}_{cf}(0)]$  is  $(1+2r)/2$  [\[30\].](#page-5-1) There are three<br>distinct scaling behaviors depending on the value of r distinct scaling behaviors depending on the value of r.

For weak dissipation,  $r < 1/2$ ,  $V_{LR}$  grows. As in the  $r = 0$  case [\[35,36\],](#page-5-12) the cosine gets pinned at a particular value. The fixed point Hamiltonian is obtained by changing the boundary condition on  $\phi_{cf}$  at  $x = 0$  from Dirichlet [for open boundary conditions on the fermionic fields in Eq. [\(1\)](#page-0-1)] to Neumann [\[56\].](#page-5-16) IFP<sub>2</sub> is the corresponding fixed point; it develops from the  $\delta = \pi/4$  Fermi-liquid fixed point [\[35\]](#page-5-12) of the dissipationless case.

The leading irrelevant operator at  $IFP<sub>2</sub>$  is, because of the change in boundary condition, simply the dual of the relevant operator at IFP<sub>1</sub> that causes  $V_{LR}$  to grow [\[4,30\]](#page-4-8). Its scaling dimension is  $2/(1 + 2r)$ —the inverse of that of the cosine operator above. The temperature dependence of the conductance is therefore expected to be [\[38\]](#page-5-5)

$$
G \sim G_0[1 - \gamma T^{2(1-2r)/(1+2r)}] \quad \text{(at IFP}_2), \tag{9}
$$

with  $\gamma$  a nonuniversal constant. We see that modification of the boundary interaction by dissipation introduces a Luttinger-liquidlike character. In addition to the conductance, the non-Fermi liquid nature of this fixed point is also manifest in its residual boundary entropy, which can be shown to be  $\ln g_{\text{IFP}_2} = \frac{1}{4} \ln (1 + 2r)$  [\[38\].](#page-5-5)

The breakdown of scaling (i.e., when  $\tilde{V}_{LR}$  becomes of order one) defines the crossover temperature,  $T_{\text{noise}}^{LR} \approx$  $T_K(\tilde{V}_{LR}^0)^{2/(1-2r)}$ , in terms of the initial value of tunneling<br>from left to right  $\tilde{V}_{LR}^0$  [57]. For higher temperatures from left to right  $\tilde{V}_{LR}^0$  [\[57\].](#page-5-17) For higher temperatures,  $T_{LR}^{LR} < T < T_K$ , the physics is controlled by the  $\tilde{V}_{LR} = 0$ , fixed point IED, so  $\tilde{V}_{LR}$  is initially small. For laws 0 fixed point IFP<sub>1</sub> as  $\tilde{V}_{LR}$  is initially small. For lower temperatures,  $T < T_{\text{noise}}^{LR}$ , the physics is controlled by IFP<sub>2</sub>.

To study the effect of deviations of the antiferromagnetic coupling K from  $K_c$ , we follow the discussion in Refs. [\[36,42\]](#page-5-8) and define the crossover temperature  $T_{\delta K} = a(K - K_c)^2 / T_K$ , where a is a dimensionless constant. If  $T_{noise}^{LR} < T_{\delta K}$ , the low-energy physics will be governed by the KFP or LSFP. However, for  $T_{\delta K} < T_{\text{noise}}^{LR}$ , an experiment would initially observe a rise in the conductance due to proximity to  $IFP<sub>2</sub>$  before the crossover to the Kondo or local-singlet physics took over (for which  $G \rightarrow 0$ ). Using the remarkable tunability of quantum dots, access to the regime  $T_{\delta K} \ll T_{\text{noise}}^{LR}$  is possible, in which case the power law approach of the conductance to the quantum limit  $G_0$ , given above, should be observable. Indeed, a strong-coupling fixed point with similar properties has recently been studied experimentally in a single dissipative quantum dot [\[27,28\].](#page-5-0)

In sharp contrast, for *strong* dissipation,  $r > 1/2$ ,  $V_{LR}$ shrinks, and the properties of the system are controlled by IFP<sub>1</sub>. The boundary condition on the field  $\phi_{cf}$  remains the Dirichlet condition. The scaling dimension of the boundary sine-Gordon term implies that the conductance decreases at low temperature according to [\[38\]](#page-5-5)

$$
G \sim T^{2r-1} \quad \text{(at IFP}_1). \tag{10}
$$

The non-Fermi liquid nature of this fixed point is further shown by the residual boundary entropy,

 $\ln g_{\text{IFP}_1} = \frac{1}{4} \ln[4/(1+2r)]$  [\[38\]](#page-5-5), and by the decoupling of the  $b$  Majorana in the dots as the last term in Eq.  $(8)$  flows to zero.

