Exacerbating the Cosmological Constant Problem with Interacting Dark Energy Models

M. C. David Marsh*
Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB3 0WA, United Kingdom
(Received 22 June 2016; revised manuscript received 15 August 2016; published 5 January 2017)

Future cosmological surveys will probe the expansion history of the Universe and constrain phenomenological models of dark energy. Such models do not address the fine-tuning problem of the vacuum energy, i.e., the cosmological constant problem (CCP), but can make it spectacularly worse. We show that this is the case for "interacting dark energy" models in which the masses of the dark matter states depend on the dark energy sector. If realized in nature, these models have far-reaching implications for proposed solutions to the CCP that require the number of vacua to exceed the fine-tuning of the vacuum energy density. We show that current estimates of the number of flux vacua in string theory, $N_{\rm vac} \sim \mathcal{O}(10^{272\,000})$, are far too small to realize certain simple models of interacting dark energy *and* solve the cosmological constant problem anthropically. These models admit distinctive observational signatures that can be targeted by future gamma-ray observatories, hence making it possible to observationally rule out the anthropic solution to the cosmological constant problem in theories with a finite number of vacua.

DOI: 10.1103/PhysRevLett.118.011302

The Universe expands at an accelerating rate [1,2], apparently driven by a negative pressure "dark energy" with an energy density of [3,4],

$$\rho_0 \approx (2.3 \times 10^{-3}\ \text{eV})^4 \left(\frac{\Omega_\Lambda}{0.69}\right)\left(\frac{h}{0.68}\right)^2. \qquad (1)$$

There are many computable contributions to the vacuum energy density, $\rho_{\rm vac}$, but unfortunately these tend to be comparatively large: quantum zero-point fluctuations of a field of mass M generically contributes by $\sim[1/(4\pi)^2]M^4$ after renormalization [5], which then needs to be canceled order by order in perturbation theory to an accuracy of one part in

$$\mathfrak{f}_\Lambda = \frac{M^4}{(4\pi)^2 \rho_{\rm obs}}, \qquad (2)$$

to yield an effective vacuum energy compatible with Eq. (1). The scale M is at least as large as the top-quark mass $m_{\rm top} = 173$ GeV (say, if supersymmetry is realized at the TeV scale), and may be as large as the Planck mass, $M_{\rm Pl} = 2.4 \times 10^{18}$ GeV, thus leading to a fine-tuning of

$$\mathfrak{f}_\Lambda = \begin{cases} 2 \times 10^{53} & M = m_{\rm top}, \\ 9 \times 10^{117} & M = M_{\rm Pl}. \end{cases} \qquad (3)$$

Hence, $\rho_{\rm vac}$ is extremely sensitive to high-scale physics, and the required cancellations are perturbatively unstable. This is the cosmological constant problem (CCP).

Over the coming decades, several large cosmological surveys such as DESI, LSST, Euclid, and WFIRST will measure the cosmic expansion history and structure growth, and produce stringent constraints on phenomenological models of dark energy. This will make it possible to distinguish between the simplest of these models, ΛCDM, and several more general models involving time-dependent vacuum energy densities or modifications of general relativity. It is important to note that these models do not address nor solve the CCP, but in many cases make this problem worse by introducing additional unprotected relevant operators (e.g., potential gradients or masses) that need to be fine-tuned, and/or by requiring that $\rho_{\rm vac} \ll (10^{-3}\,\text{eV})^4$ to make room for other mechanisms to generate an accelerated expansion.

In this Letter, we illustrate this point by considering models of "interacting dark energy" in which the dark matter fields are coupled to the dark energy sector, leading to varying dark matter masses. Such models may *a priori* appear quite natural as couplings between dark matter and dark energy are not in general forbidden, and suitable interactions may potentially explain the cosmic coincidence: why $\Omega_\Lambda/\Omega_m \sim \mathcal{O}(1)$. Interacting dark energy has been shown to be compatible, or even favored, by current observations [6–11]. For reviews see Refs. [9,12].

