Chaos in Chiral Condensates in Gauge Theories

Koji Hashimoto,[1] Keiju Murata,[2] and Kentaroh Yoshida[3]
[1]Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan
[2]Keio University, 4-1-1 Hiyoshi, Yokohama 223-8521, Japan
[3]Department of Physics, Kyoto University, Kyoto 606-8502, Japan

(Received 26 June 2016; revised manuscript received 9 September 2016; published 30 November 2016)

Assigning a chaos index for dynamics of generic quantum field theories is a challenging problem because the notion of a Lyapunov exponent, which is useful for singling out chaotic behavior, works only in classical systems. We address the issue by using the AdS/CFT correspondence, as the large N_c limit provides a classicalization (other than the standard $\hbar \to 0$) while keeping nontrivial quantum condensation. We demonstrate the chaos in the dynamics of quantum gauge theories: The time evolution of homogeneous quark condensates $\langle \bar{q}q \rangle$ and $\langle \bar{q}\gamma_5 q \rangle$ in an $\mathcal{N}=2$ supersymmetric QCD with the $SU(N_c)$ gauge group at large N_c and at a large 't Hooft coupling $\lambda \equiv N_c g_{\rm YM}^2$ exhibits a positive Lyapunov exponent. The chaos dominates the phase space for energy density $E \gtrsim (6 \times 10^2) \times m_q^4 (N_c/\lambda^2)$, where m_q is the quark mass. We evaluate the largest Lyapunov exponent as a function of (N_c, λ, E) and find that the $\mathcal{N}=2$ supersymmetric QCD is more chaotic for smaller N_c.

DOI: 10.1103/PhysRevLett.117.231602

Revealing a hidden relation between generic quantum field theories and chaos is a long-standing problem. The solution can ignite novel quantitative study of the complexity of particle physics. The problem is how one can define a quantity like a Lyapunov exponent, which measures chaos, in generic quantum dynamics of field theories. The Lyapunov exponent can be defined only in classical systems—once the systems are quantized, because the strong dependence on initial values is lost due to the quantum effect. Then how can one measure the chaos of purely quantum phenomena, such as the chiral condensate of QCD?

In the history concerned with this issue, the chaos of the classical limit of the Yang-Mills theory was first found [1–6] and was applied to an entropy production process of heavy ion collisions [7–12] together with a color glass condensate [13–15]. However, the produced quark gluon plasma is strongly coupled, and a transition from the classical Yang-Mills to quantum states is yet an open question. On the other hand, the recent study [16] of out-of-time-ordered correlators of quantum fields [17] defined a quantum analog of the Lyapunov exponent and opened a new direction about the problem [18–27]. The problem of chaos in quantum dynamics could be addressed along the line of this development.

We here provide a solution of the problem. A key observation is that there exist several ways to relate quantum field theories to classical ones, although the standard method is the semiclassical limit $\hbar \to 0$. In fact, a large N limit of strongly coupled gauge theories is another classical limit. We use the AdS/CFT correspondence [28] as a tool to resolve the problem and to map the strongly coupled theories to a classical gravity, which enables us to calculate the Lyapunov exponents of expectation values of operators directly probing the quantum dynamics. The idea is supported by recent analyses of chaotic motion of classical strings in AdS-like spacetimes [29–37] (see also [38–43]).

In this Letter, we first show that the linear σ model of low-energy QCD exhibits chaos of the chiral condensate, which serves as a toy model of chaos of a quantum phenomenon. Then we concretely study an $\mathcal{N}=2$ supersymmetric QCD with the $SU(N_c)$ $\mathcal{N}=2$ gauge group at large N_c and at strong coupling [44]. By using the AdS/CFT, we calculate Lyapunov exponents of the time evolution of a homogeneous quark condensate. The analysis shows how the complexity of the quantum dynamics depends on N_c and λ: The theory is more chaotic for a larger λ or a smaller N_c. The discovered chaos is a quantum analog of the butterfly effect. We discuss that our Lyapunov exponent is also described by a generalized out-of-time-ordered correlator.

Chaos in a linear σ model.—The most popular effective action for the chiral condensate of QCD is the linear σ model. It describes a universal class of theories governed by a chiral symmetry via a spontaneous and an explicit breaking. The former comes from the QCD strong coupling dynamics, while the latter comes from a quark mass term. We find below that the model exhibits chaos.

