

Probing Sub-GeV Dark Matter with Conventional Detectors

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The direct detection of dark matter particles with mass below the GeV scale is hampered by soft nuclear recoil energies and finite detector thresholds. For a given maximum relative velocity, the kinematics of elastic dark matter nucleus scattering sets a principal limit on detectability. Here, we propose to bypass the kinematic limitations by considering the inelastic channel of photon emission from bremsstrahlung in the nuclear recoil. Our proposed method allows us to set the first limits on dark matter below 500 MeV in the plane of dark matter mass and cross section with nucleons. In situations where a dark-matter–electron coupling is suppressed, bremsstrahlung may constitute the only path to probe low-mass dark matter awaiting new detector technologies with lowered recoil energy thresholds.

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Introduction.—Weakly interacting massive particles (WIMPs) are among the theoretically best motivated and experimentally most sought particle candidates for dark matter (DM) [1,2]. The efforts are driven by a broad expectation that physics beyond the standard model should enter near the electroweak scale, with interactions that are not too different from the weak interactions.

There has been a significant amount of experimental effort to push the sensitivity of direct detection experiments to masses below a few GeV. The efforts are hampered by the fact that light DM induces soft nuclear recoils that are difficult to detect unambiguously. In the nonrelativistic scattering of a DM particle χ and target nucleus N with mass m_N , the three-momentum transfer $\mathbf{q} = \mathbf{p}'_\chi - \mathbf{p}_\chi$ determines the kinetic recoil energy of the nucleus, $E_R = |\mathbf{q}|^2/(2m_N) \leq 2\mu_N^2 v^2/m_N$, where μ_N is the DM-nucleus reduced mass and v is the relative velocity, bounded by the finite gravitational potential of the Galaxy. New avenues have therefore been suggested to probe DM below the GeV scale, such as looking for DM-electron scattering [3] in existing data [4], employing semiconductor targets [5–7], using superconductors or superfluids [8–11], nanotubes [12], or 2D graphenelike targets [13], and exploiting a nonvirialized velocity component of DM [14].

In this Letter we propose a method of probing sub-GeV DM in direct detection by going to the inelastic channel of photon emission from the nucleus in the form of bremsstrahlung—an irreducible contribution that accompanies the elastic reaction

$$\chi + N \rightarrow \chi + N(E_R), \quad (\text{elastic}), \quad (1a)$$

$$\chi + N \rightarrow \chi + N(E'_R) + \gamma(\omega), \quad (\text{inelastic}). \quad (1b)$$

(Photon emission from the excitation of low-lying nuclear levels has been considered in Refs. [15–18]. The process requires considerable momentum transfer and concerns

electroweak scale DM masses.) The virtue of considering Eq. (1b) is that the available photon energy is bounded by the energy of the relative motion of DM and the target, $\omega \leq \mu_N v^2/2$, so that we observe a hierarchy for light dark matter

$$E_{R,\max} = 4(m_\chi/m_N)\omega_{\max} \ll \omega_{\max} \quad (m_\chi \ll m_N). \quad (2)$$

As we will see, the larger energy deposition in photon emission allows us to lower the sensitivity to nuclear recoils to the sub-GeV DM mass regime in present-day detectors. The signal will be part of the “electron recoil band” and subject to backgrounds, yet amply detectable: whereas, say, $E_R = 0.5$ keV is experimentally easily missed, a photon of energy $\omega = 0.5$ keV is hardly ever missed.

