Integrable Deformations of T-Dual σ Models

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We present a method to deform (generically non-Abelian) T duals of two-dimensional σ models, which preserves classical integrability. The deformed models are identified by a linear operator ω on the dualized subalgebra, which satisfies the 2-cocycle condition. We prove that the so-called homogeneous Yang-Baxter deformations are equivalent, via a field redefinition, to our deformed models when ω is invertible. We explain the details for deformations of T duals of principal chiral models, and present the corresponding generalization to the case of supercoset models.

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Introduction.—Integrable models in two dimensions have played a pivotal role in the understanding of (quantum) field theory, have numerous applications in condensed matter theory, and have recently attracted attention also in the context of the AdS/CFT correspondence [1], which relates certain string theories on $(d+1)$-dimensional anti–de Sitter (AdS) backgrounds to conformal field theories in d dimensions. The most studied example that exhibits integrable structures is that of the superstring on $AdS_5 \times S^5$ [2] and its dual $\mathcal{N}=4$ super Yang-Mills theory in four dimensions [3], see Refs. [4,5] for reviews. On the string side the two-dimensional world sheet theory is classically integrable; i.e., there is a Lax pair whose flatness condition is equivalent to the equations of motion of the σ model. The Lax pair depends on an auxiliary spectral parameter z, and its expansion around a fixed z_0 yields an infinite set of conserved charges, see Ref. [6] for a review. Integrability has provided the most stringent tests of AdS/CFT, culminating with the possibility of computing the spectrum of the quantum theory in the large N limit exactly [7–10].

Given this tremendous success it is natural to ask whether other theories that are not maximally (super)symmetric are still integrable. Integrability could then also be a guiding principle to discover new models that are interesting in their own right. The β deformation [11–13] or certain gravity duals of noncommutative gauge theories [14,15] are examples that are integrable but reduce to the maximally symmetric case only when a deformation parameter is sent to zero. These instances actually fall into a larger class that goes under the name of Yang-Baxter (YB) models [16–19], sometimes also called η deformations after the deformation parameter. A YB model is identified by an R matrix solving the classical Yang-Baxter equation (CYBE), which in general has a rich set of solutions. Each R generates a background that reduces to the undeformed model (e.g., $AdS_5 \times S^5$) in the $\eta \to 0$ limit. Here, we will not consider the case of the "modified" CYBE.

In this Letter we explore another possibility; we deform the original σ model by adding a topological term (a closed B field) and then apply non-Abelian T duality (NATD) [20] with respect to a subgroup $\tilde{G}$ of the isometry group G. The special case when $\tilde{G}$ is Abelian gives so-called TsT transformations [11–13]. We refer to the resulting actions as deformed T dual (DTD) models, since sending the deformation parameter $\zeta \to 0$ they reduce to NATD. DTD models are in one-to-one correspondence with the 2-cocycles ω of the Lie algebra of $\tilde{G}$. The cocycle condition (3) guarantees that integrability is preserved, and plays the same role as the CYBE for YB models.

The analogy goes even further. When ω is invertible its inverse $R=\omega^{-1}$ solves the CYBE, and each solution of the CYBE corresponds to an invertible 2-cocycle [21]. We use this identification to show that the action of YB models can be recast in the form of DTD models, where the two deformation parameters are simply related by $\eta=\zeta^{-1}$. As explained later, this translates into our language a recent conjecture by Hoare and Tseytlin [22]. We prove it by providing the explicit field redefinition that relates YB to DTD models. The field redefinition is local, albeit in general nonlinear, and it allows us to interpolate between a certain σ model ($\zeta \to \infty$) and its NATD ($\zeta \to 0$). In the case when ω is degenerate, the DTD model is equivalent to a combination of YB deformation and NATD.

We first construct the DTD of the principal chiral model (PCM), since it provides a simpler setup where all the essential features already appear. Later, we generalize it to the case of supercosets, which is more relevant to the study of deformations of superstrings. The supercoset case will be described in more detail elsewhere [23].

