Britto-Cachazo-Feng-Witten–Type Recurrent Relations for Tree Superamplitudes of $D = 11$ Supergravity

Igor Bandos*
Department of Theoretical Physics, University of the Basque Country UPV/EHU, P.O. Box 644, 48080 Bilbao, Spain and IKERBASQUE, Basque Foundation for Science, 48011 Bilbao, Spain

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We propose the on-shell superfield description for tree amplitudes of $D = 11$ supergravity and the Britto-Cachazo-Feng-Witten–type recurrent relations for these superamplitudes.

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In recent years, we have witnessed great progress in calculations of multiloop amplitudes (see, e.g., [1–4], and references therein), an important part of which is related to the applications and development of the Britto-Cachazo-Feng-Witten (BCFW) approach [5]. This first allowed us to obtain Britto-Cachazo-Feng (BCF) recursion relations for tree amplitudes in $D = 4$ Yang-Mills and $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory [6–8] and then was developed for the case of superamplitudes of $\mathcal{N} = 4$ SYM theory [9,10], loop (super)amplitudes, and $\mathcal{N} = 8$ supergravity [9–12] (see [11,12] for more references). To lighten the text, below we will mainly omit "super" in superamplitudes, calling them amplitudes.

This approach was generalized for the tree amplitudes of the $D = 10$ SYM model in Ref. [13] but then mainly used in the context of type IIB supergravity [14–17], where the presence of complex structure allowed us to lighten the "Clifford superfield" description of the amplitudes in Ref. [13]. The observation that the constrained bosonic spinor helicity variables used in Ref. [13] can be identified with spinor moving frame variables of Refs. [18–20] (or, equivalently, with Lorentz harmonics of Refs. [21,22]) [23] allowed us to simplify it('s $\mathcal{N} = 1$ version) [26] and also to generalize it to the case of $D = 11$ supergravity [27]. The results of this 11D generalization of the on-shell superfield description of tree amplitudes and of the BCFW recurrent relations for these will be reported in this Letter.

The BCFW recursion relations [5] are written for n-particle tree amplitudes $\mathcal{A}^{(n)}(p_{(1)}, \varepsilon_{(1)}; \ldots; p_{(n)}, \varepsilon_{(n)})$ in spinor helicity formalism, in which the information on the (lightlike) momentum $p_{\mu(i)}$ and on the helicity of the i-th external particle are encoded in the bosonic spinor $\lambda^A_{(i)} = (\bar{\lambda}^{\dot{A}}_{(i)})^*$. The lightlike momentum is defined by the Cartan-Penrose representation (see [31] and references therein)

$$p_{\mu(i)}\sigma^\mu_{A\dot{A}} = 2\lambda_{A(i)}\bar{\lambda}_{\dot{A}(i)} \Leftrightarrow p_{\mu(i)} = \lambda_{(i)}\sigma_\mu\bar{\lambda}_{(i)}, \tag{1}$$

where $\sigma^\mu_{A\dot{A}}$ are relativistic Pauli matrices, $A = 1, 2$ and $\dot{A} = 1, 2$ are Weyl spinor indices, and $\mu = 0, \ldots, 3$.

All n-particle amplitudes for the fields of the $\mathcal{N} = 4$ SYM theory can be described by a superfield amplitude (superamplitude) [9,10] $\mathcal{A}^{(n)}(\lambda_{(1)}, \bar{\lambda}_{(1)}, \eta_{(1)}; \ldots; \lambda_{(n)}, \bar{\lambda}_{(n)}, \eta_{(n)})$ depending, besides $\lambda^A_{(i)}$ and $\bar{\lambda}^{\dot{A}}_{(i)}$, on the set of n complex fermionic coordinates $\eta^q_{(i)} = (\bar{\eta}_{q(i)})^*$ (first introduced in Ref. [32]), $\eta^q_{(i)}\eta^p_{(j)} = -\eta^p_{(j)}\eta^q_{(i)}$, $\bar{\eta}_{q(i)}\eta^p_{(j)} = -\eta^p_{(j)}\bar{\eta}_{q(i)}$, carrying the index $q = 1, \ldots, 4$ of the fundamental representation of $SU(4)$. These superfield amplitudes are multiparticle counterparts of the so-called on-shell superfield

