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Dark Energy from Violation of Energy Conservation

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In this Letter, we consider the possibility of reconciling metric theories of gravitation with a violation of the conservation of energy-momentum. Under some circumstances, this can be achieved in the context of unimodular gravity, and it leads to the emergence of an effective cosmological constant in Einstein's equation. We specifically investigate two potential sources of energy nonconservation—nonunitary modifications of quantum mechanics and phenomenological models motivated by quantum gravity theories with spacetime discreteness at the Planck scale—and show that such locally negligible phenomena can nevertheless become relevant at the cosmological scale.

Ever since the discovery of the acceleration in the Universe's expansion [1,2], almost two decades ago, there has been a puzzlement about the strange value of the corresponding cosmological constant Λ, the simplest, and so far most successful, theoretical model that could account for the observed behavior. The origin of this puzzle is that, within the usual framework, the only seemingly natural values that Λ could take are either zero or a value which is 120 orders of magnitude larger than the one indicated by observations $\Lambda^{obs} \approx 1.1 \times 10^{-52}$ m^{-2} [3].

In this Letter, we present a scenario where something very similar to a cosmological constant emerges from certain sources of violations of energy-momentum conservation and their influence on the space-time geometry. We will consider here two different sets of ideas, motivating such violations, but these could also be introduced at a purely phenomenological level. On the one hand, such violations are commonplace in the context of certain nonunitary modifications of Schrödinger's equation [4] proposed as a way to address the measurement problem in quantum mechanics [5]. On the other hand, they are natural in quantum gravity approaches where fundamental spacetime discreteness could lead to small violations of translational invariance. A concrete model of phase space diffusion due to Planckian granularity (proposed in the context of causal sets in Refs. [6,7]) will be used here as an example.

One of the most serious difficulties faced by such proposals relates to their consistency (or lack thereof) with the gravitational interaction. We point out that this tension can be resolved in the framework of unimodular gravity in the cosmological setting: This resolution leads to the appearance of an effective cosmological constant that registers the cumulative effect of the lack of energy-momentum conservation. In the cosmological setting, we estimate the contribution to the effective cosmological

constant arising from violations of energy-momentum conservation predicted by modified quantum mechanics models as well as by the proposal based on the causal set approach to quantum gravity. We show that these contributions can be comparable in size with the value of the cosmological constant inferred from current observations.

Thus, and on a more general ground, our work proposes a new paradigm for analyzing the dark energy puzzle in cosmology, that identifies potential violations of energy-momentum conservation in the past (that could be postulated on a simply phenomenological ground) as a source of dark energy today.

In general relativity, local energy-momentum conservation $\nabla^b \langle T_{ab} \rangle = 0$ is a consequence of the field equations, at both the classical and semiclassical levels. This is obvious from the semiclassical version of Einstein's equation

$$R_{ab} - \frac{1}{2}Rg_{ab} = \frac{8\pi G}{c^4}\langle T_{ab} \rangle, \qquad (1)$$

where $\langle T_{ab} \rangle$ is the expectation value of the (renormalized) energy-momentum tensor operator in the corresponding quantum state of the matter fields—and the fact that the Bianchi identities make the geometric side divergence free. (For other views on the issue, we refer the reader to the arguments claiming that semiclassical general relativity is simply unviable [8], a dissenting opinion [9], and for an alternative way of looking at such a theory [10].)

The previous restriction can be circumvented by considering a simple modification of general relativity, already evoked by Einstein in 1919 when trying to construct a geometric account for elementary particles in terms of radiation fields [11]. He proposed the trace-free equation

$$R_{ab} - \frac{1}{4}Rg_{ab} = \frac{8\pi G}{c^4}\left(T_{ab} - \frac{1}{4}Tg_{ab}\right), \qquad (2)$$

which has been rediscovered several times and is now called unimodular gravity (see [12] and references therein). Unimodular gravity can be derived from the Einstein-Hilbert action by restricting to variations preserving the volume form, i.e., those for which $g_{ab}\delta g^{ab} = 0$.