The simplest linear σ model is with a chiral $U(1)$ symmetry with an explicit breaking (quark mass) term:

$$S = \int d^4x \left\{ -\frac{1}{2}[(\partial_\mu \sigma)^2 + (\partial_\mu \pi)^2] - V \right\}, \tag{1}$$

$$V \equiv \frac{\mu^2}{2}(\sigma^2 + \pi^2) + \frac{g_4}{4}(\sigma^2 + \pi^2)^2 + a\sigma + V_0. \tag{2}$$

Here for simplicity we consider only a single flavor and ignore the axial anomaly. $\sigma(x^\mu)$ and $\pi(x^\mu)$ are fields whose fluctuation provides a sigma meson field with the mass m_σ and a neutral pion field with the mass m_π, respectively. The vacuum expectation value of σ minimizing the potential V defines the chiral condensate: $\langle\sigma\rangle = f_\pi$. A constant V_0 is introduced just for shifting the vacuum energy to zero. Relations to observable parameters are found as $2\mu^2 = -m_\sigma^2 + 3m_\pi^2$, $g_4 = (m_\sigma^2 - m_\pi^2)/(2f_\pi^2)$, and $a = -m_\pi^2 f_\pi$ [45].

Let us consider a homogeneous motion [46] of the σ model fields $\sigma(t)$ and $\pi(t)$, which is a time evolution of the chiral condensate. In a Hamiltonian language, there are four dynamical variables $(\sigma, \pi, \dot\sigma, \dot\pi)$ while the conserved quantity is only the total energy, so there may exist chaos. We search the chaos by varying the total energy density E and find a chaotic behavior of the chiral condensate. At an intermediate scale of the energy density, the Poincaré section (a cross section of orbits in the phase space with sampled initial conditions sharing a chosen conserved energy) exhibits a scattered plot, which is chaos; see Fig. 1. For the numerical simulations, we have chosen $m_\pi = 135\,[\mathrm{MeV}]$, $m_\sigma = 500\,[\mathrm{MeV}]$, and $f_\pi = 93\,[\mathrm{MeV}]$.

In this model, the chaos emerges consequently due to the existence of a saddle point in the potential V as shown in Fig. 2. In general, separatrices (boundaries between phase space domains with distinct dynamical behavior) are associated with saddle points. They are broken under weak perturbations and become a seed of chaos. The separatrix in the potential V is generated by the combination of the explicit and the spontaneous symmetry breaking terms. For example, a potential with no $a\sigma$ term makes the system integrable due to the Poincaré-Bendixon theorem.

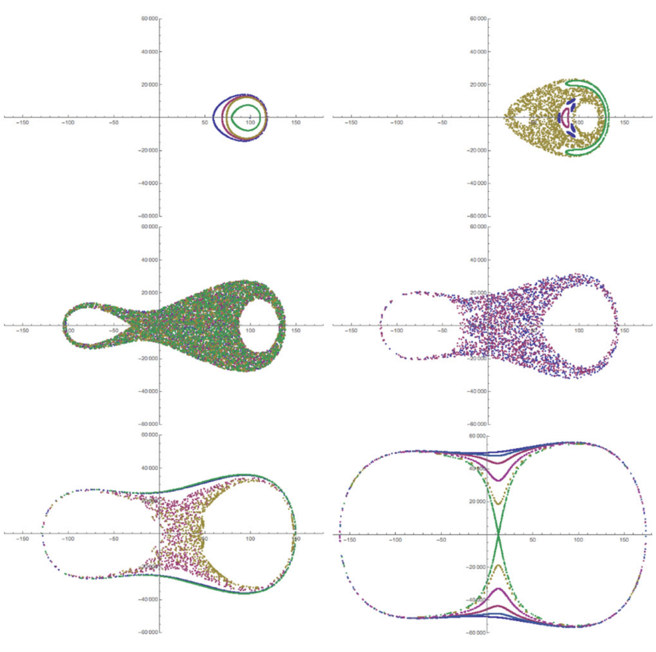

FIG. 1. The Poincaré sections for the linear sigma model. The horizontal axis is $\sigma(t)$, while the vertical axis is $\dot\sigma(t)$. The section is chosen by $\pi(t) = 0$. The energy density is chosen as $E^{1/4} = 100$, 130, 140, 150, 160, and 200 [MeV] in the top-left, top-right, middle-left, middle-right, lower-left, and lower-right figures, respectively.