We here show that cosmological models with varying dark matter masses are far from natural and can in fact exacerbate the CCP to fantastic levels. The reason is simple: the vacuum energy density depends on any mass, M, and even a small variation δM in the spectrum can induce an enormous variation in the vacuum energy density,

$$\delta\rho_{\rm vac} \sim \delta M M^3 + \cdots \qquad (4)$$

Ensuring that the scalar potential of the dark energy field is flat enough to give a viable cosmology then requires additional, fine-tuned cancellations. We here show that the total fine-tuning,

$$f_{\rm tot} = f_\Lambda f_{\delta M}, \qquad (5)$$

is unbounded from above and may be as large as $10^{10^{10}}$ in simple examples. We note that this argument is not specific to interacting dark energy models but applies more generally to any model with cosmologically varying fundamental parameters (cf. Refs. [13,14]).

Whatever the ultimate solution of the CCP is, it is not expected to set *all* potential energies to zero, and hence, the additional fine-tuning from varying masses raises an additional challenge for any proposed solution of the problem. Here, we point out that the extreme unnaturalness of these models, if realized in nature, would have significant conceptual implications, in particular for anthropic solutions to the CCP, as we now discuss.

In theories with a very large number of vacua (say, $N_{\rm vac} \gg \mathfrak{f}_\Lambda$) between which $\rho_{\rm vac}$ takes on different vacuum expectation values, it is plausible that a small subset of the vacua have $\rho_{\rm vac} \lesssim \rho_{\rm vac,obs}$ by chance. Assuming a mechanism that could realize each of these vacua with some probability, the CCP may be solved through environmental selection, as solutions with $\rho_{\rm vac} \gg \rho_0$ would not permit cosmological structure formation or intelligent life, and hence have a vanishing probability of being observed [15–18].

In this context, any dark energy model that requires additional tuning beyond that of the CCP is disfavored on statistical grounds [19]. Here, we note that such models cannot be expected to be realized at all unless $N_{\rm vac} \gg f_{\rm tot}$, and hence, a simultaneous realization of certain interacting dark energy models *and* the anthropic solution to the CCP is only justified in theories with an exceptionally large number of vacua.

String theory appears to have many four-dimensional vacuum solutions [20–23]. While the complete vacuum structure of string theory remains a distant goal, the number of vacua in particular constructions has been estimated, with the largest numbers arising from compactifications in which generalized electromagnetic fluxes wrap nontrivial cycles of the compactification manifold. Rough estimates of the number of such "flux vacua" on particular compactification manifolds include [24,25]

$$N_{\rm vac} \approx \begin{cases} \mathcal{O}(10^{506}) & \text{IIB on } \mathbb{CP}_{1,1,1,6,9}\,, \\ \mathcal{O}(10^{272\,000}) & F \text{ theory on } \mathcal{M}_{\rm max}. \end{cases} \quad (6)$$

Reference [25] argued that flux compactifications on $\mathcal{M}_{\rm max}$ dominate the total number of flux vacua by roughly a factor of $\mathcal{O}(10^{3000})$. As the vacuum number estimates [Eq. (6)] are larger than the required fine-tuning in Eq. (2), it is possible that string theory flux vacua admit an anthropic solution to the CCP. We here show that this is no longer the case if certain models of interacting dark energy are realized in nature.

We finally point out that these extremely fine-tuned models may give rise to observational signals that could be targeted by upcoming experiments. Observational evidence for such a model could then be interpreted as evidence against the anthropic solution of the CCP in any theory with a finite number of vacua.

Varying α and the CCP.—We begin by illustrating how cosmologically varying "constants" exacerbate the CCP by considering models in which the fine-structure constant evolves over cosmological scales. Such models have been considered in great detail in the literature (see Refs. [14,26–32] and references therein), largely motivated by the claimed detection of a variation in α inferred from observations from distant quasars [33–36]. However, while field-dependent coupling constants are commonplace in high-energy physics, quantum effects make varying constants unnatural: this point was made independently in Ref. [13], which argued that variations $(\delta\alpha/\alpha) \gtrsim 10^{-37}$ would require additional fine-tuning of the vacuum energy, hence, making quintessence models with varying constants unnatural, and in Ref. [14] (which, however, furthermore argued that varying constants should be expected if the anthropic principle is realized in nature. The strong assumptions leading to this conclusion have not been realized in string theory).