DTD of the PCM.—We start from a PCM parametrized by a group element $g \in G$, with the familiar action $S[g]=-\frac{1}{2}\int \mathrm{Tr}(g^{-1}\partial_+ g g^{-1}\partial_- g)$. Since we want to dualize a $\tilde{G}$ subgroup of the left copy of G [24] we rewrite [25]

$$S[f,\tilde{A},\nu]=-\frac{1}{2}\int \mathrm{Tr}((\tilde{A}_+ + J_+)(\tilde{A}_- + J_-)+\nu \tilde{F}_{+-}). \quad (1)$$

Here, $J = dff^{-1}$ is a right-invariant Maurer-Cartan form for $f \in G$, depending on fields that remain spectators under NATD. At the same time $\tilde{A} \in \tilde{\mathfrak{g}}$ and $\nu \in \tilde{\mathfrak{g}}^*$ identify each of the two T-dual frames. If T_i are generators for $\tilde{\mathfrak{g}}$, a basis for the dual algebra $\tilde{\mathfrak{g}}^*$ is given by T^i, where $\mathrm{Tr}(T_i T^j) = \delta_i^j$. The curvature of $\tilde{A}$ is $\tilde{F}_{+-} = \partial_+ \tilde{A}_- - \partial_- \tilde{A}_+ + [\tilde{A}_+, \tilde{A}_-]$. The original PCM is recovered upon integrating out ν since $\tilde{F}_{+-} = 0$ implies that $\tilde{A}$ is pure gauge, i.e., $\tilde{A} = \bar{g}^{-1} d\bar{g}$ for a $\bar{g} \in \tilde{G}$, and we get the desired action with $g = \bar{g} f$. The NATD with respect to $\tilde{G}$, on the other hand, is obtained by integrating out $\tilde{A}$.

We now add a deformation with parameter ζ given by

$$S'[f, \tilde{A}, \nu] = S[f, \tilde{A}, \nu] + \frac{\zeta}{2} \int \mathrm{Tr}(\tilde{A}_+ \omega \tilde{A}_-). \tag{2}$$

Here, ω: $\tilde{\mathfrak{g}} \to \tilde{\mathfrak{g}}^*$ is a linear antisymmetric [i.e., $\mathrm{Tr}(x\omega y) = -\mathrm{Tr}(\omega x y)$] map satisfying the cocycle condition [26]

$$\omega \mathrm{ad}_x y = \mathrm{ad}_x \omega y - \mathrm{ad}_y \omega x, \quad \forall\ x, y \in \tilde{\mathfrak{g}}. \tag{3}$$

This property is needed to have local $\tilde{G}$ invariance also for $\zeta \neq 0$, which ensures that # d.o.f. $= \dim(G)$ [27]. Equations of motion for $\tilde{A}$ give $\int \mathrm{Tr}(\delta \tilde{A}_\mp \mathcal{E}_\pm) = 0$, where

$$\mathcal{E}_\pm \equiv (1 \pm \mathrm{ad}_\nu \pm \zeta\omega)\tilde{A}_\pm \mp \partial_\pm \nu + J_\pm. \tag{4}$$

This implies $\tilde{P}^T \mathcal{E}_\pm = 0$, where $\tilde{P}$ projects onto $\tilde{\mathfrak{g}}$, $\tilde{P}^T$ onto $\tilde{\mathfrak{g}}^*$. We solve these equations by defining the linear operator $\tilde{\mathcal{O}} = \tilde{P}^T (1 - \mathrm{ad}_\nu - \zeta\omega)\tilde{P}$, which is a map $\tilde{\mathfrak{g}} \to \tilde{\mathfrak{g}}^*$

$$\tilde{A}_- = \tilde{\mathcal{O}}^{-1}(-\partial_- \nu - J_-), \qquad \tilde{A}_+ = \tilde{\mathcal{O}}^{-T}(\partial_+ \nu - J_+) \tag{5}$$

and $\tilde{\mathcal{O}}^{-T}$ is the inverse of its transpose. Note that $\tilde{\mathcal{O}}^{-1}\tilde{\mathcal{O}} = \tilde{P}$ as the lhs is defined only on $\tilde{\mathfrak{g}}$. Evaluating S' on the solution we get the DTD action

$$S'[f, \nu] = -\frac{1}{2} \int \mathrm{Tr}(J_+ J_- + (\partial_+ \nu - J_+)\tilde{\mathcal{O}}^{-1}(\partial_- \nu + J_-)). \tag{6}$$