Cross section.—It is well known that in the limit of soft photon emission off an electromagnetically charged particle the matrix element for bremsstrahlung factorizes into the matrix element of elastic scattering M_{el} times a manifestly gauge invariant piece. For this to hold in the nonrelativistic limit of the emitting particle, the three-momentum transfer \mathbf{q} in the elastic scattering must be much larger than the change of it due to the additional emission of the photon with momentum \mathbf{k} , $\delta\mathbf{q} = (\mathbf{p}'_N - \mathbf{p}_N - \mathbf{k}) - (\mathbf{p}'_N - \mathbf{p}_N)_{\omega=0}$. Hence, imposing $|\delta\mathbf{q}| \ll |\mathbf{q}|$ yields the soft-photon limit,

$\omega \ll |\mathbf{q}|v = \sqrt{2m_N E_R}v \approx O(10 \text{ keV})\sqrt{\frac{A}{130}}\sqrt{\frac{E_R}{1 \text{ keV}}}$, where A is the atomic mass number of N . The latter condition holds well away from the kinematic endpoint of minimum momentum transfer, $|\mathbf{q}_{\min}| \approx \omega/v$, and the soft photon limit will be respected. Since $\omega_{\max} \approx \mu_N v^2/2 \lesssim |\mathbf{q}_{\max}|v$ holds parametrically, we can further take the approximation $E'_R \approx E_R$ in Eq. (1).

The factorization of the matrix element is universal and does not depend on the spin of the nucleus. Summing over the photon polarization, and assuming no directional sensitivity yields a double differential cross section of

$$\left. \frac{d^2\sigma}{dE_R d\omega} \right|_{\text{naive}} = \frac{4Z^2\alpha}{3\pi} \frac{1}{\omega} \frac{E_R}{m_N} \times \frac{d\sigma}{dE_R} \Theta(\omega_{\max} - \omega). \quad (3)$$

Here, $d\sigma/dE_R$ is the WIMP-nucleus elastic scattering cross section for Eq. (1a); Z is the atomic number.

The cross section for bremsstrahlung emission off a recoiling nucleus gets modified at low photon energies by the fact that it is in a neutral bound state with electrons. The process of photon emission can be viewed as in Fig. 1 where the double line represents the nucleus in the initial (final) atomic state of electrons i (f), with intermediate state n . The matrix element for the transition can be put into the following form:

$$|V_{fi}|^2 = 2\pi\omega |M_{\text{el}}|^2 \left| \sum_{n \neq i, f} \left(\frac{(\mathbf{d}_{fn} \cdot \hat{\mathbf{e}}^*) \langle n | e^{-i(m_e/m_N)\mathbf{q} \cdot \sum_{\alpha} \mathbf{r}_{\alpha}} | i \rangle}{\omega_{ni} - \omega} + \frac{(\mathbf{d}_{ni} \cdot \hat{\mathbf{e}}^*) \langle f | e^{-i(m_e/m_N)\mathbf{q} \cdot \sum_{\alpha} \mathbf{r}_{\alpha}} | n \rangle}{\omega_{ni} + \omega} \right) \right|^2. \quad (4)$$

Here, M_{el} is the matrix element for the elastic DM-nucleus collision, $\mathbf{d}_{kl} = e \sum_{\alpha} \mathbf{r}_{\alpha, kl}$ is the atomic dipole moment (with sum over the positions \mathbf{r}_{α} of all electrons with elementary charge e), $\hat{\mathbf{e}}^*$ is the polarization vector of the photon in three-dimensional transverse gauge, and $\omega_{kl} = \omega_k - \omega_l$ is the atomic transition frequency between states $|k\rangle$ and $|l\rangle$. The cross section for photon emission will then be given by

$$d\sigma = \frac{|V_{fi}|^2 \omega^2 d\omega d\Omega_{\mathbf{k}}}{|M_{\text{el}}|^2 (2\pi)^3} \times d\sigma_{\text{el}}. \quad (5)$$