$$\Phi(\lambda,\bar{\lambda},\eta^q) = f^{(-)}(\lambda,\bar{\lambda}) + \eta^q\chi_q + \frac{1}{2}\eta^q\eta^p s_{pq} + \frac{1}{3!}\eta^q\eta^p\eta^r\epsilon_{rpqs}\bar{\chi}^s + \frac{1}{4!}\eta^q\eta^p\eta^r\eta^s\epsilon_{rpqs}f^{(+)} \tag{2}$$

describing all the states of the linearized SYM *provided* it obeys the so-called helicity constraint [31,32]

$$\hat{h}\Phi(\lambda,\bar{\lambda},\eta) = \Phi(\lambda,\bar{\lambda},\eta), \tag{3}$$

$$2\hat{h} \coloneqq -\lambda^A\frac{\partial}{\partial\lambda^A} + \bar{\lambda}^{\dot{A}}\frac{\partial}{\partial\bar{\lambda}^{\dot{A}}} + \eta^q\frac{\partial}{\partial\eta^q}. \tag{4}$$

The n-particle on-shell superfield amplitudes of 4D $\mathcal{N} = 4$ SYM theory, $\mathcal{A}^{(n)}(\lambda_{(1)}, \bar{\lambda}_{(1)}, \eta_{(1)}; \ldots; \lambda_{(n)}, \bar{\lambda}_{(n)}, \eta_{(n)}) \equiv \mathcal{A}^{(n)}(\ldots; \lambda_i, \eta_i; \ldots)$, should obey the set of n helicity constraints

$$\hat{h}_{(i)}\mathcal{A}^{(n)}(\ldots;\lambda_i,\bar{\lambda}_i,\eta_i;\ldots) = \mathcal{A}^{(n)}(\ldots;\lambda_i,\bar{\lambda}_i,\eta_i;\ldots), \tag{5}$$

with $2\hat{h}_{(i)} \coloneqq -\lambda^A_{(i)}\partial/\partial\lambda^A_{(i)} + \bar{\lambda}^{\dot{A}}_{(i)}\partial/\partial\bar{\lambda}^{\dot{A}}_{(i)} + \eta^q_{(i)}\partial/\partial\eta^q_{(i)}$.

We refer to Refs. [9,10] for the superfield generalization of the original $D = 4$ BCFW recurrent relations [5] and pass to the 11D generalization of the spinor helicity formalism.

Spinor helicity formalism in $D = 11$.—Let us denote the $D = 11$ vector indices by $a, b, c = 0, 1, \ldots, 9, 10$, spinor indices of $SO(1, 10)$ by $\alpha, \beta, \gamma, \delta = 1, \ldots, 32$, and $D = 11$ Dirac matrices by $\Gamma_{a\alpha}{}^\beta$. In our mostly minus notation, $\eta^{ab} = \mathrm{diag}(+1, -1, \ldots, -1)$, both $\Gamma_{a\alpha}{}^\beta$ and the charge conjugation matrix $C^{\alpha\beta} = -C^{\beta\alpha}$ are imaginary. We will also use the real symmetric matrices $\Gamma^a_{\alpha\beta} = \Gamma^{a\gamma}_{\alpha} C_{\gamma\beta} = \Gamma^a_{\beta\alpha}$ and $\tilde{\Gamma}^{\alpha\beta}_a = C^{\alpha\gamma}\Gamma^{\alpha\beta}_{\gamma} = \tilde{\Gamma}^{\beta\alpha}_a$.