It is interesting that the chaotic phase (at which the Poincaré section is covered mostly by an ergodic chaos pattern) appears only at an intermediate scale of the energy density: $1.3 \times 10^2\,[\mathrm{MeV}] < E^{1/4} < 1.7 \times 10^2\,[\mathrm{MeV}]$. It is roughly equal to the height of the separatrix $\sim m_\pi^2 f_\pi^2$. The measure of the chaos is provided by the Lyapunov exponent

$$L(E) \equiv \lim_{t\to\infty} \lim_{d(0)\to 0} \frac{1}{t} \log \frac{d_{\langle\bar{q}q\rangle}(t)}{d_{\langle\bar{q}q\rangle}(0)}, \tag{3}$$

where $d_{\langle\bar{q}q\rangle}(t)$ is the distance between the two time evolution orbits of the quark condensate $\sigma(t)$ and $d_{\langle\bar{q}q\rangle}(0)$ is taken to be infinitesimally small. The energy dependence of the calculated Lyapunov exponent of the linear σ model is given in Fig. 3. We observe that chaos appears only at the intermediate energy scale. It suggests that the thermal entropy of QCD might be related to the Lyapunov exponent and to an entropy production of the thermal history of the Universe in some manner.

Chaotic chiral condensate.—Our analysis suggests that generically chaos appears in the time evolution of chiral condensates, because the linear σ model is just based on a symmetry and its breaking. Although there are various σ models of QCD, they contain the simplest linear σ model (1) as a subsector. Generic σ models concern non-Abelian chiral symmetries $U(N_f)$ with N_f quark flavors, and the hidden local symmetry [47,48] can be used to formulate vector meson actions. Generically, non-Abelianization accompanies a specific nonlinearity due to the hidden symmetry, which is another possible nest of chaos.

Unfortunately, linear or nonlinear σ models are toy models in which a classical treatment is not simply justified, and, furthermore, they describe only a universality class, so a precise relation to QCD is lost. Only with the large N_c limit is classicalization certified and an explicit

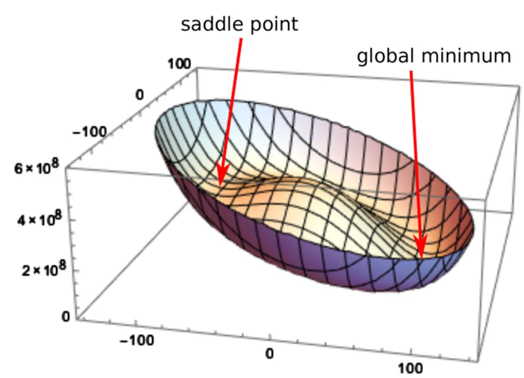

FIG. 2. The potential V of the linear σ model. The horizontal axes are for σ and π. The potential bottom is at $(\langle\sigma\rangle, \langle\pi\rangle) = (f_\pi, 0)$. Because of the quark mass term, there appears a separatrix on the negative axis of σ.

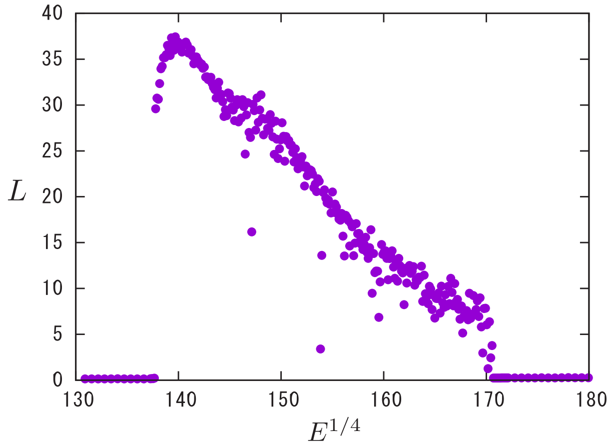

FIG. 3. The Lyapunov exponent L [MeV] of the linear σ model as a function of the energy density $E^{1/4}$ [MeV]. The initial condition is chosen as $\sigma = f_\pi$ and $\dot{\sigma} = \dot{\pi} = 0$.

connection found. In the following, we resort to the AdS/CFT correspondence with which the large N_c and the large λ limits lead to an exact classical theory of mesons and chiral condensates. Using the AdS/CFT correspondence, a chaotic index such as the Lyapunov exponent can be calculated as a function of theories.