We here review how theories with varying α exacerbate the CCP, and we extend the results of Refs. [13,14] by computing the additional fine-tuning required to keep the vacuum energy sufficiently small in such theories. Hence, we take α to depend on a scalar field $\chi(t,\mathbf{x})$ subject to a quantum effective potential with both an explicit and implicit dependence on χ,

$$V_{\rm eff}(\mu,\chi,\alpha(\chi)) = V_{0,r}(\chi) + \frac{M^4}{(4\pi)^2}\sum_{k=1}^{\infty} f_k(\mu)\frac{\alpha^{k-1}}{(4\pi)^{k-1}}, \quad (7)$$

where $V_{0,r}$ denotes the renormalized classical potential, M is a large mass scale corresponding to charged particles running in loops, and μ the renormalization group scale. Equation (1) is then satisfied by imposing the renormalization condition,

$$V_{\rm eff}(\bar{\mu},\bar{\chi},\bar{\alpha}) = \rho_0, \quad (8)$$

where $\bar{\chi}$ denotes the local value of the field, and $\bar{\alpha}$ the low-energy, laboratory value of α at the renormalization scale $\bar{\mu}$. The fine-tuning associated with Eq. (8) is just that of Eq. (2).

Now consider a variation in the field χ which induces a small, space-time dependent variation $\delta\alpha(\chi(t,\mathbf{x}))$ of maximal magnitude $\delta\alpha_{\rm m}$ [37]. We first consider the field variation $\delta\chi(t,\mathbf{x}) \le \delta\chi_{\rm m}$ to be small compared to the relevant cutoff scale, Λ, so that we can expand α as $\delta\alpha/\bar{\alpha} = c(\delta\chi/\Lambda)$. The vacuum energy density is given by

$$V_{\rm eff} = \rho_0 + \delta V_{0,r}(\chi) + \frac{M^4}{(4\pi)^2}\sum_{k=1}^{\infty} c_k\left(\frac{\delta\alpha_{\rm m}}{4\pi}\right)^k\left(\frac{\delta\chi}{\delta\chi_{\rm m}}\right)^k, \quad (9)$$

where the coefficients c_k generically are $\mathcal{O}(1)$. $V_{\rm eff}$ should not to exceed some observationally inferred value $\rho_{\rm m}$ (which will depend on the redshift range over which α varies) over the entire field range $[\bar{\chi},\bar{\chi}+\delta\chi_{\rm m}]$. We introduce the notation

$$\mathfrak{B} = \frac{M^4}{(4\pi)^2\rho_{\rm m}}, \qquad \delta = \left(\frac{\delta\alpha_{\rm m}}{4\pi}\right), \quad (10)$$

and note that ensuring the flatness of the effective potential requires fine-tuned cancellations between loop corrections and terms in $\delta V_{0,r}$: for terms of $\mathcal{O}(\delta\chi^k)$, the accuracy of the cancellation can be estimated to one part in $\mathfrak{B}\delta^k$, requiring some amount of fine-tuning up to order $k_{\rm max} = \text{floor}(\ln(\mathfrak{B})/\ln(1/\delta))$.

In direct analogy with Eq. (2), the exacerbation of the CCP caused by varying α is given by, $\mathfrak{f}_{\delta\alpha} = \prod_{k=1}^{k_{\max}} \mathfrak{B}\delta^k$, and the total fine-tuning of the CCP in theories with varying α is then given by

$$\mathfrak{f}_{\rm tot} = \mathfrak{f}_\Lambda \mathfrak{f}_{\delta\alpha} = r\prod_{k=0}^{k_{\max}} \mathfrak{B}\delta^k = r\mathfrak{B}^{(k_{\max}+1)(1-\frac{k_{\max}\ln(\delta^{-1})}{2\ln(\mathfrak{B})})} \approx r\mathfrak{B}^{\frac{1}{2}(k_{\max}+1)}, \tag{11}$$

where $r = \rho_{\rm m}/\rho_0$, and where in the last step we have approximated $k_{\max} \approx [\ln \mathfrak{B}/\ln(1/\delta)]$.