The lightlike momentum of a massless 11D particle can be expressed by relations similar to (1):

$$k_a \Gamma^a_{\alpha\beta} = 2\rho^{\#} v^-_{\alpha q} v^-_{\beta q}, \qquad \rho^{\#} v^-_q \tilde{\Gamma}_a v^-_p = k_a \delta_{qp}, \tag{6}$$

in terms of "energy variable" $\rho^{\#}$ and a set of 16 constrained bosonic 32-component spinors $v^-_{\alpha q}$, $q, p = 1, \ldots, 16$, which can be identified with $D = 11$ spinor moving frame variables [33–35] or Lorentz harmonics [36]. Essentially, the constraints on $v^-_{\alpha q}$ are given by Eq. (6) supplemented by $v^-_{\alpha q} C^{\alpha\beta} v^-_{\beta q} = 0$ and by the requirement that the rank of 32×16 matrix $v^-_{\alpha q}$ is equal to 16. We refer to Refs. [34,35] for the complete description and discussion of the constraints and gauge symmetries of the spinor moving frame formalism for the 11D massless superparticle and only notice that, taking all these into account, the variables $v^-_{\alpha q}$ can be considered as homogeneous coordinates on $\mathbb{S}^9$, the celestial sphere of a $D = 11$ observer,

$$\{v^-_{\alpha q}\} = \mathbb{S}^9. \tag{7}$$

The sign superindices $^-$ and $^{\#} \equiv ^{++}$, carried by $v^-_{\alpha q}$ and $\rho^{\#}$, characterize their scaling properties with respect to the $SO(1,1)$ gauge symmetry of the spinor moving frame (or Lorentz harmonic) approach to a massless (super)particle.

One can check that, due to (6) and $v^-_q C v^-_p = 0$, the momentum vector k_a is lightlike, $k_a k^a = 0$, and moreover the spinor moving frame variables $v^-_{\alpha q}$ obey the massless Dirac equation (in momentum representation)

$$k_a \tilde{\Gamma}^{a\alpha\beta} v_{\beta q}{}^- = 0 \Leftrightarrow k_a \Gamma^a_{\alpha\beta} v_q^{-\beta} = 0. \tag{8}$$

The 11D counterparts of the 10D spinor helicity variables of Ref. [13] are $\lambda_{\alpha q} = \sqrt{\rho^{\#}} v^-_{\alpha q}$; the counterpart of the polarization spinor of the 10D fermionic field in $D = 11$ is given by the same helicity spinor but with the risen spinor index, $\lambda^\alpha_q = \sqrt{\rho^{\#}} v_q^{-\alpha} = -iC^{\alpha\beta}\lambda_{\beta q}$ $[= (\lambda^\alpha_q)^*]$.

One notices that Eqs. (6) can be written as $\Gamma^a_{\alpha\beta} k_a = 2\lambda_{\alpha q}\lambda_{\beta q}$ and $\lambda_q \tilde{\Gamma}_a \lambda_p = k_a \delta_{qp}$. However, the energy variable $\rho^{\#}$ and its canonically conjugate coordinate $x^{=}$ play an important role in our construction below. In particular, the $D = 11$ counterparts of the on-shell superfields are defined on superspace

$$\Sigma^{(10|16)}: \{(x^{=}, v^-_{\alpha q}; \theta^-_q)\}, \tag{9}$$

with bosonic sector $\mathbb{R} \otimes \mathbb{S}^9$ [see (7)] including $\mathbb{R} = \{x^{=}\}$.

$D = 11$ *on-shell superfields.*—The description of linearized 11D supergravity multiplet by superfields in the on-shell superspace (9) was proposed in Ref. [36] (and can be reproduced when quantizing the massless 11D superparticle [26]). It was given in terms of a bosonic antisymmetric tensor superfield $\Phi^{IJK} = \Phi^{[IJK]}(x^{=}, \theta^-_q, v_{\alpha q}{}^-)$ which obeys

$$D^+_q \Phi^{IJK} = 3i\gamma^{[IJ}_{qp}\Psi^{K]}_p, \qquad \gamma^I_{qp}\Psi^I_p = 0. \tag{10}$$

Here $I, J, K = 1, \ldots, 9$, $q, p = 1, \ldots, 16$, $\gamma^I_{qp} = \gamma^I_{pq}$ are $d = 9$ Dirac matrices, $\gamma^I\gamma^J + \gamma^J\gamma^I = \delta^{IJ}\mathbb{I}_{16\times16}$, and

$$D^+_q = \partial^+_q + 2i\theta^-_q \partial_{=} \equiv \frac{\partial}{\partial\theta^-_q} + 2i\theta^-_q \frac{\partial}{\partial x^{=}} \tag{11}$$

is the fermionic covariant derivative obeying the $d = 1$, $\mathcal{N} = 16$ supersymmetry algebra $\{D^+_q, D^+_p\} = 4i\delta_{qp}\partial_{=}$.