Action for the quark condensates from AdS/CFT.—In AdS/CFT correspondence [28], chiral condensates at large λ and large N_c can be seen at the asymptotic behavior of bulk fields corresponding to the gauge-invariant operators such as $\langle \bar{q}q \rangle$ and $\langle \bar{q}\gamma_5 q \rangle$. In this Letter, as a first step, we analyze the most popular holographic model, the $\mathcal{N} = 2$ supersymmetric QCD. In the theory with N_f hypermultiplets of fundamental quarks coupled to the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory, the quark sector is introduced as N_f probe $D7$-branes [44] in the geometry of $\mathrm{AdS}_5 \times \mathrm{S}^5$ (see [49] for a review). The static quark condensates vanish due to the supersymmetries. We are interested in the time-dependent dynamics of the condensates, which is directly encoded in the $D7$-brane action we calculate in the following.

Any chaos needs nonlinear terms, and the $D7$-brane action suffices the need. For multiflavor case $N_f \geq 2$, the action possesses a non-Abelian symmetry $U(N_f)$ and is effectively described by a massive $SU(N_f)$ Yang-Mills theory. There are two adjoint scalar fields which measure the fluctuation of the N_f $D7$-brane world volume in the transverse directions, and those vacuum expectation values are the condensates $\langle \bar{q}q \rangle$ and $\langle \bar{q}\gamma_5 q \rangle$. Let us evaluate the non-Abelian D-brane action proposed in Ref. [50]:

$$S_{D7} = -T_{D7} \int d^8\xi \mathrm{STr} \sqrt{-\det \tilde{G}_{rs}} \sqrt{\det Q^a_b}, \qquad (4)$$

where $\tilde{G}_{rs} \equiv G_{rs} + G_{ra}(Q^{-1} - \delta)^{ab} G_{sb} (r, s = 0, \ldots, 7)$ and $Q^a_b \equiv \delta^a_b + i[X^a, X^c] G_{cb}/2\pi\alpha' (a, b = 8, 9)$. We took a static gauge and have ignored gauge fields on the $D7$-branes, and $G_{rs} \equiv g_{rs}(X) + \partial_r X^a \partial_s X^b g_{ab}(X)$ is the induced metric on the $D7$-branes. The $\mathrm{AdS}_5 \times \mathrm{S}^5$ metric $g(X)$ is

$$ds^2 = \frac{r^2}{R^2}(dx^\mu)^2 + \frac{R^2}{r^2}[d\rho^2 + \rho^2 d\Omega_3^2 + (dX^8)^2 + (dX^9)^2],$$

where X^8 and X^9 are directions transverse to the $D7$-brane and $r^2 \equiv \rho^2 + (X^8)^2 + (X^9)^2$. Here $R \equiv (2\lambda)^{1/4}(\alpha')^{1/2}$ is the AdS radius, and $T_{D7} \equiv (2\pi)^{-6}(\alpha')^{-4} g_{\mathrm{YM}}^{-2}$ is the $D7$-brane tension. "STr" means a symmetrized trace in the $U(N_f)$ adjoint indices.

A static classical solution of the $D7$-brane action was found in Ref. [44] as $(X^8, X^9) = (c, 0)$ in which c is related to the quark mass as $c = 2\pi\alpha' m_q$. The $D7$-brane solution independent of ρ means vanishing condensates $\langle \bar{q}q \rangle = \langle \bar{q}\gamma_5 q \rangle = 0$, since these expectation values are coefficients of $1/\rho^2$ appearing in $X^8(\rho)$ and $X^9(\rho)$ at $\rho \sim \infty$. Fluctuations $(w^8, w^9) \equiv (X^8 - c, X^9)$ correspond to towers of scalar or pseudoscalar mesons of the theory, and the action quadratic in the fluctuations provides the spectra of the mesons [51]. A linear analysis of the action (4) concerning a part of the commutator term was found in Ref. [52]. We need the full structure of the commutator term. Expanding the action (4) around the classical solution up to a quadratic order in ∂X and also up to a single commutator term $[X, X]^2$, we obtain

$$S = -T_{D7} \int \rho^3 d^4x d\rho d\Omega_3 \mathrm{STr} \left[1 + \frac{R^4 (\partial_\mu w^a)^2}{2(\rho^2 + c^2)^2} + \frac{(\partial_\rho w^a)^2}{2} - \frac{R^4 [w^8, w^9]^2}{2(2\pi\alpha')^2 (\rho^2 + c^2)^2} \right]. \qquad (5)$$

The expansion is valid for $w^a \ll c$ and $|\partial_\mu w^a| \ll c^2/R^2$. We assumed that w^a is independent of Ω_3 for simplicity.