This breaks the diffeomorphism symmetry down to volume-preserving diffeomorphism, whose infinitesimal version is given by divergence-free vector fields ξ^a, i.e.,

$$\nabla_a \xi^a = 0. \qquad (3)$$

This restriction on general covariance allows for violations of energy-momentum conservation of a certain form. To see this, consider an action for matter S_m invariant under volume-preserving diffeomorphisms and introduce the stress-energy tensor $T_{ab} \equiv -2|g|^{-1/2}\delta S_m/\delta g^{ab}$ and its energy-momentum violation current $J_a \equiv \nabla^b T_{ab}$. The variation of the action under an infinitesimal diffeomorphism (of compact support) ξ^a is

$$\delta S_m = -\int T_{ab}\nabla^a \xi^b \sqrt{-g}dx^4 = \int J_a\xi^a \sqrt{-g}dx^4, \qquad (4)$$

where the matter field equations are assumed to hold. Inserting the general solution of (3) $\xi^a = \epsilon^{abcd}\nabla_b\omega_{cd}$— for an arbitrary two-form ω—the requirement that the action is invariant under volume-preserving diffeomorphisms ($\delta S_m = 0$) implies $dJ = 0$. Hence, violations of energy-momentum conservation are allowed in unimodular gravity as long as they are of such an integrable type.

For simply connected spacetimes, this condition reduces to

$$J_a = \nabla_a Q, \qquad (5)$$

for some scalar field Q. Thus, if the matter action is invariant only under volume-preserving diffeomorphisms, then $J \neq 0$ will introduce deviations from general relativity. We will discuss later the naturalness of such symmetry breaking in quantum field theory.

An important feature of unimodular gravity in the semiclassical framework is that vacuum fluctuations of the energy-momentum tensor do not gravitate [3]. This removes the need to contemplate the enormous discrepancy between the observed value of the cosmological constant and the standard estimates from the vacuum energy [12–14]. It has also been argued that unimodular gravity may not suffer from the problem of time [15,16]; however, this view has been criticized and clarified in Ref. [17].

Let us move on and consider the semiclassical version of Eq. (2), where the energy-momentum tensor and its trace are now replaced by the corresponding expectation values in a quantum state of the matter fields. Using Bianchi identities, one then deduces that

$$\frac{1}{4}\nabla_a R = \frac{8\pi G}{c^4}\left(\nabla^b\langle T_{ab}\rangle - \frac{1}{4}\nabla_a\langle T\rangle\right), \qquad (6)$$

which, after integration, can be used to recast (2) as

$$R_{ab} - \frac{1}{2}Rg_{ab} + \left(\Lambda_{-\infty} + \frac{8\pi G}{c^4}Q\right)g_{ab} = \frac{8\pi G}{c^4}\langle T_{ab}\rangle, \qquad (7)$$

where $\Lambda_{-\infty}$ is a constant of integration and Q is defined by (5) [18]. As expected, when the stress-energy tensor is conserved, i.e., $Q = 0$, Eq. (7) simply reduces to Einstein's equation, with a cosmological constant equal to $\Lambda_{-\infty}$.

We emphasize that both semiclassical general relativity and its unimodular version are regarded here as an effective and emergent description of more fundamental degrees of freedom (just like the Navier-Stokes description of a fluid). The violation of energy-momentum conservation, in our scenario, would have to admit a description in terms of the more fundamental (presumably) quantum gravitational degrees of freedom.

Specializing to cosmology and considering an homogeneous, isotropic, and spatially flat Friedmann-Lemaître-Robertson-Walker universe, $ds^2 = -c^2dt^2 + a^2d\vec{x}^2$, the modified Friedmann equation reads

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\rho(t) + \frac{\Lambda^{\text{eff}}(t)c^2}{3}, \qquad (8)$$

where the effective cosmological "constant"

$$\Lambda^{\text{eff}}(t) \equiv \Lambda_{-\infty} + \frac{8\pi G}{c^4}\int^t J \qquad (9)$$

registers the possible violations of energy-momentum conservation in the past history of the Universe. We have reexpressed $Q = \int^t J$, as it will be more convenient for explicit calculations in the following paragraphs. As we shall see later, small violations of energy-momentum conservation—that might remain inaccessible to current tests of local physics—can nevertheless have important cosmological effects at late times, in the form of a nontrivial contribution to the present value of the cosmological constant.