The consistency of Eq. (10) requires that fermionic superfield Ψ^I_q satisfies, besides $\gamma^I_{qp}\Psi^I_p = 0$,

$$D^+_q \Psi^I_p = \frac{1}{18}(\gamma^{IJKL}_{qp} + 6\delta^{I[J}\gamma^{KL]}_{qp})\partial_{=}\Phi^{JKL} + 2\partial_{=} H_{IJ}\gamma^J_{qp}, \tag{12}$$

with symmetric traceless $SO(9)$ tensor superfield $H_{IJ} = H_{((IJ))}$ (below, to simplify notation, we will write $_{(IJ)}$ instead of $_{((IJ))}$ indices), obeying

$$D^+_q H_{IJ} = i\gamma^{(I}_{qp}\Psi^{J)}_p, \qquad H_{IJ} = H_{JI}, \qquad H_{II} = 0. \tag{13}$$

The leading component of this bosonic superfield, $h_{IJ}(x^{=}, v^-_{\alpha q}) = H_{IJ}|_{\theta^-_q = 0}$, describes the on-shell degrees of freedom of the 11D graviton (see [36] for more details).

One can collect all the above on-shell superfields in

$$\Psi_Q(x^{=}, v^-_{\alpha q}; \theta^-_q) = \{\Psi_{Iq}, \Phi_{[IJK]}, H_{(IJ)}\}, \tag{14}$$

with multi-index Q taking 128 ($= 144 - 16$) "fermionic" and $128 = 84 + 44$ "bosonic values," $Q = \{Iq, [IJK], (IJ)\}$. The set of equations (12), (10), and (13) can be unified in

$$D^+_q \Psi_Q = \Delta_{QqP}\Psi_P, \tag{15}$$

where the operator Δ_{QqP} can be easily read off Eqs. (12), (10), and (13). It contains differential operator $\partial_{=}$ when $Q = Iq$ and is purely algebraic otherwise. This difference is diminished when passing to the Fourier images of the superfields with respect to the $x^{=}$ coordinate, $\Psi_Q(\rho^{\#}, v^-_{\alpha q}; \theta^-_q) = (1/2\pi)\int dx^{=} \exp(i\rho^{\#}x^{=})\Psi_Q(x^{=}, v^-_{\alpha q}; \theta^-_q)$. These obey the same equation (15) but with $\partial_{=} \mapsto -i\rho^{\#}$ and

$$D^+_q = \partial^+_q + 2\rho^{\#}\theta^-_q. \tag{16}$$

As we have already noticed, the set of Eqs. (12), (10), and (13), collected in (15), are dependent. We can choose any of them and reproduce two others from its consistency conditions. Passing to the Fourier image makes it natural to choose the fermionic superfield as fundamental and to describe the linearized 11D supergravity by the equation

$$D^+_q \Psi^I_p = -\frac{i\rho^{\#}}{18}(\gamma^{IJKL} + 6\delta^{I[J}\gamma^{KL]})_{qp}\Phi^{JKL} - 2i\rho^{\#} H_{IJ}\gamma^J_{qp}. \tag{17}$$

Equations (15) [i.e., the set of Eqs. (10), (12), and (13)] and $\gamma^I_{qp}\Psi^I_p = 0$ play the role of $D = 4$ helicity constraint (3). Then it is natural to expect that an on-shell tree superfield amplitude should satisfy essentially the same set of equations for each of the scattered particles.

