Comment on "Nature of Long-Range Order in Stripe-Forming Systems with Long-Range Repulsive Interactions"

In a recent Letter, Mendoza-Coto *et al.* [1] presented a study focusing on the nature of phase transitions in two-dimensional (2D) stripe-forming systems with competing short-range attractive and long-range ($1/r^{\alpha}$) repulsive interactions. In particular, they conclude that for dipolar interactions ($\alpha = 3$), the isotropic-nematic phase transition is in the Kosterlitz-Thouless (KT) universality class. The authors support their findings by mapping an effective Hamiltonian into models which behavior resembles the 2D XY model at low temperatures [2], and a finite-size scaling analysis from Langevin simulations. However, the validity of their conclusion is hindered by the lack of numerical evidence due to the relative small lattice sizes [3], and more generally because a nonuniversal behavior is expected for such 2D systems [4]. As we illustrate here, tiny lattice effects are enough to alter the transition scenario completely.

Below we present a comparative analysis between the microcanonical caloric curves $\beta(E)$ obtained for the 2D dipolar Ising model [5], which displays an isotropic-nematic transition, and the 2D XY model [6], which shows a KT transition. The Hamiltonians $H_{\rm dip}^{\rm 2D}$ and $H_{XY}^{\rm 2D}$ are given in terms of the parameters δ and J as in [5,6], respectively. We evaluate $\beta(E)$ via the statistical temperature-weighted histogram analysis method (ST-WHAM) [7] from data produced by Monte Carlo simulations with cluster updates for the XY model and replica exchange method for the dipolar Ising model.

Figure 1 clearly shows that caloric curves for the two models exhibit distinct behaviors. While there are two S-shaped curves in $\beta(E)$ for the dipolar Ising model, a monotonic decreasing behavior is observed for the XY model. The two S-shaped curves in Fig. 1(a) corresponds to two transitions that separates the isotropic phase (which is stable for $E/N \gtrsim -1.086$ and temperatures $T > 1/\beta_{\rm IN} \simeq 0.8$), the nematic phase (snapshot in the middle), and the striped phase (which is stable for $E/N \lesssim -1.187$ and $T < 1/\beta_{\rm NS} \simeq 0.767$, and displays ground-state configurations with 18 stripes for $N = 72^2$). The S-shaped curves in $\beta(E)$ are due to the presence of first-order phase transitions [8]. The results for the XY model in Fig. 1(b) shows the region near the KT transition at $T = 1/\beta_{\rm KT} \simeq 0.893$. Accordingly, there is no signal of S-shaped curves in $\beta(E)$ because the KT transition is an infinite order transition [6].

In conclusion, our analysis indicates that the isotropic-nematic transition in the dipolar Ising model is a first-order phase transition instead of a KT transition. By considering this example and the expected nonuniversality, we argue that both mapping and numerical results in [1] are insufficient to determine the nature of isotropic-nematic transition in 2D systems with competing short-range and dipolar interactions.

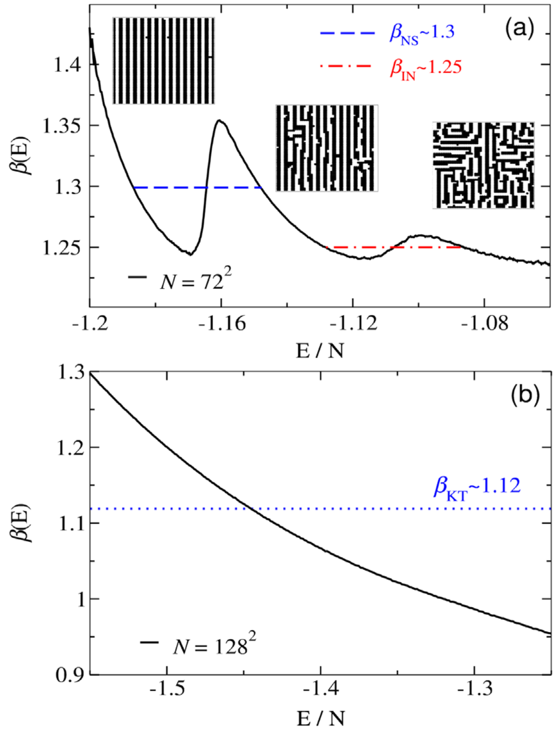

FIG. 1. Caloric curves $\beta(E)$ vs energy per spin E/N. (a) dipolar Ising model for $\delta = 2$, where a nematic phase is observed [5]. Horizontal lines denote transition temperatures obtained by Maxwell's construction. From left to right: configurations in the striped, nematic, and isotropic phases, respectively. (b) XY model for $J = 1$ at the KT transition.

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