The first scenario—leading to a violation of energy-momentum conservation—that we explore is the one offered by nonunitary modifications of quantum dynamics. In order to recover Born's rule for probabilities of experimental outcomes, these modifications of the quantum theory involve nonlinearity and stochasticity, which for a wide class of models can be described by a Markovian evolution equation for the density matrix $\hat{\rho}$: the so-called Kossakowski-Lindblad equation [19,20]

$$\dot{\hat{\rho}} = -i[\hat{H}, \hat{\rho}] - \frac{1}{2}\sum_\alpha \lambda_\alpha[\hat{K}_\alpha, [\hat{K}_\alpha, \hat{\rho}]], \qquad (10)$$

where $\hat{H}$ is the standard Schrödinger Hamiltonian operator, $\{\hat{K}_\alpha\}$ are Hermitian operators characterizing the modified dynamics, and $\{\lambda_\alpha\}$ are suitable parameters determining the strength of the new effects.

Such an equation has been used to describe a possible nonunitary evolution induced by the creation and evaporation of black holes [21,22], in the context of Hawking's information puzzle [23]. It also appears in the description of modifications of quantum mechanics with a spontaneous stochastic collapse [4]. It has been argued by Penrose [24] that the two apparently different contexts could actually be related in a more fundamental description of quantum gravitational phenomena (for a recent development, see [25]). Finally, the previous equation would also arise in the description of decoherence with an underlying discrete spacetime [26–28]. In all these cases, a generic feature of Eq. (10) is that the average energy $\langle E \rangle \equiv \text{Tr}[\hat{\rho}\hat{H}]$ is not constant.

One of the prominent models of this type, for non-relativistic particles, is the so-called mass-proportional continuous spontaneous localization (CSL) model [29–32], obtained when $\hat{K}_\alpha$ are smeared mass-density operators. It exhibits a ceaseless creation of energy proportional to the mass of the object collapsing [33]. Thus, in the cosmological context, the CSL of baryons leads to an energy-momentum violation current

$$J = -\xi_{\text{CSL}}\rho^b dt, \qquad (11)$$

where ρ^b is the energy density of the baryonic fluid and the parameter ξ_{CSL} is constrained by current experiments according to $3.3 \times 10^{-42}\,\text{s}^{-1} < \xi_{\text{CSL}} < 2.8 \times 10^{-29}\,\text{s}^{-1}$ (see Fig. 4 in Ref. [34]). Choosing hadronization ($z_h \approx 7 \times 10^{11}$) as the initial time, (9) and (11) lead to

$$\Delta\Lambda_{\text{CSL}}^{\text{eff}} \approx -\frac{3\Omega_0^b H_0 \xi_{\text{CSL}}}{\sqrt{\Omega_0^r} c^2} z_h \approx -\frac{\xi_{\text{CSL}}}{4.3 \times 10^{-31}\text{s}^{-1}}\Lambda^{\text{obs}}, \quad (12)$$

where Λ^{obs} is the observed value of the cosmological constant and we used standard values for the cosmological parameters [35]. For simplicity, the contribution to Ω^r of particles like electrons, muons, or pions, which were relativistic at the hadronization epoch, has been neglected. This would affect the estimate (12) by a numerical factor of the order of 1. As the effect is linear in the matter density, Λ^{eff} becomes quickly a constant (Fig. 1).