Tree on-shell amplitudes in $D = 11$.—The tree on-shell n-particle scattering amplitudes can be described as a function in a direct product of n copies of the on-shell superspace (9)"

$$\mathcal{A}^{(n)}_{Q_1\ldots Q_n}(k_1,\theta^-_1;\ldots;k_n,\theta^-_n) \equiv \mathcal{A}^{(n)}_{\ldots Q_l\ldots}(\ldots;k_l,\theta^-_l;\ldots) \equiv \mathcal{A}^{(n)}_{\ldots Q_l\ldots}(\ldots;\rho^{\#}_{(l)},v^-_{q(l)},\theta^-_{q(l)};\ldots), \tag{18}$$

carrying n multi-indices $Q_l = \{I_l q_l, [I_l J_l K_l], (I_l J_l)\}$ [see (14)]. As indicated in (18), for shortness we often write the bosonic argument of the amplitude as $k^a_{(l)}$ instead of $\rho^{\#}_{(l)}; v^-_{q(l)}$ [implying that $k^a_{(l)}$ is expressed in terms of these by (6), where $\rho^{\#}_{(l)}$ is allowed to be negative]. We will also omit the arguments of the amplitude when this does not produce confusion.

The set of equations for the 11D amplitudes, playing the role of $D = 4$ helicity constraints (5), includes, besides the γ-tracelessness on every fermionic multi-index $I_l q_l$,

$$\gamma^{I_l}_{p_l q_l}\mathcal{A}_{\ldots I_{(l)} q_{(l)}\ldots} = 0, \tag{19}$$

the equation

$$D^+_{q(l)}\mathcal{A}_{\ldots Q_{(l)}\ldots} = (-)^{\Sigma_l}\Delta_{Q_l q P_{(l)}}\mathcal{A}_{\ldots P_{(l)}\ldots}, \tag{20}$$

where $\Delta_{Q_l q P_{(l)}}$ is the same as in (15) [i.e., can be read off (17), (10), and (13)], but acting on variables and indices corresponding to the lth particle, and Σ_l can be defined as the number of fermionic, $I_j q_j$, indices among $Q_1, \ldots Q_{(l-1)}$, i.e.,

$$\Sigma_l = \sum_{j=1}^{l-1}\frac{(1-(-)^{\varepsilon(Q_j)})}{2}, \quad \begin{cases}\varepsilon([I_jJ_jK_j]) = 0 = \varepsilon(I_jJ_j),\\ \varepsilon(I_jq_j) = 1.\end{cases} \tag{21}$$

In particular, when $Q_l = I_l p_l$, Eq. (20) reads

$$(-)^{\Sigma_l}D^{+(l)}_{q_l}\mathcal{A}^{(n)}_{Q_1\ldots I_l p_l\ldots Q_n} = -2i\rho^{\#}_{(l)}\gamma_{J_l qp}\mathcal{A}^{(n)}_{Q_1\ldots(I_lJ_l)\ldots Q_n} - \frac{i}{18}\rho^{\#}_{(l)}(\gamma^{I_lJ_lK_lL_l}_{qp} + 6\delta^{I_l[J_l}\gamma^{K_lL_l]}_{qp})\mathcal{A}^{(n)}_{Q_1\ldots[J_lK_lL_l]\ldots Q_n}. \tag{22}$$

Generalized BCFW deformation in $D = 11$.—To write the generalized BCFW recurrent relations in $D = 11$, we have to define the generalized BCFW deformation of bosonic and fermionic variables of the above described 11D on-shell superfield formalism.

As in the original 4D construction [5], the deformation of, say, the first and the nth particle variables should imply the opposite shift of their lightlike momenta:

$$k^{\hat{a}}_{(1)} = k^a_{(1)} - zq^a, \qquad k^{\hat{a}}_{(n)} = k^a_{(n)} + zq^a, \tag{23}$$

on a lightlike vector q^a orthogonal to both $k^a_{(1)}$ and $k^a_{(n)}$:

$$q_a q^a = 0, \qquad q_a k^a_{(1)} = 0, \qquad q_a k^a_{(n)} = 0, \tag{24}$$

multiplied by an arbitrary complex number $z \in \mathbb{C}$ [5] (the 10D construction of Ref. [13] used real $z \in \mathbb{R}$). Equations (24) guarantee that the deformed momenta remain lightlike:

$$(k_{(1)})^2 = 0 = (k_{(n)})^2 \Rightarrow (k_{\hat{(1)}})^2 = 0 = (k_{\hat{(n)}})^2. \tag{25}$$

Thus, the amplitude depending on these, instead of original $k^a_{(1)}$ and $k^a_{(n)}$, $\mathcal{A}_{zQ_1\ldots Q_n}(k_{\hat{(1)}},\theta^-_{\hat{(1)}};\ldots k_{\hat{(n)}},\theta^-_{\hat{(n)}})$, remains an on-shell amplitude.