To illustrate the severity of the additional fine-tuning, we take $\delta\alpha_m/\bar{\alpha} = 10^{-6}$, as motivated by the observations [34–36], and impose that the vacuum energy is subleading to the matter energy density at $z = 3$ (around which time light from the distant quasars was emitted) so that $\rho_m = 4^3\Omega_{m,0}\rho_c$. We then have,

$$\delta = 6\times 10^{-10}, \quad \mathfrak{B} = 9\times 10^{50}\times\left(\frac{M}{100\ \mathrm{GeV}}\right)^4, \tag{12}$$

giving

$$\mathfrak{f}_{\rm tot} = \begin{cases} 2\times 10^{174} & M = m_{\rm top}, \\ 3\times 10^{795} & M = M_{\rm Pl}, \end{cases} \tag{13}$$

which should be compared with the corresponding estimates for the ordinary CCP in Eq. (2).

Hence, even minute variations of α substantially worsen the CCP, but the required fine-tunings of Eq. (13) remain far smaller than the F-theory vacuum number estimate of Eq. (6).

The linear dependence of $\delta\alpha$ on χ is not crucial to our argument, and the same conclusions could be reached for a more general monomial form of the variation, $\delta\alpha \sim (\bar{\alpha}\delta\chi/\Lambda)^q$ for $q \geq 1$. Subleading terms in the expansion will in general increase the fine-tuning somewhat, but will only become important if the variation in α is caused by a large-field variation of the dark energy field, $\chi > \Lambda$. In this case, $k_{\max} \to \infty$ and $f_{\rm tot} \to \infty$.

Interacting dark energy.—Are there models that could make the fine-tuning of the CCP massively worse than what is possible in theories with varying α? We will now show that the answer to this question is yes: in multifield models of interacting dark energy, keeping the effective potential flat over a multiple dimensional domain in field space (cf. $[\bar{\chi}, \bar{\chi}+\delta\chi_m]^p$ for $p > 1$) requires fantastic amounts of fine-tuning. In the section "Observational prospects" we discuss the observational prospects of such models.

Studies of interacting dark energy have primarily focussed on deriving cosmic microwave background and large-scale structure (LSS) constraints on the form and magnitude of the energy transfer between the dark matter and dark energy sectors. Microscopic realizations of such models with any form of varying parameters exacerbate the CCP (see also Ref. [39]). Here, we present a particular example of a broad class of microscopic models that can realize a variety of phenomenological scenarios. For concreteness, we consider a dark matter sector with p real scalar fields, ϕ^i, subject to the classical potential,

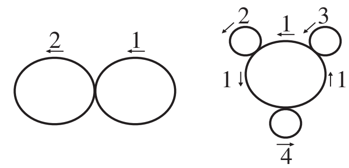

FIG. 1. Contributions to the vacuum energy.

$$V_{0,\phi,r} = \frac{1}{2}m_i^2(\phi^i)^2 + \frac{1}{4!}\lambda_i(\phi^i)^4 + \sum_{i\neq j}\frac{1}{(2!)^2}\lambda_{ij}(\phi^i)^2(\phi^j)^2.$$

Quantum effects correct this potential at each loop order: using dimensional regularization and $\overline{\rm MS}$ renormalization, the one-loop Coleman-Weinberg potential is given by

$$V_{1l} = \sum_i^n \frac{m_i^4}{64\pi^2}\left(\ln\left(\frac{m_i^2}{\mu^2}\right) - \frac{3}{2}\right), \tag{14}$$

for where μ denotes the renormalization scale. Higher loop orders induce a more complicated dependence on the masses and couplings, as we here illustrate by the processes depicted in Fig. 1. At two loop order, cross terms of the form,

$$V_{2l} \supset \frac{\lambda_{12}}{(4\pi)^4}m_1^2 m_2^2 L\left(\frac{m_1^2}{\mu^2}\right)L\left(\frac{m_2^2}{\mu^2}\right), \tag{15}$$

are induced while at four loops $V_{\rm eff}$ receives contributions of the form,

$$V_{4l} \supset \frac{\lambda_{13}\lambda_{14}\lambda_{12}}{2\times 3!(4\pi)^8}\frac{m_2^2 m_3^2 m_4^2}{m_1^2}\prod_{i=2}^4 L\left(\frac{m_i^2}{\mu^2}\right). \tag{16}$$

Here $L(m_i^2/\mu^2) = \ln(m_i^2/\mu^2) - 2$.