We are interested in low-energy region, so we excite only the lowest meson eigenstate $w^a = [\mathcal{N}/(\rho^2 + c^2)]\phi_a(t)$ and substitute it to (5). The normalization $\mathcal{N}$ is fixed to have a canonical kinetic term for the lightest scalar or pseudoscalar mesons ϕ_a. The resultant action for spatially homogeneous meson fields is

$$S = \int d^4x \mathrm{Tr} \left[\frac{1}{2} \dot{\phi}_a^2 - \frac{8\pi^2 m_q^2}{\lambda} \phi_a^2 + \frac{36\pi^2}{5N_c} (\phi_8, \phi_9)^2 \right]. \qquad (6)$$

The matrix elements of the expectation value of the mesons are the condensates of the flavor $i, j (= 1, \ldots, N_f)$:

$$(\phi_8^{ij}(t), \phi_9^{ij}(t)) \propto (\langle \bar{q}^i q^j(t) \rangle, \langle \bar{q}^i \gamma_5 q^j(t) \rangle). \qquad (7)$$

At the static vacuum, the condensates vanish in this $\mathcal{N} = 2$ supersymmetric QCD.

Chaotic behavior.—To extract the simplest nonlinearity, we consider $N_f = 2$ and a subsector $\phi_8 = x(t)\sigma_1/\sqrt{2}$

and $\phi_9 = y(t)\sigma_2/\sqrt{2}$. This respects the equations of motion of (6). Then the system reduces to a classical mechanics of a quartic oscillator with the action

$$S_0 = \int dt \left[\frac{1}{2}(\dot{x}^2 + \dot{y}^2) - \frac{m^2}{2}(x^2 + y^2) - g x^2 y^2\right]. \quad (8)$$

Here $m \equiv 4\pi m_q/\sqrt{\lambda}$ is the meson mass, and the quartic coupling is $g \equiv 72\pi^2/(5N_c)$. This is a well-known model of chaos [2,3] (see [4] for a review). In Fig. 4, we show six numerical plots of Poincaré sections of the system as an example to visualize the chaos [53]. As the energy density increases, the system clearly shows an order-chaos phase transition. Indeed, it is known that, in this system (8), above a certain energy chaos dominates the phase space [2,3,54]. On the other hand, for a lower energy, the system is in an ordered phase and the motion is regular. So, we conclude that the time evolution of the homogeneous quark condensate of the $\mathcal{N} = 2$ supersymmetric QCD at strong coupling and at large N_c has a chaotic phase.

Let us study the strength of the chaos as a function of the theory, λ, N_c, and m_q. The system is invariant [2,3] under the following scaling symmetry: $x \to \alpha x$, $y \to \alpha y$, $t \to \beta t$, and $\lambda \to \beta^2\lambda$, $N_c \to \alpha^2\beta^2 N_c$, $m_q \to m_q$, $E \to (\alpha^2/\beta^2)E$. So, a scale-invariant combination $E\lambda^2/N_c$ governs the dynamical phase of the system. The chaos-order phase transition occurs at a critical energy scale $E_{\rm chaos}$ above which the Poincaré section is covered mostly by the ergodic chaos pattern. Our numerical calculation shows $E_{\rm chaos} \sim 0.6 m_q^4$ for $\lambda = 100$ and $N_c = 10$, so together with the scaling argument we obtain

$$E_{\rm chaos} \sim (6 \times 10^2) \times m_q^4 \frac{N_c}{\lambda^2}. \quad (9)$$

Therefore, the energy region for chaos increases for smaller N_c or larger λ. We conclude that our $\mathcal{N} = 2$ supersymmetric QCD is more chaotic for smaller N_c or larger λ.

By the scaling transformation described above, the Lyapunov exponent, which has the dimension of inverse time, is scaled as $L \to \beta^{-1}L$. In Fig. 5, our numerical evaluation of the Lyapunov exponent is shown. In Fig. 5 (left), $(N_c/E)^{1/4}L$ is plotted as a function of $(E/N_c)^{1/2}\lambda$, where the horizontal and vertical axes are taken as scaling-invariant combinations. This figure is convenient to see the λ dependence of the Lyapunov exponent for fixed N_c and E. For $(E/N_c)^{1/2}\lambda \lesssim 10$, one can see that there is no chaos, i.e., $L = 0$. For $10 \lesssim (E/N_c)^{1/2}\lambda \lesssim 200$, the Lyapunov exponent increases linearly as a function of $\log\lambda$. Fitting the plots in this region, the following formula is obtained:

$$L \simeq \left(\frac{E}{N_c}\right)^{1/4}\left\{0.90 \log\left[\left(\frac{E}{N_c}\right)^{1/2}\lambda\right] - 2.03\right\}. \quad (10)$$

Slightly above the critical energy scale $E_{\rm chaos}$, the Lyapunov exponent can be approximated by this simple expression. For large λ, it deviates from (10) and has a maximum value $L \simeq 1.6 \times (E/N_c)^{1/4}$ at $\lambda \simeq 200 \times (N_c/E)^{1/2}$. For $(E/N_c)^{1/2}\lambda \gtrsim 200$, the Lyapunov exponent decreases, because the mass term disappears and chaos is expected to be saturated by that of the pure massless Yang-Mills.

