Dependence of the Spectrum of Shock-Accelerated Ions on the Dynamics at the Shock Crossing

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Diffusive shock acceleration (DSA) of ions occurs due to pitch-angle diffusion in the upstream and downstream regions of the shock and multiple crossing of the shock by these ions. The classical DSA theory implies continuity of the distribution at the shock transition and predicts a universal spectrum of accelerated particles, depending only on the ratio of the upstream and downstream fluid speeds. However, the ion dynamics at the shock front occurs within a collision-free region and is gyrophase dependent. The ions fluxes have to be continuous at the shock front. The matching conditions for the gyrophase-averaged distribution functions at the shock transition are formulated in terms of the transition and reflection probabilities. These probabilities depend on the shock angle and the magnetic compression as does the power spectrum of accelerated ions. Their spectral index is expressed in terms of the reflectivity. The spectrum is typically harder than the spectrum predicted by the classical DSA theory.

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Introduction.—Collisionless shocks are known to be very efficient charged-particle accelerators [1–9]. During the course of the acceleration, the charged particles are scattered by magnetic irregularities in the upstream and downstream regions and cross the shock front many times [see, e.g., Ref. [10]. Sufficiently far from the shock transition, it is reasonable to consider the gyrophase-averaged particle distributions and describe their evolution in terms of pitch-angle diffusion [11]. If the shock transition is gradual, the diffusive approximation may be valid throughout (see, e.g., Ref. [12]). It has been shown that the drifts in the macroscopic fields of the shock have to be taken into account, even in the diffusive approximation [13,14]. However, typically, the shock transition is narrow. Since the mean free path of the particles, with respect to collisions with magnetic irregularities, is much larger than the shock width and the ion gyroradius, the ion dynamics at the shock crossing is collision free [5,12,15]. The spectrum of the accelerated particles is determined by the matching conditions at the shock front [6,7]. These conditions relate the upstream and downstream distributions, which should otherwise satisfy the upstream and downstream diffusion-convection equations. In parallel shocks, where the magnetic field does not change throughout the shock, the matching conditions reduce to the continuity requirement of the distribution function (see, e.g., Ref. [16]), since ions cross the shock freely in the absence of the magnetic field change. For weakly anisotropic distributions, the distribution function continuity requirement leads to the universal power spectrum of accelerated particles in nonrelativistic shocks [6,17], $F \propto p^{-s}$, $s = 3V_u/(V_u - V_d)$, where V_u and V_d are, respectively, the upstream and downstream fluid speeds in the shock frame, and p is the ion momentum, $p/m \gg V_u$, where m is the ion mass. Here, $F(p) = \int f(p,\mu)d\mu$ is the omnidirectional phase space density, that is, pitch-angle cosine-integrated distribution function. More than 80% of the observed shocks are oblique, with the angle between the shock normal and upstream magnetic field $\theta > 30°$ [18]. In oblique shocks, the magnetic jump has to be taken into account. The magnetic compression has been shown to significantly affect the ion motion even at $\theta = 10°$ [19]. As a result, a part of the ions are reflected and the continuity requirements have to be replaced with the equality of the fluxes across the shock [12,17]. The widely accepted approximation of the magnetic moment conservation during the ion motion in the shock front reduces to the previously applied continuity requirement for the transmitted particles [6,12,17]. Magnetic moment conservation places a hard threshold on the pitch angle of the incident ions: all ions with larger pitch angles are reflected, while all ions with smaller pitch angles are transmitted. Yet, the ion dynamics in the shock front are gyrophase dependent and the eventual fate of an ion; that is, whether it is reflected or transmitted depends not only on the pitch angle, but also on the gyrophase of the ion at the entry to the shock [20,21]. For gyrophase-averaged distributions, these dynamics can be described in terms of the probabilities of being reflected or transmitted [19,21]. These probabilities are not sensitive to the details of the shock structure, but depend on the magnetic compression and the angle between the shock normal and the upstream magnetic field [19]. In general, the flux continuity is not equivalent to the distribution function continuity. Since the spectral index s is obtained from the matching conditions, it may depend on the probabilities of the reflection and transmission. In this Letter, we provide the matching conditions at the shock front without invoking the approximation of the magnetic moment conservation. We derive the spectral index of the accelerated ions in terms of the reflectivity.