The dark matter masses are now assumed to be functions of dark energy fields $\chi_\alpha = \chi_\alpha(t, \mathbf{x})$, for $\alpha = 1\ldots p$. The renormalization condition of the vacuum energy is given by

$$V_{\rm eff}(\bar{\mu}, \bar{m}_i) = V_{0,\phi,r} + V_{0,\chi,r} + \sum_{n=1} V_{nl} = \rho_0, \tag{17}$$

with ρ_0 as in Eq. (1).

For simplicity, we here take the fields χ_α to induce independent variations of the masses of the form $\delta m_i/\bar{m}_i \sim \chi_i/\Lambda$, and we again assume that the variation of the field is small with respect to the cutoff Λ (we will return to large-field models in the section "Observational prospects").

Given some fractional variations of the dark matter masses, $\delta_i \equiv \delta m_i/\bar{m}_i$, the change in the effective potential is given by

$$\delta V_{\rm eff} = \delta V_{0,\chi,r} + M^4 \sum_{\substack{k_i=0 \\ \text{not all } k_i=0}}^{\infty} \frac{c_{\underline{k}}}{(4\pi)^{2l_{\underline{k}}}}\delta_1^{k_1}\ldots\delta_p^{k_p}, \tag{18}$$

where $\underline{k} = (k_1, \ldots, k_p)$, and M denotes a suitable mass scale [40]. From Eqs. (14)–(16) we see that cross terms between n distinct masses arise at n-loop order, and hence, $l_{\underline{k}}$, which denotes the first loop order at which the contribution with index $\underline{k}$ appears, is given by $l_{\underline{k}} = p - \sum_{i=1}^p \delta_{k_i}^0$ (a rather good, simple lower bound on the fine-tuning can be obtained by taking $l_{\underline{k}} = p$).

For concreteness, we specialize to fractional variations that are independent but of the same amplitude, $\max(\delta_i) = \delta_m$ and field variations $\chi_i \in [0, \chi_m]$. We will furthermore consider dark matter masses around $\bar{m} = M = 100$ GeV and take all quartic interactions $\lambda_i, \lambda_{ij} \sim \mathcal{O}(1)$ so that $c_{\underline{k}} \sim \mathcal{O}(1)$. We note that this gives $\sigma/\bar{m} \sim 1/\bar{m}^3 \ll 1\,\mathrm{cm}^2/\mathrm{s}$, consistent with the bound on dark matter self-interactions from the Bullet cluster [41]. The upper bound on the energy density, ρ_m, is model dependent as it is sensitive to how and when the dark energy evolves. Here, we conservatively note that supernovae data are consistent with ΛCDM [42], and take $\rho_m = 10\rho_0$ for models with masses varying between $0 < z \lesssim 1.5$. We then have $\mathfrak{B} = M^4/\rho_m = 4 \times 10^{53}$.

Keeping the effective potential sufficiently flat over the entire p-dimensional domain $[0, \chi_m]^p$ now requires accurate cancellations not only of terms of the form χ_i^k for some fixed species with index i, but of cross terms with other fields as well. Proceeding as in the single-field case, the additional fine-tuning is given by

$$f_{\delta m} \equiv \prod_{\substack{k_1,\ldots,k_p=0 \\ \text{not all } k_i=0}}^{\sum_i^p k_i \le k_{\max}} \mathfrak{B} \frac{1}{(4\pi)^{2 l_{\underline{k}}}} \delta^{(\sum_{i=1}^p k_i)}, \qquad (19)$$

where we take $k_{\max} = \ln(\mathfrak{B})/\ln(\delta^{-1}) + p\ln(16\pi^2)/\ln(\delta)$. Evaluating the product, the total fine-tuning is given by