In Fig. 5 (right), the same result is shown in a different normalization, $N_c/(E\lambda^2)$ vs $\lambda^{1/2}L$. This is convenient to see N_c dependence for fixed λ and E. From the figure, we can find that Lyapunov exponent is a decreasing function of N_c for fixed E and λ. Therefore, we conclude that the strongly coupled large N_c $\mathcal{N} = 2$ supersymmetric QCD is more chaotic for a smaller N_c.

Outlook.—We have explicitly showed that Lyapunov exponents can be calculated for chiral condensates by

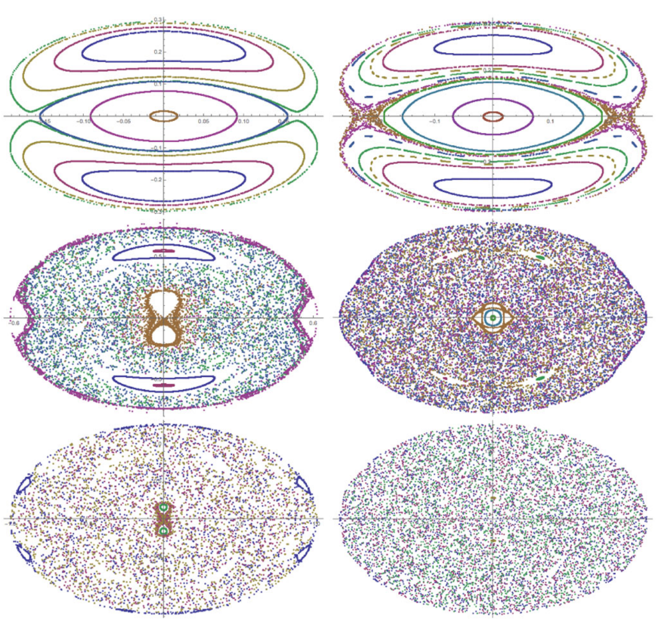

FIG. 4. The Poincaré sections for $\lambda = 100$ and $N_c = 10$. The horizontal axis is $y(t)$, while the vertical axis is $\dot{y}(t)$. The section is chosen by $x(t) = 0$. The energy is chosen as $E = 0.05$, 0.1, 0.3, 0.6, 0.8, 1 in the top-left, top-right, middle-left, middle-right, lower-left, and lower-right figures, respectively, in the unit $m_q = 1$.

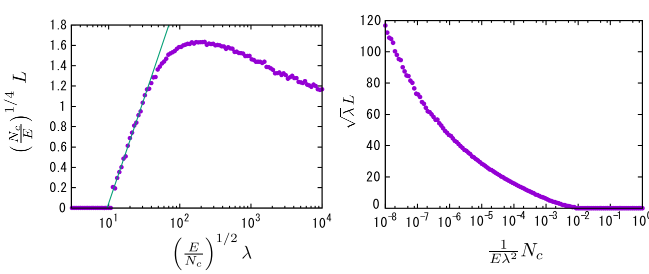

FIG. 5. The Lyapunov exponent L as functions of λ (left) and N_c (right) in the unit of $m_q = 1$.

using the large N_c limit, which amounts to solving the problem of assigning a chaos index to quantum dynamics.

Let us note that our Lyapunov exponent L can be written as an out-of-time-ordered correlator

$$e^{2Lt} \sim \langle E, \mathcal{M}|[Q(t), P(0)]^2|E, \mathcal{M}\rangle_{N\gg 1, \lambda\gg 1}, \quad (11)$$

where $Q(t) \equiv \bar{\psi}\psi(t)$ is the chiral condensate operator inserted at time t and P is for its shift, $[Q(x), P(x')] = i\delta(x - x')$. The state $|E, \mathcal{M}\rangle$ is an energy eigenstate of the supersymmetric QCD Hamiltonian, with a degeneracy index $\mathcal{M}$ [55]. The original out-of-time-ordered correlator uses a thermal partition [16,17], while ours is an energy eigenstate, so the temperature scale of the former roughly corresponds to our energy E.