A few comments regarding Eq. (4) are in order. First, the factors $\mathbf{d}_{kl} \cdot \hat{\mathbf{e}}^*$ are part of the dipole transition element $V_{kl}^{(\gamma)} = -\mathbf{d}_{kl} \cdot \partial_t \mathbf{A}_{\mathbf{e}, \omega}^*$ with $\mathbf{A} = (2\pi/\omega)^{1/2} e^{-i\omega t} \hat{\mathbf{e}}$ (we work in unrationalized units of $e^2 = \alpha$), responsible for the emission of a photon of energy ω . Here, the spatial dependence entering the photon wave function through $|\mathbf{k} \cdot \mathbf{x}| \leq \omega R_{\text{Atom}}$ has been neglected; this is a good approximation, unless one considers the kinematic photon endpoint and substitutes for R_{Atom} the entire atomic radius, for which the product can become $O(1)$. Second, the matrix elements $\langle k | e^{-i(m_e/m_N)\mathbf{q} \cdot \sum_{\alpha} \mathbf{r}_{\alpha}} | l \rangle$ describe the motion of the electron cloud relative to the nucleus with velocity $|\mathbf{v}_N| = |\mathbf{q}|/m_N$ after the latter receives an impulse \mathbf{q} from DM. It is assumed that the kick is to good approximation instantaneous; i.e., the DM-nucleus interaction time

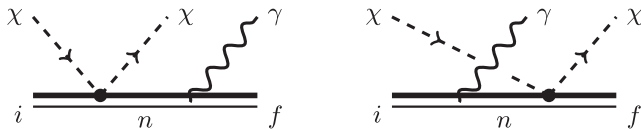


FIG. 1. Photon emission resulting from DM-nucleus scattering. The thick line represents the nucleus in the atomic initial (final) state i (f) with intermediate state n , represented by the thin solid line.

$\tau_{\chi} \sim R_N/v_{\chi}$ is smaller than the time it takes electrons in orbit to adjust to the perturbation, $\tau_{\alpha} \sim |\mathbf{r}_{\alpha}|/v_{\alpha}$. Taking for the nuclear radius $R_N = 1.3 \text{ fm } A^{1/3}$, a typical DM velocity $v_{\chi} = 10^{-3}$, and an inner shell electron with radius $|\mathbf{r}_{\alpha}| = 1/(Z\alpha m_e)$ and velocity $v_{\alpha} \sim Z\alpha$, we get $\tau_{\chi}/\tau_{\alpha} \approx 10^{-4} A^{1/3} Z^2$. Hence our approximation is well justified for light elements; for heavier targets such as xenon, the ratio can become $O(1)$, but only for the innermost electrons. Going beyond the mentioned approximations requires a dedicated atomic physics calculation, which is certainly welcome but well beyond the scope of this Letter. Finally, in the denominators of Eq. (4) we neglect any dependence on E_R based on the fact that $E_R \ll \omega_{ni}$.

On similar grounds as for the dipole matrix element for photon emission, we can make use of the dipole approximation in the boosted matrix elements

$$\langle k | e^{-i(m_e/m_N)\mathbf{q} \cdot \sum_{\alpha} \mathbf{r}_{\alpha}} | l \rangle \approx \frac{-i m_e}{e m_N} \mathbf{q} \cdot \mathbf{d}_{kl} \quad (k \neq l). \quad (6)$$

The limit is well justified, since $(m_e/m_N)\mathbf{q} \cdot \mathbf{r}_{\alpha} \ll 1$ for all practical purposes. This expansion brings about a major simplification when we consider the special case $i = f$:

$$|V_{ii}|^2 = \frac{4\pi\omega m_e^2 E_R}{\alpha m_N} |M_{\text{el}}|^2 \times |\hat{\mathbf{e}}^* \hat{\mathbf{q}}_s \alpha_{rs}(\omega)|^2. \quad (7)$$

Here, $\alpha_{rs}(\omega)$ (r, s are Cartesian coordinates) denotes the polarizability of an individual atom. In the limit of spherical symmetry, which we will assume henceforth, $\alpha_{rs}(\omega) = \alpha(\omega)\delta_{rs}$. The latter function $\alpha(\omega)$ can be related to the atomic scattering factors $f(\omega) = f_1(\omega) + if_2(\omega)$, which are tabulated, $\alpha(\omega) = -\frac{\alpha}{m_e\omega^2} f(\omega)$. By taking the limit in which the atom stays in the ground state, $i = f$, we neglect further contributions to the photon yield. Our derived limits must therefore be considered as conservative; we leave more detailed calculations of the atomic processes as future work.