In $D = 4$, the deformation of the momenta (25) results from the following deformation of the bosonic spinors entering the Penrose representation (1):

$$\lambda^{\hat{A}}_{(n)} = \lambda^A_{(n)} + z\lambda^A_{(1)}, \qquad \bar{\lambda}^{\hat{\dot{A}}}_{(1)} = \bar{\lambda}^{\dot{A}}_{(1)} - z\bar{\lambda}^{\dot{A}}_{(n)}, \tag{26}$$

In $D = 11$, (25) results from the following deformation of the associated spinor moving frame variables:

$$v^{-}_{\alpha q\hat{(n)}} = v^-_{\alpha q(n)} + zv^-_{\alpha p(1)}\mathbb{M}_{pq}\sqrt{\rho^{\#}_{(1)}/\rho^{\#}_{(n)}}, \tag{27}$$

$$v^{-}_{\alpha q\hat{(1)}} = v^-_{\alpha q(1)} - z\mathbb{M}_{qp}v^-_{\alpha p(n)}\sqrt{\rho^{\#}_{(n)}/\rho^{\#}_{(1)}}, \tag{28}$$

which enter the Penrose-like constraints (6),

$$k^a_{(i)}\Gamma_{a\alpha\beta} = 2\rho^{\#}_{(i)}v^-_{\alpha q(i)}v^-_{\beta q(i)}, \qquad k_{a(i)}\delta_{qp} = \rho^{\#}_{(i)}v^-_{q(i)}\tilde{\Gamma}_a v^-_{p(i)}. \tag{29}$$

The energy variables $\rho^{\#}_{(i)}$ are not deformed. The matrix $\mathbb{M}_{qp}$ is constructed from the lightlike vector q^a of (25):

$$\mathbb{M}_{qp} = -q^a(v^-_{q(1)}\tilde{\Gamma}_a v^-_{p(n)})\sqrt{\rho^{\#}_{(1)}\rho^{\#}_{(n)}}/(k_{(1)}k_{(n)}) \tag{30}$$

(cf. with 10D relations in Ref. [13]), with $16k^a_{(i)} = \rho^{\#}_{(i)}v^-_{q(i)}\tilde{\Gamma}^a v^-_{q(i)}$ [see (29)], and is nilpotent:

$$\mathbb{M}_{rp}\mathbb{M}_{rq} = 0, \qquad \mathbb{M}_{qr}\mathbb{M}_{pr} = 0, \tag{31}$$

due to (24). This nilpotent matrix enters also the deformation rules of the fermionic coordinates:

$$\theta^-_{p\hat{(n)}} = \theta^-_{p(n)} + z\theta^-_{q(1)}\mathbb{M}_{qp}\sqrt{\rho^{\#}_{(1)}/\rho^{\#}_{(n)}}, \tag{32}$$

$$\theta^-_{q\hat{(1)}} = \theta^-_{q(1)} - z\mathbb{M}_{qp}\theta^-_{p(n)}\sqrt{\rho^{\#}_{(n)}/\rho^{\#}_{(1)}}. \tag{33}$$

These can be also written as

$$\theta^-_{p\hat{(i)}} = e^{-zD^+_{(1)}\mathbb{M}\theta^-_{(n)} - z\theta^-_{(1)}\mathbb{M}D^+_{(n)}}\theta^-_{p(i)}, \tag{34}$$

where the covariant fermionic derivatives $D^+_{q(i)}$ are defined in (16). Their deformation

$$D^{\hat{+}}_{q(i)} = e^{-zD_{(1)}\mathbb{M}\theta_{(n)} - z\theta_{(1)}\mathbb{M}D_{(n)}} D^+_{q(i)} e^{zD_{(1)}\mathbb{M}\theta_{(n)} + z\theta_{(1)}\mathbb{M}D_{(n)}} \quad (35)$$

is similar to the deformation of 8d Clifford algebra valued variables in the 10D construction of Ref. [13].