Our method can assign a Lyapunov exponent to quantum dynamics of gauge theories and opens broad applications of chaos to particle physics. Possible arenas may include entropy production (see [57]), anarchy neutrino masses [58], and Higgs criticality [59] and related inflations [60–62]. It would be interesting to find some relations between fundamental physics and chaos.

We have greatly benefited from the advice and comments by S. Sasa and are grateful to him. We also thank valuable discussions with L. Pando Zayas, S. Heusler, A. Ohnishi, K. Fukushima, B.-H. Lee, and H. Fukaya. The work of K. H. was supported in part by JSPS KAKENHI Grants No. 15H03658 and No. 15K13483. The work of K. M. was supported by JSPS KAKENHI Grant No. 15K17658. The work of K. Y. was supported by the Supporting Program for Interaction-based Initiative Team Studies (SPIRITS) from Kyoto University and by a JSPS Grant-in-Aid for Scientific Research (C) No. 15K05051. This work was also supported in part by the JSPS Japan-Russia Research Cooperative Program and the JSPS Japan-Hungary Research Cooperative Program.

[1] S. G. Matinyan, G. K. Savvidy, and N. G. Ter-Arutunian Savvidy, Zh. Eksp. Teor. Fiz. **80**, 830 (1981) [Sov. Phys. JETP **53**, 421 (1981)].
[2] S. G. Matinyan, G. K. Savvidy, and N. G. Ter-Arutunian Savvidy, Pis'ma Zh. Eksp. Teor. Fiz. **34**, 613 (1981) [JETP Lett. **34**, 590 (1981)].
[3] G. K. Savvidy, Nucl. Phys. **B246**, 302 (1984).
[4] T. S. Biro, S. G. Matinyan, and B. Muller, Lect. Notes Phys. **56**, 1 (1994).
[5] B. Muller and A. Trayanov, Phys. Rev. Lett. **68**, 3387 (1992).
[6] C. q. Gong, Phys. Lett. B **298**, 257 (1993).
[7] T. Kunihiro, B. Muller, A. Ohnishi, and A. Schafer, Prog. Theor. Phys. **121**, 555 (2009).
[8] T. Kunihiro, B. Muller, A. Ohnishi, A. Schafer, T. T. Takahashi, and A. Yamamoto, Phys. Rev. D **82**, 114015 (2010).
[9] B. Muller and A. Schafer, Int. J. Mod. Phys. E **20**, 2235 (2011).
[10] H. Iida, T. Kunihiro, B. Muller, A. Ohnishi, A. Schafer, and T. T. Takahashi, Phys. Rev. D **88**, 094006 (2013).
[11] H. Tsukiji, H. Iida, T. Kunihiro, A. Ohnishi, and T. T. Takahashi, Prog. Theor. Exp. Phys. **2015**, 083A01 (2015).
[12] H. Tsukiji, H. Iida, T. Kunihiro, A. Ohnishi, and T. T. Takahashi, arXiv:1603.04622 [Phys. Rev. D (to be published)].
[13] L. D. McLerran and R. Venugopalan, Phys. Rev. D **49**, 2233 (1994).
[14] L. D. McLerran and R. Venugopalan, Phys. Rev. D **49**, 3352 (1994).
[15] L. D. McLerran and R. Venugopalan, Phys. Rev. D **50**, 2225 (1994).
[16] J. Maldacena, S. H. Shenker, and D. Stanford, J. High Energy Phys. 08 (2016) 106.
[17] A. I. Larkin and Y. N. Ovchinnikov, J. Exp. Theor. Phys. **28**, 1200 (1969).
[18] J. Polchinski, arXiv:1505.08108.
[19] T. Hartman, S. Jain, and S. Kundu, J. High Energy Phys. 05 (2016) 099.
[20] A. R. Brown, D. A. Roberts, L. Susskind, B. Swingle, and Y. Zhao, Phys. Rev. Lett. **116**, 191301 (2016).
[21] D. Berenstein and A. M. Garcia-Garcia, arXiv:1510.08870.
[22] P. Hosur, X. L. Qi, D. A. Roberts, and B. Yoshida, J. High Energy Phys. 02 (2016) 004.
[23] G. Gur-Ari, M. Hanada, and S. H. Shenker, J. High Energy Phys. 02 (2016) 091.
[24] D. Stanford, J. High Energy Phys. 10 (2016) 009.
[25] A. L. Fitzpatrick and J. Kaplan, J. High Energy Phys. 05 (2016) 070.
[26] E. Perlmutter, J. High Energy Phys. 10 (2016) 069.
[27] G. Turiaci and H. Verlinde, arXiv:1603.03020.
[28] J. M. Maldacena, Int. J. Theor. Phys. **38**, 1113 (1999) [Adv. Theor. Math. Phys. **2**, 231 (1998)].
[29] L. A. Pando Zayas and C. A. Terrero-Escalante, J. High Energy Phys. 09 (2010) 094.
[30] P. Basu and L. A. Pando Zayas, Phys. Lett. B **700**, 243 (2011).
[31] P. Basu, D. Das, and A. Ghosh, Phys. Lett. B **699**, 388 (2011).
[32] P. Basu and L. A. Pando Zayas, Phys. Rev. D **84**, 046006 (2011).
[33] P. Basu, D. Das, A. Ghosh, and L. A. Pando Zayas, J. High Energy Phys. 05 (2012) 077.
[34] D. Z. Ma, J. P. Wu, and J. Zhang, Phys. Rev. D **89**, 086011 (2014).
[35] X. Bai, B. H. Lee, T. Moon, and J. Chen, J. Korean Phys. Soc. **68**, 639 (2016).
[36] T. Ishii and K. Murata, J. High Energy Phys. 06 (2015) 086.
[37] Y. Asano, D. Kawai, H. Kyono, and K. Yoshida, J. High Energy Phys. 08 (2015) 060.
[38] I. Y. Aref'eva, P. B. Medvedev, O. A. Rytchkov, and I. V. Volovich, Chaos Solitons Fractals **10**, 213 (1999).
[39] L. A. Pando Zayas and D. Reichmann, J. High Energy Phys. 04 (2013) 083.
[40] P. Basu and A. Ghosh, Phys. Lett. B **729**, 50 (2014).
[41] D. Giataganas, L. A. Pando Zayas, and K. Zoubos, J. High Energy Phys. 01 (2014) 129.