Taking the polarization sum, integrating over the photon directions $d\Omega_{\mathbf{k}}$, and averaging over the direction $\hat{\mathbf{q}}$ of the momentum transfer, we arrive at the final result for the photon-emission cross section

$$\begin{aligned} \frac{d^2\sigma}{d\omega dE_R} &= \frac{4\omega^3 E_R m_e^2 |\alpha(\omega)|^2}{3\pi m_N \alpha} \times \frac{d\sigma}{dE_R} \Theta(\omega_{\max} - \omega) \\ &= \frac{4\alpha E_R}{3\pi\omega m_N} |f(\omega)|^2 \times \frac{d\sigma}{dE_R} \Theta(\omega_{\max} - \omega). \end{aligned} \quad (8)$$

A comparison with Eq. (3) exposes nicely the atomic physics modification to the naive cross section of unscreened bremsstrahlung emission from the bare nucleus. At low photon energy, the process weakens as ω^3 as is typical for dipole emission (the dipole created between the nucleus and electrons). At large energies, $f_1 \rightarrow Z \gg f_2$, the atomic state becomes irrelevant, and Eq. (8) approaches Eq. (3).

Event rates.—The main idea is to tap the electron recoil that is induced by bremsstrahlung, when (reliable) experimental sensitivity to nuclear recoils fails at low recoil energy. To arrive at a convenient expression for the differential event rate we neglect the energy deposition E_R since the respective maximum energies fulfill $E_{R,\max} \ll \omega_{\max}$, and take the photon energy ω as the only detectable signal, with rate $d\sigma/d\omega = \int_{E_{R,\min}}^{E_{R,\max}} dE_R (d\sigma/dE_R d\omega)$. The boundaries of the recoil energy integration are found from three-body kinematics

$$E_{R,\max/\min} = \frac{\mu_N^2 v^2}{m_N} \left[\left(1 - \frac{\omega}{\mu_N v^2} \right) \pm \sqrt{1 - \frac{2\omega}{\mu_N v^2}} \right].$$

Consider now a standard DM-nucleus recoil cross section, $d\sigma/dE_R = \sigma_0^{\text{SI}} m_N / (2\mu_N^2 v^2) F^2(|\mathbf{q}|)$ with spin-independent DM-nucleus cross section $\sigma_0^{\text{SI}} \approx A^2 \sigma_n (\mu_N / \mu_n)^2$, where σ_n is the DM-nucleon elastic cross section and μ_n is the DM-nucleon reduced mass. Making the excellent approximation that the nuclear form factor at low recoil is unity, $F^2 \approx 1$, the differential cross section can be integrated to yield

$$\frac{d\sigma}{d\omega} = \frac{4\alpha |f(\omega)|^2 \mu_N^2 v^2 \sigma_0^{\text{SI}}}{3\pi\omega m_N^2} \sqrt{1 - \frac{2\omega}{\mu_N v^2}} \left(1 - \frac{\omega}{\mu_N v^2} \right). \quad (9)$$

In a final step, we take the average of the cross section over the velocity distribution of DM in the frame of the detector and compute the event rate

$$\frac{dR}{d\omega} = N_T \frac{\rho_\chi}{m_\chi} \int_{|\mathbf{v}| \geq v_{\min}} d^3\mathbf{v} v f_v(\mathbf{v} + \mathbf{v}_e) \frac{d\sigma}{d\omega}. \quad (10)$$

Here, N_T is the number of target nuclei per unit detector mass and $\rho_\chi = 0.3 \text{ GeV}/\text{cm}^3$ is the local DM mass density. For $f_v(\mathbf{v})$ we take a truncated Maxwellian with escape speed $v_{\text{esc}} = 544 \text{ km/s}$ [19] and most probable velocity $v_0 = 220 \text{ km/s}$; \mathbf{v}_e is the velocity of the Earth relative to the Galactic rest frame and $v_{\min} = \sqrt{2\omega/\mu_N}$.