A second interpretation of DTD comes from integrating out ν rather than $\tilde{A}$ from Eq. (2), which gives again $\tilde{A} = \bar{g}^{-1} d\bar{g}$. The resulting action is a topological deformation of the PCM, since the cocycle condition implies that $B = \zeta\omega(\bar{g}^{-1} d\bar{g}, \bar{g}^{-1} d\bar{g})$ is closed. At the classical level adding this term has no effect, and in fact this picture of a deformation that is trivial in the dual frame is reminiscent of YB models: in some cases they correspond to TsT transformations [22,28–30], which are just field redefinitions in a T-dual frame. Since DTD is a NATD of a topological deformation of the PCM, it is classically integrable, where NATD can be applied thanks to closure of B. In fact, the equation $\tilde{A} = \bar{g}^{-1} d\bar{g}$ with $\tilde{A}$ given in Eq. (5) allows us to relate the variables of the deformed model to those of the original PCM. In the special case of Abelian subalgebra $\tilde{\mathfrak{g}}$ the relation simplifies and the deformed model becomes equivalent to the PCM with twisted boundary conditions, consistent with the TsT interpretation [12].

A third interpretation of DTD comes from the possibility of applying NATD to a centrally extended subalgebra. This idea first appeared in Ref. [22] and was the original motivation for considering the deformation (2). One can indeed replace $\tilde{A}$ in Eq. (1) with $\tilde{A}' \in \tilde{\mathfrak{g}}_{\mathrm{c.e.}} = \tilde{\mathfrak{g}} \oplus \mathfrak{c}$ and $\mathfrak{c}$ central; similarly $\nu' \in \tilde{\mathfrak{g}}^*_{\mathrm{c.e.}}$. We decompose $\tilde{A}' = \tilde{A} + \tilde{A}^{\mathfrak{c}}$, $\nu' = \nu + \nu^{\mathfrak{c}}$ with obvious notation, and extend the definition of the trace $\mathrm{Tr}(\mathfrak{c}^2) = 1$, $\mathrm{Tr}(\mathfrak{c}\mathfrak{g}) = 0$. Equations for $\tilde{A}^{\mathfrak{c}}$ imply that $\nu^{\mathfrak{c}}$ is constant, $\nu^{\mathfrak{c}} = \zeta\mathfrak{c}$. At this point $\mathrm{Tr}(\nu' \tilde{F}'_{+-}) = \mathrm{Tr}(\nu \tilde{F}_{+-}) + \zeta \mathbf{f}_{ab} \tilde{A}_+^a \tilde{A}_-^b$, where $\mathbf{f}_{ab}$ are the structure constants introduced by the central extension $[T_a, T_b] = f_{ab}^c T_c + \mathbf{f}_{ab}\mathfrak{c}$. Introducing a map ω whose components are $\omega_{ab} = -\mathbf{f}_{ab}$ we just notice that it is antisymmetric and satisfies the cocycle condition, a consequence of the Jacobi identity in $\tilde{\mathfrak{g}}_{\mathrm{c.e.}}$ projected on $\mathfrak{c}$.

For some ω's DTD reduces to just NATD; i.e., the deformation parameter can be removed by a field redefinition. This happens when ω is a coboundary, i.e., $\omega(x, y) = f([x, y])$ for some function f. Therefore, nontrivial deformations are in one-to-one correspondence with 2-cocycles modulo coboundaries, i.e., with elements of the second cohomology group $H^2(\tilde{\mathfrak{g}})$. The same holds also for nontrivial central extensions. In particular, there are none for semisimple $\tilde{\mathfrak{g}}$. Trivial deformations are equivalently described as adding an exact B field to the PCM.