$$\begin{aligned} f_{\mathrm{tot}} &= f_\Lambda f_{\delta m} \\ &= r\left(\frac{1}{16\pi^2}\right)^{[(k_{\max}+p-1)!/(p-1)!(k_{\max}-1)!]+1} \\ &\quad \times \mathfrak{B}^{[(k_{\max}+p)!/p!k_{\max}!]\{1-(p/p+1)[k_{\max}\ln(\delta^{-1})]/[\ln(\mathfrak{B})]\}}, \end{aligned} \qquad (20)$$

where, again, $r = \rho_m/\rho_0$. Equation (20) is the main result of this Letter and is numerically evaluated in Fig. 2. The fine-tuning diverges as $\delta \to 1$, and is extremely large already for modest variations of multiple masses. In particular, for 25% variations of 3 masses, $f_{\mathrm{tot}} \approx 10^{10^6}$, exceeding the vacuum number estimates of Eq. (6). Fine-tuning of 1 part in $10^{10^{10}}$ is required, e.g., for models with 4 masses and $\delta = 0.75$.

Observational prospects.—Of crucial importance is whether the class of highly tuned interacting dark energy models discussed in this Letter can be observationally distinguished from less fine-tuned models. Obviously, multiple dark matter masses can vary due to couplings to a single field χ, and keeping the effective potential flat over a $p = 1$ curve in field space may require much less fine-tuning than over a $p > 1$ domain. Hence, simply observing multiple varying masses cannot be regarded as evidence for the most fine-tuned models discussed here. Nevertheless, we will now show that observational evidence for highly tuned models may in principle be in reach by future experiments.

Current and future cosmic microwave background and LSS experiments will constrain the energy transfer between the dark matter states and the dark energy sector, thereby constraining the masses, couplings, and abundances Ω_{ϕ_i} in the models considered here. However, such observations are unlikely to by themselves determine the details of the underlying microscopic model.

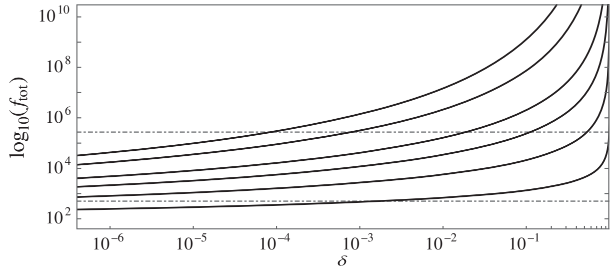

FIG. 2. From bottom and up: black lines correspond to $p = 1$–4, 6, 8; grey horizontal lines to $\log_{10}(f_{\mathrm{tot}}) = 506$ and 272 000.

The "smoking gun" signal of these models instead arises through very weak dark matter couplings to photons, leading to the decay $\phi_i \to \gamma\gamma$ that may be observable from dark matter dense astrophysical objects. The variation in the dark matter masses would cause a variation in the energy of the photon line, $E_i(t, \mathbf{x}) = m_i(\chi_\alpha(t, \mathbf{x}))/2$, over the sky, which could be mapped out to very high accuracy if the signal is detectable from individual galaxies or galaxy clusters. For dark matter masses of $\sim$100 GeV, such signals could in principle be observed by future gamma ray observatories.

To establish that the signal arises from a very fine-tuned model, we must in addition show that it corresponds to either a large-field variation ($\delta\chi/\Lambda > 1$) or a variation in a $p > 1$ dimensional domain of field space. It suffices to consider a simple example to see that this may indeed be possible.