Basic equations.—In what follows, subscripts u and d refer to the upstream and downstream regions, respectively. Primed variables are in the fluid frame and nonprimed are in the de Hoffman–Teller (HT) frame. There is no uniform electric field in the HT frame, so that in the scatter-free region, the particle energy is conserved. In the diffusive region, the energy change is due to scattering, which is elastic in the fluid frame. The HT velocity relative to the plasma frame, $V = \tilde{V}/\cos\theta$, is along the magnetic field both upstream and downstream. Here, $\tilde{V}$ is the fluid velocity along the shock normal. Since the tangential component of the HT velocity is constant throughout the shock, one has $V_u \sin\theta_u = V_d \sin\theta_d$ and $\tilde{V}_u \tan\theta_u = \tilde{V}_d \tan\theta_d$. The relations between momentum and energy in the fluid and HT frames, respectively, are

$$p'\mu' = \Gamma(p\mu - mV\gamma), \qquad \gamma = (1 + p^2/m^2c^2)^{1/2} \quad (1)$$

$$\gamma' = \Gamma(\gamma - Vp\mu/mc^2), \qquad \Gamma = (1 - V^2/c^2)^{-1/2} \quad (2)$$

Here, $\mu = \cos(\widehat{pB})$ is the particle pitch-angle cosine. For $V \ll v$, $\Gamma = 1$, by Taylor expanding (2) up to the first order in V/v, one has $p' = p - mV\mu\gamma$. We assume that upstream and downstream distributions $f(p', \mu')$ obey the diffusive acceleration equation (see, e.g., Ref. [17])

$$\Gamma\cos\theta(p'\mu'/\gamma' + V)\frac{\partial f}{\partial x} = \frac{\partial}{\partial\mu'}D(p', \mu')\frac{\partial}{\partial\mu'}f. \quad (3)$$

The equations are written in mixed variables, where x is in the HT frame, while p' and μ' are in the fluid frame.

Matching conditions.—Let us define the probabilities $t(\mu, \mu_1)$, $r(\mu, \mu_2)$, and $l(\mu_1, \mu_2)$ as follows: a) $t(\mu, \mu_1)$ (transmission) is the probability of an incident ion, with a pitch-angle cosine $\mu > 0$ to be transmitted (appear downstream) with a pitch-angle cosine $\mu_1 > 0$, b) $r(\mu, \mu_2)$ (reflection) is the probability of an incident ion, with the initial $\mu > 0$ to be reflected (appear upstream) with $\mu_2 < 0$, and c) $l(\mu_1, \mu_2)$ (leakage) is the probability of a downstream ion, with the initial $\mu_1 < 0$ to be transmitted (appear upstream) with $\mu_2 < 0$. It has been shown that for $v \gg V_u$, these probabilities do not depend on the particle momentum [19]. The probabilities are normalized as follows: $\int_{-1}^{0} r(\mu, \mu_2)d\mu_2 = \chi(\mu)$, $\int_{-1}^{0} l(\mu_1, \mu_2)d\mu_2 = 1$, $\int_0^1 t(\mu, \mu_1)d\mu_1 = \tau(\mu) = 1 - \chi(\mu)$, where the reflectivity $\chi(\mu)$ is the fraction of the initial flux of the upstream incident ions with a pitch-angle cosine μ, which are reflected. When crossing from upstream to downstream, ion reflection occurs because of the increase of the angle between the shock normal and the magnetic field, which enhances the gyration of an ion at the expense of the motion along the magnetic field, thus allowing some ions to return to the shock front. When crossing from downstream to upstream, the angle between the shock normal and the magnetic field decreases, and gyration is not sufficient for an ion to return to the shock front. Thus, all downstream ions moving toward the shock cross it without being reflected, as was shown analytically in the approximation of the magnetic moment conservation [5] and also by numerically tracing ions without this approximation [22]. The matching conditions at the shock front require equality of the fluxes [5,12,15,17]. The upstream and downstream energies are, in general, slightly different because of the cross-shock potential: $\gamma_d = \gamma_u - e\phi/mc^2$. In the HT frame, the cross-shock potential is typically, by about an order of magnitude, smaller than the energy of the ion moving with the speed $V_u \cos\theta_u$ [23], $e\phi \sim 0.1mV_u^2\cos^2\theta_u/2$. For the high-energy ions considered here, the relative effect of the cross-shock potential is of second order in V_u/v and will be neglected. Thus, in what follows, $p_u = p_d = p$, $dp_d = dp_u$, and $\gamma_u = \gamma_d = \gamma$. Consider now the particles with the momenta in the interval p, $p + dp$ and downstream pitch-angle cosine in the interval μ_1, $\mu_1 + d\mu_1$. The flux of these particles in the downstream region is