An example.—Before continuing our general discussion, let us provide an explicit example: a PCM on $U(2)$. We use generators $T_j = i\sigma_j \in \mathfrak{su}(2)$ and $T_4 = i\mathbf{1}$, with duals $T^j = -(i/2)\sigma_j$ and $T^4 = -(i/2)\mathbf{1}$. We parametrize the group element by $g = \exp(i\theta\mathbf{1})\exp(i\phi_+\sigma_1)\check{g}(\xi)\exp(i\phi_-\sigma_2)$, where $\phi_\pm = (\phi_1 \pm \phi_2)/2$ and $\check{g}(\xi) = \mathrm{diag}(i^{-1/2}e^{i\xi}, i^{1/2}e^{-i\xi})$. The PCM action yields the metric of $\mathrm{S}^3 \times \mathrm{S}^1$

$$ds^2 = d\xi^2 + \sin^2\xi d\phi_1^2 + \cos^2\xi d\phi_2^2 + d\theta^2. \tag{7}$$

Suppose we want to dualize the coordinates ϕ_+ in S^3 and θ in S^1, corresponding to the Abelian subalgebra $\tilde{\mathfrak{g}} = \mathrm{span}\{T_1, T_4\}$. We take $f = \check{g}(\xi)\exp(i\phi_-\sigma_2)$ and $\nu = 2(\tilde{\phi}_+ T^1 + \tilde{\theta} T^4)$, where $\tilde{\phi}_+$, $\tilde{\theta}$ are dual coordinates. We deform the dual theory by taking $\omega = 2T^1 \wedge T^4$; namely, $\omega T_1 = -2T^4$, $\omega T_4 = 2T^1$. From Eq. (6) we find the action of DTD $S' = \int \partial_+ X^i (G_{ij} - B_{ij}) \partial_- X^j$, with the metric and B field

$$ds^2 = d\xi^2 + (1+\zeta^2)^{-1}(d\tilde{\phi}_+^2 + (\zeta^2 + \sin^2 2\xi)d\phi_-^2 + d\tilde{\theta}^2 + 2\zeta\cos 2\xi d\tilde{\theta}d\phi_-),$$
$$B = (1+\zeta^2)^{-1}(\cos 2\xi d\phi_- - \zeta d\tilde{\theta}) \wedge d\tilde{\phi}_+. \tag{8}$$

The $\zeta \to 0$ limit yields the T-dual model of $S^3 \times S^1$ with respect to $\tilde{\mathfrak{g}}$. To relate this simple example to a YB model it is enough to take $\nu = \eta^{-1}R(\vartheta T^4 + \varphi_+ T^1)$ with $R = \frac{1}{2}(T_4 \wedge T_1)$. However, when $\tilde{\mathfrak{g}}$ is non-Abelian, the field redefinition is more complicated, see Eq. (13).

Integrability.—Above we argued that DTD models must be integrable; however, it is instructive to show this explicitly to see how the cocycle condition enters and write a Lax connection. We will show that the equations of motion formally resemble those of the PCM, for which a Lax pair is known. Suppose we consider a PCM with group element $g = \bar{g}f$, with $\bar{g} \in \tilde{G}$, $f \in G$. We prefer to rewrite its on-shell equations in terms of the left and right currents $\tilde{A} = \bar{g}^{-1}d\bar{g}$ and $J = dff^{-1}$. To start, the flatness condition for $A = g^{-1}dg$ is equivalent to $\mathcal{F}^J = 0$, $\mathcal{F}^{\tilde{A}} = 0$

$$\mathcal{F}^J \equiv \partial_+ J_- - \partial_- J_+ - [J_+, J_-],$$
$$\mathcal{F}^{\tilde{A}} \equiv \partial_+ \tilde{A}_- - \partial_- \tilde{A}_+ + [\tilde{A}_+, \tilde{A}_-]. \tag{9}$$

Moreover, the equations of motion for the PCM, i.e., conservation of A, become $\mathcal{C} = 0$,

$$\mathcal{C} \equiv \partial_+(J_- + \tilde{A}_-) + \partial_-(J_+ + \tilde{A}_+) + [\tilde{A}_+, J_-] + [\tilde{A}_-, J_+]. \tag{10}$$

Let us now rederive the above equations for DTD models, where now importantly $\tilde{A}$ is identified as in Eq. (5). To start, the flatness condition $\mathcal{F}^J = 0$ still follows from the definition of J. Flatness for $\tilde{A}$, instead, now arises as the equations of motion for ν, which are $\delta_\nu S'[f,\nu] = -\frac{1}{2}\int \mathrm{Tr}(\delta\nu \mathcal{F}^{\tilde{A}}) = 0$. It is nice that the known mechanism familiar from T duality of trading flatness for an equation of motion still holds for DTD models.