For concreteness, we consider a $p = 3$ model in which the profiles of the dark energy fields in the local Universe can be approximated by $\chi_1 = \chi_0 x/L$, $\chi_2 = \chi_0 y/L$, $\chi_3 = \chi_0 z/L$ for some $\chi_0 \ll \Lambda$ and for some cartesian coordinates $0 < |x|, |y|, |z| < L$. We here neglect the explicit time dependence of the dark energy profiles as these are expected to vary slowly on observational time scales, and we again take the dark matter mass m_i to depend only on χ_i, so that $m_1(x)$, $m_2(y)$ and $m_3(z)$. By detecting the corresponding photon lines from a large number of astrophysical objects, we can establish that a three-dimensional domain of *mass-parameter space* is sampled. Can a single field model give rise to the same observational signal? Surely so, as long as the one-parameter curve $m_i(\chi)$ covers the entire three-dimensional mass-parameter space sufficiently densely to be consistent with observational accuracy. However, such a single-field model is too complicated to be described by a small-field model under perturbative control, and hence, must correspond to a large field model, and hence $f_{\mathrm{tot}} \to \infty$. Thus, the $p = 3$ small-field model with excessively large fine-tuning provides the most natural explanation of such observations.

By this example and Eq. (20), we have shown that there exists possible future observations for which models with a fine-tuning larger than the vacuum number estimates of Eq. (6) provide the simplest explanation.

We note, however, that the dimension of the image of the map $\mathbf{x} \to \chi_i(\mathbf{x})$ is not greater than 3 and hence less than the dimension of the target space for $p > 3$. This means that the profiles $\chi_i(\mathbf{x})$ need to be very complicated to densely sample a p-dimensional domain, and we expect that such models will be hard to realize in cosmological models and, moreover, to observationally distinguish from the $p = 3$ case.

Conclusions.—We have shown that models in which fundamental parameters vary over cosmological scales generically spoil the delicate cancellations of vacuum energies required by the observed smallness of the cosmological constant, in effect making the cosmological constant problem substantially worse. Models of interacting dark energy gives a stark illustration of this point: we have for the first time shown that there exists rather simple cosmological models with characteristic observable signatures which are too fine-tuned to be compatible with an anthropic solution of the cosmological constant problem in theories with any finite number of vacua. Our argument applies in particular to 'flux vacua' of string theory, which according to current estimates (6) are numerous but finite.

We note in closing that varying constants can be made natural if a mechanism dictates cancellations of the various contributions to the effective potential. Supersymmetry provides an example of such a mechanism, but supersymmetry is not realized in nature below the TeV scale and, hence, cannot solve the fine-tuning problem of interacting dark energy. Other mechanisms to the same effect may in principle exist, but none have so far been identified nor shown to be realized among the effective theories arising from flux compactifications for which the vacuum number estimates Eq. (6) applies.

I'm very grateful for stimulating conversations with Thomas Bachlechner, John Barrow, Marcus Berg, Raphael Bousso, Frederik Denef, Liam McAllister, and Joshua Schriffin. I acknowledge support from a Starting Grant of the European Research Council (ERC STG Grant No. 279617).