$$dN_d = v\mu_1 \cos\theta_d f_d(p, \mu_1)dV_p d\mu_1, \quad (4)$$

where $v = p/m\gamma$ and $dV_p = 2\pi p^2 dp$. Here, $v\mu_1 \cos\theta_d$ is the x component of the velocity of the guiding center in the downstream region. This flux should be equal to the flux of the particles transmitted from upstream

$$dN_u = \left(\int_0^1 v\mu\cos\theta_u f_u(p, \mu)t(\mu, \mu_1)d\mu\right)dV_p d\mu_1. \quad (5)$$

Thus, for the forward moving ions, μ_1, $\mu > 0$, one gets

$$\int_0^1 \mu\cos\theta_u f_u(p, \mu)t(\mu, \mu_1)d\mu = \mu_1\cos\theta_d f_d(p, \mu_1). \quad (6)$$

All downstream ions leaving the shock are transmitted from the upstream region. Integration over μ_1 gives

$$\int_0^1 \mu\cos\theta_u \tau(\mu) f_u(p,\mu)d\mu = \int_0^1 \mu\cos\theta_d f_d(p,\mu)d\mu. \quad (7)$$

For the backward moving ions, μ_1, $\mu_2 < 0$, one gets

$$\mu_2\cos\theta_u f_u(p, \mu_2) = -\int_0^1 \mu\cos\theta_u f_u(p, \mu)r(\mu, \mu_2)d\mu + \int_{-1}^0 \mu_1\cos\theta_d f_d(p, \mu_1)l(\mu_1, \mu_2)d\mu_1. \quad (8)$$

The upstream ions leaving the shock are produced by reflection or leakage or both. Integration over μ_2 gives

$$-\int_0^1 \mu\cos\theta_u f_u(p,\mu)\chi(\mu)d\mu + \int_{-1}^0 \mu\cos\theta_d f_d(p,\mu)d\mu = \int_{-1}^0 \mu\cos\theta_u f_u(p,\mu)d\mu. \quad (9)$$

We note that the probabilities are defined in HT frame.

Hard threshold.—Let there be $\chi(\mu) = 1$ for $0 < \mu < \mu_c$, and $\chi(\mu) = 0$ for $\mu > \mu_c$. Let there also be a one-to-one correspondence between the pitch-angle cosines: a) $\mu_1 = \mu_1(\mu)$ for any $\mu > \mu_c$ (transmission), b) $\mu_2 = \mu_2(\mu)$ for any $0 < \mu < \mu_c$ (reflection), and c) $\mu_2 = \mu_2(\mu_1)$ for any $\mu_1 < 0$ (leakage). In this case, an upstream backstreaming ion with certain momentum and pitch angle is either reflected or leaked from downstream. Reflection and leakage are now mutually exclusive, and there is no mixing of the reflected and leaked populations. Then, one can write $t(\mu,\mu_1) = \delta(\mu_1 - \mu_1(\mu))$, $\mu > \mu_c$, $r(\mu,\mu_2) = \delta(\mu_2 - \mu_2(\mu))$, $0 < \mu < \mu_c$, $l(\mu_1,\mu_2) = \delta(\mu_2 - \mu_2(\mu_1))$, μ_1, $\mu_2 < 0$. Respectively, the matching conditions will take the form