The equations of motion for f are $\delta_f S'[f,\nu] = +\frac{1}{2}\int \mathrm{Tr}(\delta f f^{-1}\mathcal{C}) = 0$, essentially as in the previous example of the PCM. However, in that case it is only thanks to the equations of motion for $\bar{g}$ [i.e., $\int \mathrm{Tr}(\bar{g}^{-1}\delta\bar{g}\mathcal{C}) = 0$] that one can claim $\mathcal{C} = 0$. In analogy to the PCM, it is then clear that our task is to show that $\tilde{P}^T\mathcal{C} = 0$ also for DTD models. We generalize the argument of Ref. [31] for NATD of the PCM, and consider the equations $\mathcal{E}_\pm = M_\pm^\perp$, for some $M_\pm^\perp$ for which $\tilde{P}^T M_\pm^\perp = 0$. They imply $\tilde{P}^T \mathcal{E}_\pm = 0$; i.e., they are equivalent to the solutions for $\tilde{A}$ as in Eq. (5). They obviously imply also the equation $(\partial_+ + \mathrm{ad}_{\tilde{A}_+})(\mathcal{E}_- - M_-^\perp) + (\partial_- + \mathrm{ad}_{\tilde{A}_-})(\mathcal{E}_+ - M_+^\perp) = 0$, which reads as

$$\mathcal{C} = [\partial_- + \mathrm{ad}_{\tilde{A}_-}, \partial_+ + \mathrm{ad}_{\tilde{A}_+}]\nu - (\partial_- + \mathrm{ad}_{\tilde{A}_-})M_+^\perp - (\partial_+ + \mathrm{ad}_{\tilde{A}_+})M_-^\perp + \zeta[\omega(\partial_+\tilde{A}_- - \partial_-\tilde{A}_+) + \mathrm{ad}_{\tilde{A}_+}\omega\tilde{A}_- - \mathrm{ad}_{\tilde{A}_-}\omega\tilde{A}_+].$$

The first line on the right-hand side is rewritten as $[\nu, \tilde{F}_{+-}]$, and hence vanishes thanks to the flatness of $\tilde{A}$. The second line vanishes upon projecting with $\tilde{P}^T$ [32]. Finally, the last line vanishes thanks to the cocycle condition: using Eq. (3) it is rewritten as $-\zeta\omega(\tilde{F}_{+-})$, which is again zero. Since also $\tilde{P}^T\mathcal{C} = 0$ holds, we conclude that the whole set of on-shell equations for the DTD models is formally equivalent to those of a PCM, provided the proper $\tilde{A}$ is used. We can furthermore write the Lax pair as

$$L_\pm = \frac{1}{2}(1 + z^{\mp 2})\mathrm{Ad}_f^{-1}(\tilde{A}_\pm + J_\pm) \tag{11}$$

with z a spectral parameter. In fact, the flatness condition $\partial_+ L_- - \partial_- L_+ + [L_+, L_-] = 0$ is equivalent to the on-shell equations just derived.

Relation to Yang-Baxter models.—We now prove that YB deformations for the PCM on the group G are equivalent to DTD. This was checked for many particular examples in Ref. [22]. YB models are identified by an R matrix solving the CYBE on the Lie algebra $\mathfrak{g}$. If $g \in G$

$$S_{\mathrm{YB}}[g] = -\frac{1}{2}\int \mathrm{Tr}\left(g^{-1}\partial_+ g \frac{1}{1 - \eta R_g} g^{-1}\partial_- g\right). \tag{12}$$

R is invertible on a certain subalgebra and its inverse is a 2-cocycle [21]. As anticipated, we identify $R = \omega^{-1}$, where ω is the operator defining the DTD model. Then, R: $\tilde{\mathfrak{g}}^* \to \tilde{\mathfrak{g}}$. The two deformation parameters will be related by $\eta = \zeta^{-1}$.