Generalized BCFW recurrent relations for tree amplitudes in $D = 11$.—The deformed tree amplitude is defined as an amplitude depending on deformed momenta and fermionic coordinates. We denote it by

$$\begin{aligned} \mathcal{A}_z^{(n)}{}_{\ldots Q_l \ldots} &:= \mathcal{A}_z^{(n)}{}_{Q_1 \ldots Q_l \ldots Q_n}(k_{\hat{(1)}}, \ldots; k_{\hat{(l)}}, \theta^-_{\hat{(l)}}; \ldots, \theta^-_{\hat{(n)}}) \\ &= \mathcal{A}_z^{(n)}{}_{Q_1 \ldots Q_n}(k_{\hat{(1)}}, \theta^-_{\hat{(1)}}; k_{(2)}, \ldots, \theta^-_{(n-1)}; k_{\hat{(n)}}, \theta^-_{\hat{(n)}}), \end{aligned} \quad (36)$$

where in the last line it is assumed that the deformed momenta correspond to the first and nth of the scattered particles [so that $k_{\hat{(l)}}, \theta^-_{\hat{(l)}} = k_{(l)}, \theta^-_{(l)}$ for $l = 2, \ldots, (n-1)$] and the subscript z indicates the parameter used in this deformation (27)–(33). Notice that deformed amplitudes (36) satisfy, besides the gamma-tracelessness (19), Eqs. (20) with deformed derivatives (35):

$$D^{\hat{+}}_{q(l)} \hat{\mathcal{A}}_{zQ_1 \ldots Q_{(l)} \ldots} = (-)^{\Sigma_l} \Delta_{Q_l q P_{(l)}} \hat{\mathcal{A}}_{z, Q_1 \ldots P_{(l)} \ldots}. \quad (37)$$

In particular,

$$(-)^{\Sigma_l} D^{\hat{+}}_{q_l(l)} \hat{\mathcal{A}}_{z \ldots [I_l J_l K_l] \ldots} = 3i\gamma_{[J_l K_l | q_l p_l} \hat{\mathcal{A}}_{z \ldots | I_l] p_l \ldots}, \quad (38)$$

$$(-)^{\Sigma_l} D^{\hat{+}}_{q_l(l)} \hat{\mathcal{A}}_{z \ldots (I_l J_l) \ldots} = i\gamma_{q_l p_l (I_l|} \hat{\mathcal{A}}_{z \ldots | J_l) p_l \ldots}. \quad (39)$$

The proposed BCFW-type recurrent relation for tree superfield amplitudes of 11D supergravity reads

$$\begin{aligned} \mathcal{A}^{(n)}_{Q_1 \ldots Q_n}(k_1, \theta^-_{(1)}; k_2, \theta^-_{(2)}; \ldots; k_n, \theta^-_{(n)}) &= \sum_l^n \frac{(-)^{\Sigma_{(l+1)}}}{64[\hat{\rho}^\#(z_l)]^2} D^+_{q(z_l)} \{ \hat{\mathcal{A}}^{(l+1)}_{z_l Q_1 \ldots Q_l J p}[\hat{k}_1, \theta^-_{\hat{(1)}}; k_2, \theta^-_{(2)}; \ldots; k_l, \theta^-_{(l)}; \hat{P}_l(z_l), \Theta^-] \\ &\times \frac{1}{(P_l)^2} \overleftrightarrow{D}^+{}_{q(z_l)} \hat{\mathcal{A}}^{(n-l+1)}_{z_l J p Q_{l+1} \ldots Q_n}[-\hat{P}_l(z_l), \Theta^-; k_{l+1}, \theta^-_{(l+1)}; \ldots; k_{n-1}, \theta^-_{(n-1)}; \hat{k}_n, \theta^-_{\hat{(n)}}] \}|_{\Theta^- = 0}. \end{aligned} \quad (40)$$