*m.c.d.marsh@damtp.cam.ac.uk

[1] A. G. Riess *et al.* (Supernova Search Team), Astron. J. **116**, 1009 (1998).
[2] S. Perlmutter *et al.* (Supernova Cosmology Project), Astrophys. J. **517**, 565 (1999).
[3] P. A. R. Ade *et al.* (Planck Collaboration), Astron. Astrophys. **594**, A13 (2016).
[4] E. Aubourg *et al.*, Phys. Rev. D **92**, 123516 (2015).
[5] In effective theories with a UV cutoff Λ, states with $M > \Lambda$ are integrated out and generically contribute to $\rho_{\rm vac}$ by $\sim\Lambda^4$.
[6] V. Salvatelli, N. Said, M. Bruni, A. Melchiorri, and D. Wands, Phys. Rev. Lett. **113**, 181301 (2014).
[7] E. Abdalla, E. G. M. Ferreira, J. Quintin, and B. Wang, arXiv:1412.2777.
[8] R. Murgia, S. Gariazzo, and N. Fornengo, J. Cosmol. Astropart. Phys. 04 (2016) 014.
[9] B. Wang, E. Abdalla, F. Atrio-Barandela, and D. Pavon, Rep. Prog. Phys. **79**, 096901 (2016).
[10] A. A. Costa, X.-D. Xu, B. Wang, and E. Abdalla, arXiv: 1605.04138.
[11] R. C. Nunes, S. Pan, and E. N. Saridakis, Phys. Rev. D **94**, 023508 (2016).
[12] Yu. L. Bolotin, A. Kostenko, O. A. Lemets, and D. A. Yerokhin, Int. J. Mod. Phys. D **24**, 1530007 (2015).
[13] T. Banks, M. Dine, and M. R. Douglas, Phys. Rev. Lett. **88**, 131301 (2002).
[14] J. F. Donoghue, J. High Energy Phys. 03 (2003) 052.
[15] S. Weinberg, Phys. Rev. Lett. **59**, 2607 (1987).
[16] J. D. Barrow and F. J. Tipler, *The Anthropic Cosmological Principle* (Oxford University Press, Oxford, 1988).
[17] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).
[18] H. Martel, P. R. Shapiro, and S. Weinberg, Astrophys. J. **492**, 29 (1998).
[19] R. Bousso, Gen. Relativ. Gravit. **40**, 607 (2008).
[20] R. Bousso and J. Polchinski, J. High Energy Phys. 06 (2000) 006.
[21] S. B. Giddings, S. Kachru, and J. Polchinski, Phys. Rev. D **66**, 106006 (2002).
[22] S. Kachru, R. Kallosh, A. D. Linde, and S. P. Trivedi, Phys. Rev. D **68**, 046005 (2003).
[23] S. Ashok and M. R. Douglas, J. High Energy Phys. 01 (2004) 060.
[24] F. Denef, M. R. Douglas, and B. Florea, J. High Energy Phys. 06 (2004) 034.
[25] W. Taylor and Y.-N. Wang, J. High Energy Phys. 12 (2015) 164.
[26] J. D. Bekenstein, Phys. Rev. D **25**, 1527 (1982).
[27] J. D. Barrow, H. B. Sandvik, and J. Magueijo, Phys. Rev. D **65**, 063504 (2002).
[28] H. B. Sandvik, J. D. Barrow, and J. Magueijo, Phys. Rev. Lett. **88**, 031302 (2002).
[29] J. D. Barrow, H. B. Sandvik, and J. Magueijo, Phys. Rev. D **65**, 123501 (2002).
[30] J. D. Barrow, J. Magueijo, and H. B. Sandvik, Phys. Rev. D **66**, 043515 (2002).
[31] J. D. Barrow and J. Magueijo, Mod. Phys. Lett. A **30**, 1540029 (2015).
[32] J.-P. Uzan, Living Rev. Relativ. **14**, 2 (2011).
[33] J. K. Webb, V. V. Flambaum, C. W. Churchill, M. J. Drinkwater, and J. D. Barrow, Phys. Rev. Lett. **82**, 884 (1999).
[34] J. K. Webb, J. A. King, M. T. Murphy, V. V. Flambaum, R. F. Carswell, and M. B. Bainbridge, Phys. Rev. Lett. **107**, 191101 (2011).
[35] J. A. King, J. K. Webb, M. T. Murphy, V. V. Flambaum, R. F. Carswell, M. B. Bainbridge, M. R. Wilczynska, and F. E. Koch, Mon. Not. R. Astron. Soc. **422**, 3370 (2012).
[36] M. R. Wilczynska, J. K. Webb, J. A. King, M. T. Murphy, M. B. Bainbridge, and V. V. Flambaum, Mon. Not. R. Astron. Soc. **454**, 3082 (2015).
[37] The light field χ will act as a mediator of the long-range "fifth" force and will be subject to observational constraints from the absence of violations of the weak equivalence principle [38], which, however, will not be important for our present discussion.
[38] S. M. Carroll, Phys. Rev. Lett. **81**, 3067 (1998).
[39] G. D'Amico, T. Hamill, and N. Kaloper, Phys. Rev. D **94**, 103526 (2016).
[40] For unprotected scalar masses we generically expect $M \sim \Lambda$. Here, in order not to overestimate the fine-tuning, we take $M \sim \bar{m}_i$.
[41] M. Markevitch, A. H. Gonzalez, D. Clowe, A. Vikhlinin, W. Forman, C. Jones, S. Murray, and W. Tucker, Astrophys. J. **606**, 819 (2004).
[42] N. Suzuki *et al.* (The Supernova Cosmology Project), Astrophys. J. **746**, 85 (2012).