We first split the group element parametrizing the YB model as $g = \tilde{g}f$, where $\tilde{g} \in \tilde{G}$ and $f \in G$. We identify f with the homonym appearing on the DTD side. Our proof of equivalence of the two actions will then consist in giving the field redefinition relating $\tilde{g}$ and ν. Since R is invertible, we can always take $\tilde{g} = \exp(RX)$ for some $X \in \tilde{\mathfrak{g}}^*$. One can check that taking $X = \eta\nu + (\eta^2/2)\tilde{P}^T[R\nu, \nu] + \mathcal{O}(\eta^3)$ the two actions are equivalent up to terms that are at least cubic in η. The generalization to all orders can be obtained by requiring that the $dfdf$ terms in the two actions match. This leads to the condition $(1 - \eta R_{\tilde{g}})^{-1} = 1 - \tilde{\mathcal{O}}^{-1}$ whose solution can be shown to be

$$\nu = \frac{1}{\eta}\tilde{P}^T \frac{1 - e^{-\mathrm{ad}_{RX}}}{\mathrm{ad}_{RX}} X = \frac{1}{\eta}\tilde{P}^T \frac{1 - \mathrm{Ad}_{\tilde{g}}^{-1}}{\log \mathrm{Ad}_{\tilde{g}}}\omega \log \tilde{g}. \tag{13}$$

It follows that $d\nu = (\tilde{P}^T - \tilde{\mathcal{O}})\tilde{g}^{-1}d\tilde{g}$ or, equivalently,

$$\mathbf{A}_\pm = \mathrm{Ad}_f^{-1}(J_\pm + \tilde{A}_\pm), \tag{14}$$

where we defined $\mathbf{A}_\pm = (1 \pm \eta R_g)^{-1}(g^{-1}\partial_\pm g)$ on the YB side. Using these relations it is not hard to check that the two actions are the same up to the topological term $\zeta\omega(\tilde{g}^{-1}d\tilde{g}, \tilde{g}^{-1}d\tilde{g})$, which has no effect in the classical theory as remarked earlier.

We have proven the equivalence of the DTD and YB models when ω is nondegenerate. In the case of a degenerate ω it is always possible to choose it in such a way that it is nondegenerate on a subalgebra $\hat{\mathfrak{g}} \subset \tilde{\mathfrak{g}}$ [33] and acts trivially on its complement $\check{\mathfrak{g}}$ in $\tilde{\mathfrak{g}}$, also an algebra thanks to Eq. (3). We interpret it as NATD on $\check{\mathfrak{g}}$ of the YB model corresponding to restricting ω to $\hat{\mathfrak{g}}$.

DTD of supercosets.—The construction of DTD models for supercosets follows the steps explained in the simpler case of the PCM. Here, we only present the main results, whose derivation will be collected in Ref. [23].

We still denote by G the group of superisometries, e.g., $PSU(2,2|4)$ for superstrings on $\mathrm{AdS}_5 \times \mathrm{S}^5$, see Ref. [34] for a review. Its Lie superalgebra $\mathfrak{g}$ admits a $\mathbb{Z}_4$ decomposition, and we denote by $P^{(j)}$ the projectors onto the four subspaces. They typically appear in the combination $\hat{d} = P^{(1)} + 2P^{(2)} - P^{(3)}$ or its transpose $\hat{d}^T$. The absence of $P^{(0)}$ in $\hat{d}$ is necessary for the local $\mathfrak{g}^{(0)}$ invariance of the action, i.e., local Lorentz transformations. The action for DTD models of supercosets is [35]

$$S'[f,\nu] = -\frac{T}{2}\int \mathrm{Str}(J_+\hat{d}_f J_- + (\partial_+\nu - \hat{d}_f^T J_+)\tilde{\mathcal{O}}^{-1}(\partial_-\nu + \hat{d}_f J_-)), \tag{15}$$

where $\hat{d}_f \equiv \mathrm{Ad}_f \hat{d}\mathrm{Ad}_f^{-1}$. We keep the same definitions for J, ν, which however now take values in superalgebras. Moreover, now $\tilde{\mathcal{O}} = \tilde{P}^T(\hat{d}_f - \mathrm{ad}_\nu - \zeta\omega)\tilde{P}$.