$$\mu\cos\theta_u f_u(p,\mu)|d\mu_1/d\mu|^{-1} = \mu_1\cos\theta_d f_d(p,\mu_1) \quad (10)$$

for transmission, $\mu > \mu_c$,

$$-\mu\cos\theta_u f_u(p,\mu)|d\mu_2/d\mu|^{-1} = \mu_2\cos\theta_u f_u(p,\mu_2) \quad (11)$$

for reflection, $0 \le \mu \le \mu_c$, and

$$\mu_1\cos\theta_d f_d(p,\mu_1)|d\mu_2/d\mu_1|^{-1} = \mu_2\cos\theta_u f_u(p,\mu_2) \quad (12)$$

for leakage, $\mu_1 < 0$. The continuity of the distribution functions for transmitted and leaked ions across the shock, $f_u(p,\mu_u) = f_d(p,\mu_d)$, requires $\cos\theta_u\mu_u d\mu_u = \cos\theta_d\mu_d d\mu_d$. Since the function $\mu_d(\mu_u)$ and the inverse function are single-valued functions, and $d\mu_d/d\mu_u > 0$, one has to require $\mu_u = 1 \leftrightarrow \mu_d = 1$, otherwise either upstream or downstream distribution would have a gap in the vicinity of $\mu = 1$. Integrating the last equation with this requirement, one has $g_u \equiv (1-\mu_u^2)\cos\theta_u = (1-\mu_d^2)\cos\theta_d \equiv g_d$, which is nothing but the magnetic moment conservation (cf., e.g., Refs. [12,17]). Indeed, $G = v_\perp^2/|\mathbf{B}| = v^2(1-\mu^2)\cos\theta/B_x = (v^2/B_x)g$. If the magnetic moment is conserved for reflection also, then $f_u(p,-\mu) = f_u(p,\mu)$ for reflected ions. Thus, the upstream ion population is an even function of μ in the *shock* frame but not in the *fluid* frame. Pitch-angle scattering occurs far from the shock front. Unless the diffusive mechanism is so peculiar that the distribution is even for $|\mu| < \mu_c$, but it is not even for $|\mu| > \mu_c$, the whole upstream distribution would be an even function of μ. Accordingly, the downstream distribution should be also an even function of μ in the *shock* frame, but not in the *fluid* frame. Thus, in this case, the distributions cannot be isotropic in the fluid frame.

Weakly ansiotropic distributions.—The derived Eqs. (6)–(9) are related to the changes in the pitch-angle cosine at the shock crossing, different from the usually assumed one due to the magnetic moment conservation. Therefore, it is natural to expect that anisotropic distributions will be affected by this modification. However, the first and the most important test of the significance of the new physics would be application to the isotropic part of the distribution function in the classical case of weakly isotropic distributions achieved by diffusive processes [4,6,12]. Let the scattering be isotropic and separable: $D(p',\mu') = D(p')(1-\mu'^2)$. Here, p' and μ' are the momentum and pitch-angle cosine, respectively, in the fluid frame. Following, e.g., Ref. [6], we consider the downstream distribution, which is isotropic in the fluid frame: $f_d(p',\mu') = \frac{1}{2}F_d(p')$, $F_d(p') = \int_{-1}^1 d\mu' f_d(p',\mu')$. Taylor expanding $p' = p - mV\mu\gamma$, one has in HT frame

$$f_d(p,\mu) \approx \frac{F_d(p)}{2} - \frac{mV_d\mu\gamma}{2}\left(\frac{dF_d}{dp}\right). \quad (13)$$