[42] A. Farahi and L. A. Pando Zayas, Phys. Lett. B **734**, 31 (2014).
[43] Y. Asano, D. Kawai, and K. Yoshida, J. High Energy Phys. 06 (2015) 191.
[44] A. Karch and E. Katz, J. High Energy Phys. 06 (2002) 043.
[45] The action can also be thought of as a neutral part of a non-Abelian linear σ model with three pions.
[46] The homogeneity is an assumption, but there are examples of successful homogeneous study such as the chiral random matrix theory.
[47] M. Bando, T. Kugo, S. Uehara, K. Yamawaki, and T. Yanagida, Phys. Rev. Lett. **54**, 1215 (1985).
[48] M. Bando, T. Kugo, and K. Yamawaki, Phys. Rep. **164**, 217 (1988).
[49] J. Erdmenger, N. Evans, I. Kirsch, and E. Threlfall, Eur. Phys. J. A **35**, 81 (2008).
[50] R. C. Myers, J. High Energy Phys. 12 (1999) 022.
[51] M. Kruczenski, D. Mateos, R. C. Myers, and D. J. Winters, J. High Energy Phys. 07 (2003) 049.
[52] J. Erdmenger, K. Ghoroku, and I. Kirsch, J. High Energy Phys. 09 (2007) 111.
[53] The effective mass of the field $y(t)$ in Eq. (8) is given by $m^2 - 2gx^2$, and, thus, the oscillation of $x(t)$ can cause the parametric resonance. This would be regarded as the origin of the chaos in the current system.
[54] L. Salasnich, Mod. Phys. Lett. A **12**, 1473 (1997).
[55] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.117.231602 for details about the out-of-time-ordered correlator and classical chaos, which includes Ref. [56].
[56] P. Hasenfratz, Nucl. Phys. **B828**, 201 (2010).
[57] V. Latora and M. Baranger, Phys. Rev. Lett. **82**, 520 (1999).
[58] L. J. Hall, H. Murayama, and N. Weiner, Phys. Rev. Lett. **84**, 2572 (2000).
[59] D. Buttazzo, G. Degrassi, P. P. Giardino, G. F. Giudice, F. Sala, A. Salvio, and A. Strumia, J. High Energy Phys. 12 (2013) 089.
[60] A. Salvio, Phys. Lett. B **727**, 234 (2013).
[61] Y. Hamada, H. Kawai, and K. Oda, J. High Energy Phys. 07 (2014) 026.
[62] Y. Hamada, H. Kawai, K. Y. Oda, and S. C. Park, Phys. Rev. D **91**, 053008 (2015).