The penalty for going to the inelastic channel is of course very large. Whereas a factor of α is compensated by Z^2 in Eq. (3) [or by $f_{1,2}^2$ in Eq. (9)], the factor E_R/m_N may be overcome by a quasiexponential rising event rate $dR_{\text{el}}/dE_R \sim e^{-E_R/E_0}$ with decreasing E_R , where $E_0 = \text{few} \times \text{keV}$ for WIMPs and typical target masses. The spill over from photons into the higher energy region is the key that allows us to exploit the inelastic channel in the electron recoil band experimentally.

The prospective parameter space where the method of bremsstrahlung emission yields an improvement of sensitivity is best identified by demanding that no elastic nuclear recoil event (with rate dR/dE_R) has been induced above the detector-specific nominal threshold recoil energy $E_{R,\text{th}}$, $N(E_R > E_{R,\text{th}}) = \text{exposure} \times \int_{E_{R,\text{th}}}^{\infty} dE_R (dR/dE_R) < 1$, and by computing from there the number of bremsstrahlung-induced electron recoil events via Eq. (10). It is important

to note that $N(E_R > E_{R,\text{th}}) < 1$ becomes trivially fulfilled for any value of DM-nucleon cross section once the DM mass falls below the kinematic threshold imposed by the maximum relative velocity between DM and the target nucleus, $v_{\max} = v_{\text{esc}} + v_e \approx 750 \text{ km/s}$. For example, $N(E_R > E_{R,\text{th}}) < 1$ for any value of σ_0^{SI} once the DM mass falls below 3.3 GeV in a xenon experiment with a nominal threshold of $E_{R,\text{th}} = 1.1 \text{ keV}$ such as in LUX [20,21] and before accounting for finite detector resolution. Figure 2 shows the theoretical rates for elastic scattering, dR/dE_R , and the photon emission rate $dR/d\omega$ as labeled resultant from nuclear recoils of a DM particle of mass $m_\chi = 1 \text{ GeV}$ and a DM-nucleon cross section of $\sigma_n = 10^{-35} \text{ cm}^2$. The dotted line is the rate according to the naive estimate (3).

Probing low-mass DM.—We now explore the sensitivity to bremsstrahlung in the usual (m_χ, σ_n) plane. Here, we focus on the ionization-only signal in liquid scintillator experiments, for which XENON10 [22] and most recently XENON100 [23] have presented results. The ionization threshold of xenon is $\sim 12 \text{ eV}$; hence, the emission of a 100 eV photon can already produce multiple ionized electrons.

For XENON10 the collaboration has reported the spectrum in number of electrons, and we compute the electron yield upon absorption of the bremsstrahlung photon following Refs. [4,24] and assuming that it takes on average 13.8 eV to produce an ionized electron [25]; a similar, albeit simplified program has been carried out in Refs. [26,27]. For XENON100 we convert the expected ionization signal into photoelectrons (PE) using a yield of 19.7 PE/ e^- and a width of 7 PE/ e^- [23]; the conversion corresponds to 1.43 PE/eV. Although signal formation at lowest energies is poorly understood, a recent measurement at 200 eV electron recoil energy supports such naive expectations of charge yield, with recombination of ions and electric field dependence playing little role [28,29]. We then place a limit using the “ p_{\max} ” method [22,30]. The respective sensitivities to σ_n are shown in Fig. 3 by the (blue and orange) shaded region as labeled. A thin orange line in the XENON100 region shows the limit with the *ad hoc* pessimistic choice of 30 eV/electron and resulting conversion factor 0.6 PE/eV. Finally, we note that LUX may soon improve on the XENON100 limit, because of

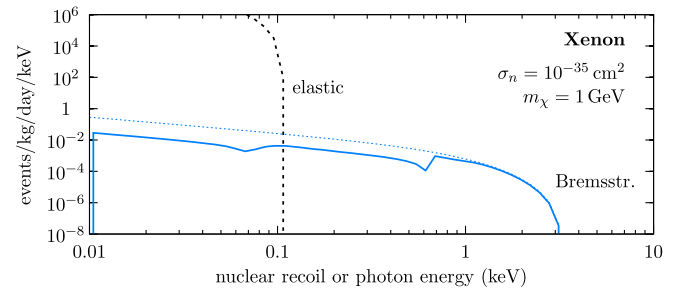


FIG. 2. Elastic (dR/dE_R) and photon-emission ($dR/d\omega$) rates in xenon. The ionization threshold is 12 eV. The dotted line is derived from the naive cross section (3).