For the upstream distribution, we restrict ourselves to the first two terms in the fluid frame [6,17], corresponding the dipole correction to the isotropic distribution in the order $V_u/v \ll 1$: $f_u(p',\mu') = (1–3V_u\mu'/v')F_u(p')/2$, which gives in the HT frame

$$f_u(p,\mu) \approx \left(1 - \frac{3V_u\mu}{v}\right)\frac{F_u(p)}{2} - \frac{mV_u\mu\gamma}{2}\left(\frac{dF_u}{dp}\right). \quad (14)$$

Substituting into the matching conditions for the forward moving ions (7), one has in the lowest order

$$\alpha F_u\cos\theta_u = F_d\cos\theta_d/2, \quad \alpha = \int_0^1 \mu(1-\chi)d\mu. \quad (15)$$

In the first order, one gets

$$3\zeta(V_u/v)\cos\theta_u F_u + \zeta mV_u\cos\theta_u\gamma\left(\frac{dF_u}{dp}\right) = \frac{1}{3}mV_d\cos\theta_d\gamma\left(\frac{dF_d}{dp}\right), \quad \zeta = \int_0^1 \mu^2(1-\chi)d\mu. \quad (16)$$

Together, these relations give

$$\left(\frac{2\alpha V_d}{3} - \zeta V_u\right)\left(\frac{dF_u}{dp}\right) = \left(\frac{3\zeta V_u}{p}\right)F_u, \quad (17)$$

which has the solution $F_u \propto p^{-s}$, with

$$s = 3[1 - (2\alpha/3\zeta)(V_d/V_u)]^{-1}. \quad (18)$$

For $\chi = H(\mu_c - \mu)$, where $H(x)$ is the Heaviside step function, and $\mu_c = \sqrt{1 - \cos\theta_d/\cos\theta_d} = \sqrt{1 - B_u/B_d}$, one has $\alpha = \frac{1}{2}(1-\mu_c^2)$, $\zeta = \frac{1}{3}(1-\mu_c^3)$, and $(2\alpha/3\zeta) = (1+\mu_c)/(1+\mu_c+\mu_c^2) < 1$. In the parallel case, $\theta_u = \theta_d = 0$ and $\mu_c = 0$, so that one returns to the classical $s_{\text{cl}} = 3r/(r-1)$, $r = V_u/V_d$. In general, if $d\chi/d\mu \le 0$ for

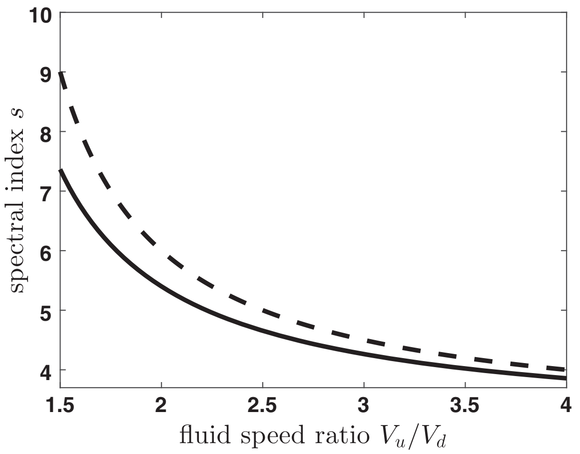

FIG. 1. Dependence of the spectral index s on the fluid speed ratio V_u/V_d for the classical DSA, $2\alpha/3\zeta = 1$ (dashed line) and for the case $2\alpha/3\zeta = 8/9$ (solid line).