Here

$$P^a_l = -\sum_{m=1}^{l} k^a_m, \quad (41)$$

$$\hat{P}^a_l(z) = -\sum_{m=1}^{l} k^{\hat{a}}_m(z) = P^a_l - zq^a, \quad (42)$$

$$z_l := P^a_l P_{la} / (2P^b_l q_b), \quad (43)$$

with q^a obeying (24) and (30) [37]. Equation (42) implies that $[\hat{P}_l(z)]^2 = (P_l)^2 - 2zP_l \cdot q$, so that $\hat{P}^a_l(z_l)$ is lightlike:

$$[\hat{P}_l(z_l)]^2 = 0, \qquad z_l := (P_l)^2 / (2P_l \cdot q). \quad (44)$$

As a result, first, both amplitudes in the rhs of (40) are on the mass shell, and, second, we can express $\hat{P}^a_l(z)$ in terms of associated spinor moving frame variables $v^-_{\alpha q}(z_l) := v^-_{\alpha q \hat{P}_l(z_l)}$ and energy $\pm\hat{\rho}^\#(z_l)$ [see (6)]:

$$\begin{aligned} \hat{P_l}^a(z_l)\Gamma_{a\alpha\beta} &= 2\hat{\rho}^\#(z_l) v^-_{\alpha q}(z_l) v^-_{\beta q}(z_l), \\ \hat{P_l}^a(z_l)\delta_{qp} &= \hat{\rho}^\#(z_l) v^-_q(z_l) \tilde{\Gamma}^a v^-_p(z_l). \end{aligned} \quad (45)$$

This $\hat{\rho}^\#(z_l)$ enters the denominator of the terms in rhs of (40) (which is needed to simplify the relation between the amplitude and superamplitude).

Actually, the bosonic arguments of the on-shell amplitudes are energies $\rho^\#_{(i)}$ and $v^-_{\alpha(i)}$ related to lightlike momenta $k^a_{(i)}$ by (29) and the above $v^-_{\alpha q}(z_l)$ and $\pm\hat{\rho}^\#(z_l)$; just for shortness in (40), following (18), we hide this, writing instead the dependence on the momenta.

Finally, $D^+_{q(z_l)}$ in (40) is the covariant derivative with respect to Θ^-_q constructed with the use of $\hat{\rho}^\#(z_l)$ of (45):

$$D^+_{q(z_l)} = \frac{\partial}{\partial \Theta^-_q} + 2\hat{\rho}^\#(z_l)\Theta^-_q. \quad (46)$$

Notice that the structure of the rhs of (40),

$$\begin{aligned} &D^+_q(\mathcal{A}_{\ldots Jp} \overleftrightarrow{D}^+{}_q \mathcal{A}_{Jp\ldots})|_{\Theta^- = 0} \\ &\equiv D^+_q(\mathcal{A}_{\ldots Jp} D^+_q \mathcal{A}_{Jp\ldots} - (-)^{\Sigma_l} D^+_q \mathcal{A}_{\ldots Jp} \mathcal{A}_{Jp\ldots})|_0, \end{aligned} \quad (47)$$

can be treated as an integration over the fermionic variable Θ^-_q in (47) with an exotic measure similar to one used in Refs. [38,39] to construct a world sheet superfield formulation of the heterotic string (see [40] for a formal discussion on superspace measures).

To argue that there is no contribution to the rhs of (40) of a pole at $|z| \mapsto \infty$, we can use the line of arguments presented in Ref. [13] for the 10D case, which refers on the case when external momenta lay in some 4d subspace of spacetime and on the original proof of Ref. [5], which was extended to $\mathcal{N} = 8$ supergravity in Refs. [9–11].

The calculation of sample tree superamplitudes of 11D supergravity with the use of the above BCFW-type recurrent relations (45) and generalization of these to loop amplitudes will be the subject of subsequent work. See Supplemental Material [41] for some technicalities needed to proceed with explicit superamplitude calculations.

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*igor.bandos@ehu.eus

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