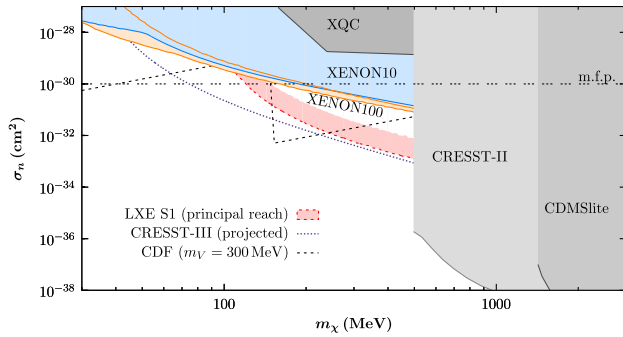


FIG. 3. (m_χ, σ_n) plane of DM mass and DM-nucleon cross section. Regions labeled CRESST-II and CDMSlite were previously excluded from elastic DM-nucleus scattering [31,32]; regions labeled XENON10 and XENON100 show newly derived constraints based on (1b). Above the line “m.f.p.” limits are invalidated as DM scatters before reaching the detector. The region XQC is excluded from rocket-based X-ray calorimetry data [33]. Projections for CRESST-III and for a dedicated liquid Xe experiment, labeled LXE, are also shown; see main text. The monojet constraint CDF is model dependent; CMB constraints can be evaded [34] and are not shown.

lower electron backgrounds [35]. We estimate (supported by our own Monte Carlo simulation, see also Refs. [36–39]) that for $\sigma_n \gtrsim 10^{-30} \text{ cm}^2$ the limits become invalidated as elastic scattering of DM inside the Earth slowly degrades energy and flux of the incident particles; the corresponding approximate demarcation line is labeled “m.f.p.”

Importantly, the next generation of dual-phase liquid scintillator direct detection experiments, such as XENON1T [40] and LZ [41], are coming online or are being planned. Although a much reduced electromagnetic recoil background may be expected, $R_{b.g.} = 10^{-4} - 10^{-5} \text{ /kg/day/keV}$ [40], such a rate requires volume fiducialization. The latter is accomplished through the much weaker scintillation signal S1 with an overall detection efficiency of only $\sim 10\%$. Hence, improvement over current limits is not guaranteed, and instead we advocate a dedicated smaller setup with both high S1 light collection efficiency and single photon sensitivity [42], see also the recent Ref. [43]. We estimate a principle limit for this technology in Fig. 3, assuming $R_{b.g.} = 10^{-4} \text{ /kg/day/keV}$ with an exposure of 600 kg yr and requiring two S1 photons produced at 10% with respect to ionized electrons and each of the S1 photons detected with 40%–100% efficiency (varying the efficiency determines the thickness of the red band). Finally, solid state scintillators have reached $O(100 \text{ eV})$ thresholds, most notably CRESST-II with the only reported DM-nucleon cross section limit below 1 GeV [31]. We exemplify the near-future reach by following the experiment’s own projections [44] with threshold 100 eV, a factor of 100 reduced backgrounds, and neglecting efficiencies for simplicity.