 $IFP<sub>1</sub>$  evolves from the intermediate fixed point of the twoimpurity Kondo model ( $r = 0$ ). Formally, however, IFP<sub>1</sub> is a distinct fixed point—the residual boundary entropy, for instance, depends on r. Nevertheless, for reasonable values of  $r \sim 1/2$ , this system can emulate the physics of the twoimpurity Kondo model: the  $SO(7)$  symmetry manifest in the Majorana fields [\[36,49\],](#page-5-8) for instance, is restored asymptotically. Any observable not directly related to charge transfer between the leads, such as the magnetic susceptibility, will have the same behavior in the two models.

The crossover temperature to the KFP or LSFP  $T_{\delta K}$  is given by the same expression as in the weak noise case. Thus, for  $T_{\delta K} < T < T_K$ , the physics of IFP<sub>1</sub>, bearing strong resemblance to that of the two-impurity Kondo model, will be experimentally accessible.

Finally, the *borderline*  $r = 1/2$  case is particularly interesting. The cosine in Eq. [\(8\)](#page-2-0) is exactly marginal [\[58\]](#page-5-18), corresponding to an SU(2) chiral symmetry. Hence, we can replace the cosine by the Abelian chiral current  $\partial_x \phi_{cf}$  [\[56\]](#page-5-16). The model becomes quadratic and the conductance can be calculated exactly  $[30,51]$ — $G$  depends on the initial value  $\tilde{V}_{LR}^0$  and so is not universal. The exactly marginal operator creates a line of fixed points connecting  $\text{IFP}_1$  to  $\text{IFP}_2$ , all with residual boundary entropy  $\frac{1}{4} \ln 2$ . The line is unstable to deviations from the critical coupling  $K_c$ ; as in the previous cases,  $T < T_{\delta K}$  leads to flow toward the KFP or LSFP. Even at  $K_c$ , corrections to the effective Hamiltonian [\(8\)](#page-2-0) will presumably cause flow away from this line at the lowest temperatures (which we have not analyzed); however, because their initial strength is very small, the crossover temperature  $T^*$  to see these effects will be very low. Thus, in a wide range of temperatures,  $T^* < T < T_K$ , the properties of the line of fixed points could be seen experimentally, varying  $V_{LR}^0$  to move among them.

Conclusions.—We have presented an example in which the introduction of a quantum environment reveals a quantum phase transition previously hidden under a crossover: the quantum noise has rescued the quantum phase transition. There are two quantum critical points (Fig. [2\)](#page-1-2): one dominant for weak dissipation (IFP<sub>2</sub>,  $r < 1/2$ ) and the other at strong dissipation (IFP<sub>1</sub>,  $r > 1/2$ )—this latter fixed point is similar to that of the two-impurity Kondo model.

A broader view is obtained by connecting to the idea of "quantum frustration of decoherence" of a qubit [\[59,60\]:](#page-5-19) a quantum system acted upon by two processes that are at cross purposes may retain more coherence than if acted upon by just one. The quantum system to be protected here is the non-Fermi-liquid quantum critical state, delicately balanced between the KFP and LSFP, a striking signature of which is the decoupled, and so completely coherent, Majorana mode. Charge transfer between the electron reservoirs associated with the leads is the first process acting on the system, one that completely destroys the delicate quantum state and the coherence of the Majorana mode. Adding the quantum noise produced by the resistive EM environment impedes the deleterious effect of the first process, rendering the coherent Majorana zero mode again manifest at  $\text{IFP}_1$ . Thus, the quantum coherence of the delicate many-body state survives due to the "quantum frustration" of these two processes.

This quantum critical state is highly nontrivial and clearly unstable toward the KFP and LSFP, but it has experimental consequences in a wide temperature range. We emphasize that measurements of the conductance near  $IFP_1$  and  $IFP_2$ are experimentally feasible at this time—similar amounts of tuning have been used successfully, for instance, in recent experiments [\[27,28\]](#page-5-0). An experimental study along these lines would directly contradict the general notion that more noise leads inevitably to less quantum many-body behavior.

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