The second scenario where violations of energy-momentum conservation have been argued to arise naturally is the causal set approach to quantum gravity [6,7]. These effects are shown to be compatible with Lorentz invariance; they are described for both massive and massless particles and controlled by a few phenomenological parameters. More precisely, for free massless particles, the physics is encoded in a phase space diffusion equation that reads

$$\frac{d\mu}{dt} = -\frac{p^i}{E}\partial_i\mu - (k_1 + k_2)\frac{\partial\mu}{\partial E} + k_1 E\frac{\partial^2\mu}{\partial E^2}, \quad (13)$$

where k_1 and k_2 have been constrained comparing the cosmic microwave background with Planck's spectrum [7]. In the cosmological context, the diffusion in phase space leads to an energy-momentum violation current of the form
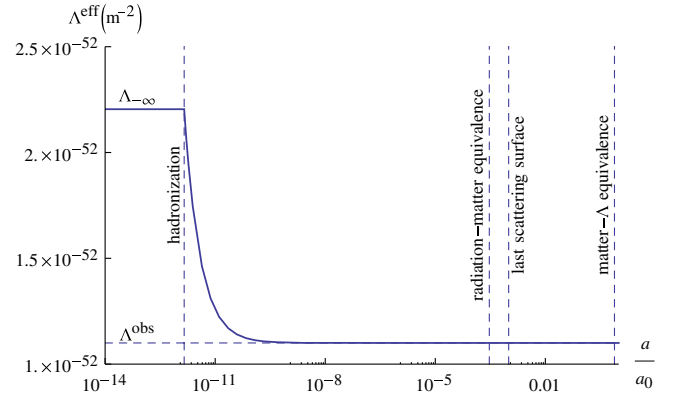

FIG. 1. Effective cosmological constant induced by a wave-function collapse of baryons, using the mass-proportional CSL model with $\xi_{\text{CSL}} = 4.3 \times 10^{-31}\,\text{s}^{-1}$.

$$J = -(3k_1 + k_2)n^\gamma dt = -\xi_{\text{CS}}\rho_0^\gamma\left(\frac{a_0}{a}\right)^3 dt, \quad (14)$$

where n^γ is the number density of photons and $-10^{-21}\,\text{s}^{-1} < \xi_{\text{CS}} < 2 \times 10^{-21}\,\text{s}^{-1}$. Interestingly, ξ_{CS} can be negative (endothermic evolution) and thus contributes positively to the effective cosmological constant. Being very conservative, we can estimate that contribution starting from when photons decoupled from electrons ($z_{\text{dec}} \approx 1100$); the result is

$$\Delta\Lambda_{\text{CS}}^{\text{eff}} \approx -\frac{2\Omega_0^\gamma H_0 \xi_{\text{CS}}}{\sqrt{\Omega_0^m} c^2} z_{\text{dec}}^{3/2} \approx -\frac{\xi_{\text{CS}}}{6 \times 10^{-19}\,\text{s}^{-1}}\Lambda^{\text{obs}}. \quad (15)$$

Both results (12) and (15) are very sensitive to the initial time at which violations of energy conservation started. In the case of CSL, a precise description of the quark-gluon plasma to hadron gas transition is difficult [36]; we simply assumed it to be instantaneous at $T_{\text{QCD}} = 2 \times 10^{12}$ K. Moreover, from an objective collapse perspective, one also expects modifications of quantum mechanics for relativistic particles and interacting systems, but, due to the lack of concrete models, it is not yet possible to determine the corresponding contribution to the cosmological constant. In the causal set example, something similar to (14) is likely to hold also before decoupling and thus may largely enhance the corresponding contribution to the effective cosmological constant. In addition, the diffusion of nonrelativistic particles [6] leads to the creation of energy of the same form as (11). For that reason, we did not include a detailed analysis here.

Finally, a very important feature of energy nonconservation in the context of unimodular gravity is that an effective cosmological constant accessible to observations like (12) does not require strong modifications of the local physics. To see this, let us consider for simplicity a universe made only of baryons (undergoing spontaneous localization) and photons. Let us assume moreover that the kinetic energy (11) created through CSL is mostly transferred to photons (because of the equipartition theorem and the large

number of photons). The backreaction on the stress-energy tensor is given by the modified continuity equation for photons $\dot{\rho}^\gamma + 4H\rho^\gamma = \xi_{CSL}\rho^b$. A solution of this equation in the radiation-dominated era can be written explicitly