The model is integrable since we can write down a Lax pair. This is more conveniently expressed in terms of $A = \mathrm{Ad}_f^{-1}(\tilde{A} + J)$, where

$$\tilde{A}_+ = \tilde{\mathcal{O}}^{-T}(+\partial_+\nu - \hat{d}_f^T J_+),$$
$$\tilde{A}_- = \tilde{\mathcal{O}}^{-1}(-\partial_-\nu - \hat{d}_f J_-). \tag{16}$$

The flatness condition $\partial_+\mathcal{L}_- - \partial_-\mathcal{L}_+ + [\mathcal{L}_+, \mathcal{L}_-] = 0$ for

$$\mathcal{L}_\pm = A_\pm^{(0)} + zA_\pm^{(1)} + z^{\mp 2}A_\pm^{(2)} + z^{-1}A_\pm^{(3)} \tag{17}$$

is equivalent to the on-shell equations of the DTD model.

DTD models of supercosets possess kappa symmetry, and therefore correspond to solutions of the generalized supergravity equations of Refs. [36,37]. Kappa symmetry transformations are $\delta f f^{-1} = \hat{d}_f^T(\delta\nu) = \rho_{1,-} + \rho_{3,+}$, where

$$\rho_{j,\pm} = \{i\mathrm{Ad}_f\kappa^{(j)}, J_\pm^{(2)} + \tilde{A}_\pm^{(2)}\} \tag{18}$$

and $\kappa^{(j)}$, $j = 1, 3$ are two local parameters of grading j. The action (15) is invariant under these transformations upon using the Virasoro constraints. If we were not fixing conformal gauge, the variation of the action would be compensated by the variation of the world sheet metric. From these kappa symmetry transformations it is possible to extract the background fields of DTD models [23].

The equivalence to YB models for invertible ω's holds also in the case of DTD models of supercosets. Remarkably, the field redefinition is still given by Eq. (13) as for the PCM. We have further verified that kappa symmetry transformations of YB models [18] take the above form under this field redefinition, when we fix the $\tilde{G}$ gauge to get $\delta f f^{-1} = \hat{d}_f^T(\delta\nu)$.

Conclusions.—We provided a unified picture of (non-Abelian) T duality and homogeneous YB deformations as DTD of σ models. As pointed out in Ref. [22], an advantage of this formulation is that it can be realized at the path integral level, giving a better handle on the quantum theory. In fact, it also explains why the condition for one-loop Weyl invariance, i.e., unimodularity of $\tilde{\mathfrak{g}}$, is the same for both the YB model and NATD [30,38,39].

Despite the close relation, it is still worth viewing the DTD models as a distinct class of deformations. In fact, the field redefinition that relates it to the YB model is singular in the two undeformed limits; the YB model becomes degenerate when taking the undeformed (i.e., $\zeta \to 0$) limit of DTD models, and vice versa. Therefore, the interpretation as deformation applies to just one of the two models in the T-dual pair. It would be interesting to understand if there is any connection to the λ model of Refs. [31,40,41], which is also a deformation of NATD and is related to the inhomogeneous YB deformation [16–18].

Although our motivation was integrability, such deformations can be applied also to nonintegrable models, which provides an interesting and potentially useful way to generate new supergravity solutions.

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[24] The construction could be generalized to include the right copy. That DTD model should then be related to the bi-YB model of Ref. [17].
[25] We omit the integration measure $d\sigma^+ d\sigma^-$, where $\sigma^\pm = \tau \pm \sigma$.
[26] We use standard notation $\mathrm{Ad}_g M = gMg^{-1}$ and $\mathrm{ad}_x M = [x, M]$. Equivalently, Eq. (3) takes the form $\omega(x, [y, z]) + \omega(y, [z, x]) + \omega(z, [x, y]) = 0$ for ω: $\tilde{\mathfrak{g}} \otimes \tilde{\mathfrak{g}} \to \mathbb{R}$.
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