$0 < \mu < 1$, one has $2\alpha < 3\zeta$. Indeed, let us approximate the reflectivity with a stepwise function of μ: $1 - \chi = \sum_i \sigma_i H(\mu - \mu_i)$, where all $\sigma_i > 0$ and $\sum_i \sigma_i \leq 1$. Substituting into (15) and (17), one has $2\alpha - 3\zeta = \sum_i \sigma_i \mu_i^2 (\mu_i - 1) < 0$. Since any monotonic function can be represented as a limit of a stepwise function with the number of steps going to infinity, one concludes that for the reflectivity, monotonically decreasing with the pitch-angle cosine, the spectrum should be less steep than the classical one, $s < s_{\rm cl}$. A rough approximation $\chi(\mu) = 1 - \mu$ was numerically obtained by Gedalin *et al.* [19] for a quasiparallel shock with the Mach number $M = 3$, the magnetic compression $B_d/B_u = 1.5$, and the shock angle $\theta = 10°$. These parameters are close to those of a low-Mach number low-β interplanetary shock, observed by Wind on Dec 16, 2006 [24]. Here, β is the ratio of the upstream kinetic pressure to the upstream magnetic pressure. In this case, $\alpha = 1/3$, $\zeta = 1/4$, and $s = 3r/(r - 8/9)$. Figure 1 compares the dependence of the spectral index s on the compression ratio $r = V_u/V_d$ in this case, with the predictions of the classical DSA. In the extreme case of a strong shock, $r = 4$, one has $s = 3.86$, instead of the classical $s = 4$. For lower compression ratios r, the difference is larger. For $r = 2$, which is more typical for interplanetary shocks, one has $s = 6$ for the classical spectrum and $s = 5.4$ for the modified one.

Conclusions.—The collision-free dynamics of high-energy ions at the oblique shock front is gyrophase dependent. In general, the magnetic moment of the ions is not conserved upon crossing the shock transition. Accordingly, the chances of being reflected depend not only on the initial pitch-angle cosine, but also on the initial gyrophase of the ion. Within the gyrophase-averaged description of the distribution functions for accelerated ions, this means that the reflection and transition should be treated using the corresponding probabilities as functions of the pitch-angle cosine. The probabilities depend parametrically on the magnetic compression and the angle between the shock normal and the upstream magnetic field, but do not depend on the particle momentum [19]. In the steady state, where there is no gradual pileup of ions at the shock itself, the fluxes of the ions entering and leaving the shock should be equal. Respectively, the matching conditions at the shock are expressed in an integral form in terms of the reflection and transition probabilities. The effect of the probabilistic shock crossing on the power spectrum on the isotropic part of the distribution is found in the classical theoretical case of the weak anisotropy. In a wide range of the particle momenta, the distributions should be self similar, with a power spectrum $f \propto p^{-s}$. The spectral index s is expressed in terms of the reflectivity, that is, the functional dependence on the pitch-angle cosine of the fraction of reflected ions. The probabilistic nature of the ion reflection at the oblique shock front results in a significant modification of the spectral index, which is no longer universal and depends on three parameters instead of one: a) the ratio of the fluid velocities V_u/V_d, b) the ratio of the magnetic fields B_d/B_u, and c) the angle θ between the shock normal and the upstream magnetic field. Our derivation provides the asymptotic spectral index for shock-accelerated particles at high energies; the spectrum at lower energies is not addressed. The spectral index is derived in the HT frame. In order to have it in any other reference frame, the corresponding Lorentz transformation should be applied. The spectral index depends both on the scattering in the diffusive region and on the scatter-free shock transition. This double dependence is clearly seen in Eq. (18), obtained in the strong upstream and downstream scattering regime, when deviations from isotropy are weak. In this expression, the dependence on V_u/V_d is due to the diffusive shaping of the distributions, and the dependence on α/ζ is due to the scatter-free dynamics in the shock front.

While DSA is probably a good approximation within the uncertainty of presently available measurements, this should not prevent theoretical developments, which study the physics of the acceleration in more depth than it is done within DSA. Although the difference between 4 and 3.86 may seem minor, the significance of the result is that even in this lowest order approximation of near isotropy, there is a modification of the classical spectrum. That is, the proposed approach does not show up only in the high-order corrections, which are not known to affect the spectrum for sure. Effects of the proposed modification of the shock acceleration theory on substantially anisotropic distributions requires a deeper analysis and is beyond the scope of this Letter.

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