$$\rho^\gamma(a) = \rho_h^\gamma \frac{a_h^4}{a^4}\left[1 + \frac{1}{2}\frac{\rho_h^b}{\rho_h^\gamma}\frac{\xi_{CSL}}{H_h}\left(\frac{a^3}{a_h^3} - 1\right)\right]^{2/3}, \quad (16)$$

where the subscript h denotes the cosmological quantities at hadronization time. The departure from the standard equation of state at the end of the radiation-dominated era ($z_{eq} \sim 3000$) can be read from the quantity

$$\frac{\rho_{eq}^\gamma a_{eq}^4}{\rho_h^\gamma a_h^4} - 1 \approx \frac{\Omega_0^b}{2(z_{eq}\Omega_0^m)^{3/2}}\frac{\xi_{CSL}}{H_0} < 10^{-17}, \quad (17)$$

which is found to be completely negligible. Physically, the above result shows that the energy created, or lost, produces effects that pile up in Λ^{eff}, while their backreaction on ordinary matter decreases together with the expansion of the Universe.

The computations of the contribution to the effective cosmological constant performed for two models (continuous spontaneous localization and causal sets) illustrate how, despite the smallness of the modifications of the dynamics at the local level, the effect on the cosmological scale can be of the order of Λ^{obs}. Moreover, it is worth noting that, although the quantitative estimates in this Letter have been obtained for some specific examples, the results of our analysis are far more general and remain valid as long as the violation of energy conservation is of integrable type (5). This framework could therefore be used to rule out nonstandard models that would lead to an effective cosmological constant that varies to much at late time.

There is, *a priori*, no reason for the energy-momentum violations produced by the type of mechanisms evoked here (or those from other hypothetical fundamental sources) to satisfy the integrability condition in a general situation. In cases where that condition is violated, a semiclassical account of the phenomenon in terms of a metric variable theory of gravity would simply not be viable. However, in the cosmological setting considered here, the cosmological principle—homogeneity and isotropy of the Universe at large scales—constrains the current J to be of the form $J_t(t)dt$ for which (5) is automatically satisfied at the relevant scales, making the framework of unimodular gravity useful despite a possible short scale break in the integrability requirement. This, together with the fact that deviations of energy-momentum conservation are strongly constrained in local experiments, is what gives phenomenological relevance to our analysis that could also be applied to other situations whenever the integrability condition can be argued to be approximately valid. In more general situations, a metric formulation (seen here as an effective description) would be precluded, and a more fundamental description would need to be found.

It is, however, interesting to point out that the breaking of diffeomorphism invariance down to volume-preserving diffeomorphisms [so that (5) is satisfied down to the local scale] is actually generic in the regime of validity of quantum field theory in curved space-times. Concretely, the renormalization of the expectation value of the energy-momentum tensor requires the subtraction of ultraviolet divergences which leads to a normal-ordered stress tensor satisfying

$$\nabla^a \langle T_{ab}\rangle_{NO} = \nabla_b Q, \quad (18)$$

where Q is a geometric, state-independent, quantity [37] (for a simple proof of this fact in 2d, see [38]). The standard view, motivated by consistency with semiclassical general relativity, is to enforce energy-momentum conservation through the redefinition $\langle \tilde{T}_{ab}\rangle \equiv \langle T_{ab}\rangle_{NO} - Qg_{ab}$. In the case of conformally coupled theories, this leads to the famous trace anomaly, interpreted as a breaking of scale invariance by quantum effects. We can instead simply deal with $\langle T_{ab}\rangle_{NO}$ in the context of unimodular gravity; the physical implications will be the same. Even if the contributions to the cosmological constant in that case would be tiny, it constitutes a clear-cut example where the type of phenomenon considered here stems from standard quantum effects.

In conclusion, we have shown that a violation of energy-momentum conservation can be reconciled with the metric theory of gravity by taking the fundamental theory of spacetime to be unimodular gravity. This change of paradigm leads to an effective cosmological constant term in Friedmann's equation, that can be seen as a record of the energy-momentum nonconservation during the history of the Universe. It decreases or increases in time, whenever energy is created or lost, yet it becomes quickly a constant (at least in the models described here) as regular matter density dilutes with the expansion.

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