Comparison to DM-electron scattering.—MeV-mass DM is already constrained through scattering on electrons

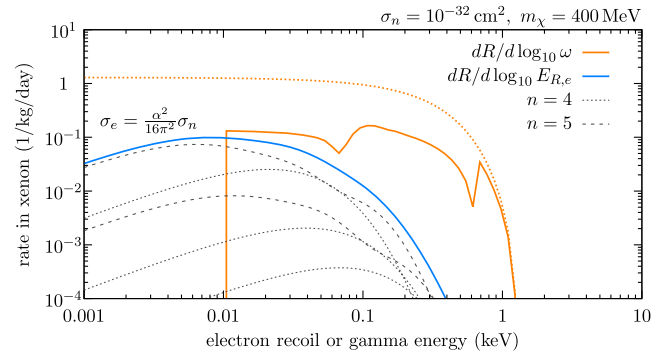


FIG. 4. Comparison of electron recoil (blue) vs photon emission (orange) events in a leptophobic model per logarithmic energy interval, $dR/d(\log_{10} E_{R,e})$ vs $dR/d(\log_{10} \omega)$, respectively, assuming that $\sigma_e = \alpha^2/(16\pi^2)\sigma_n$. The dotted (orange) line is the naïve rate. Thin gray lines break down the ionization contribution from the respective atomic shells (following previous calculations [3,4].)

and the resulting ionization signal [3,4]. Hence, best progress from our proposals can be expected in “leptophobic” models with suppressed (or absent) DM couplings to electrons. One of the simplest leptophobic models is DM coupled to the standard model through a light $U(1)_B$ gauge boson V_μ

$$\mathcal{L}_{\text{int}} = g_B(V^\mu J_\mu^B - \bar{\chi} N \chi) - \frac{\kappa}{2} V^{\mu\nu} F_{\mu\nu} \quad (11)$$

with g_B the $U(1)_B$ gauge coupling (charge); $J_B^\mu \equiv \frac{1}{3} \sum_i \bar{q}_i \gamma^\mu q_i$ is the baryon current (with the sum over all quark species). Even if $\kappa = 0$ at the tree level, DM-electron scattering may be induced radiatively, giving parametrically, if no cancellation occurs, $\kappa_{\text{rad}} \sim e g_B / (16\pi^2)$. This would lead to a ratio of cross sections of DM-electron over DM-nucleon scattering as $\sigma_e / \sigma_n = \alpha \kappa_{\text{rad}}^2 / \alpha_B \sim \alpha^2 / (16\pi^2) \sim 3 \times 10^{-7}$, which demonstrates that a large hierarchy can be achieved in this simple model; $\sigma_n \simeq 16\pi \alpha_B^2 m_\chi^2 m_V^{-4}$ for $m_V^2 \gg \mathbf{q}^2$. This is exemplified in Fig. 4 where we compare the rate of bremsstrahlung emission to the rate of DM-electron scattering $dR/dE_{R,e}$ for $m_\chi = 400 \text{ MeV}$. Any detailed analysis of electron multiplicity upon either scattering process will likely improve the sensitivity to bremsstrahlung because of a higher primary energy of the photon.

The model of gauged baryon number is constrained in a number of ways, notably from monojet production at colliders [45], from missing energy contributions to rare meson decays [34], and from cosmology; further, model-dependent constraints arising from the UV-completion of $U(1)_B$ are obtained in Ref. [46]. For illustration, in Fig. 3 we pick $m_V = 300 \text{ MeV}$, compatible with flavor constraints, and show the ensuing collider limit, which comes in this case from CDF. Finally, MiniBOONE may probe the window on large α_B and MeV scale m_V in the near future [34].

Conclusions.—In this Letter we show that the irreducible contribution of photon emission in the ordinary process of elastic DM-nucleus scattering, “bremsstrahlung,” opens up

the possibility to probe sub-GeV DM with present-day technology and conventional detectors. The photon endpoint energy is the kinetic energy of DM, and we derive the first limits on DM-nucleon scattering for $m_\chi < 500$ MeV.

Further progress along the lines suggested here can be made. First, atomic physics calculations should allow us to quantify contributions to photon emission from excited final states of the atom. Second, there is an additional contribution to the electron yield from the “shakeoff” of electrons in the elastic scattering [47]. Third, the spin-dependent case should be investigated. Fourth, signal formation in materials with band structure, where single-atom polarizability is inadequate, should be investigated. Fifth, photon emission in coherent neutrino nucleus scattering should be included in direct detection neutrino background estimates. A number of these points will be addressed in an upcoming paper [48].

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