Arbitrary Dicke-State Control of Symmetric Rydberg Ensembles

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We study the production of arbitrary superpositions of Dicke states via optimal control. We show that N atomic hyperfine qubits, interacting symmetrically via the Rydberg blockade, are well described by the Jaynes-Cummings Hamiltonian and fully controllable by phase-modulated microwaves driving Rydberg-dressed states. With currently feasible parameters, it is possible to generate states of ~ten hyperfine qubits in ~1 μs, assuming a fast microwave phase switching time. The same control can be achieved with a "dressed-ground control" scheme, which reduces the demands for fast phase switching at the expense of increased total control time.

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Creating entangled many-body states is a central challenge of quantum information science. Beyond their intrinsic interest as highly nonclassical states, such states are a resource for information processing protocols, including measurement-based quantum computation [1], error correction [2], and metrology beyond the standard quantum limit [3]. In neutral atoms, one powerful tool for generating such states is the Rydberg blockade, where the electric dipole-dipole interaction between high-lying Rydberg states suppresses the excitation of multiple Rydberg states at a time [4,5]. This effect has been used to entangle pairs of trapped atoms [6–8] and mesoscopic ensembles of atoms [9,10].

Here, we apply the Rydberg blockade to many-body quantum state control. Specifically, we study symmetric ensemble control, in which one produces a target entangled state by applying a Hamiltonian that acts on every atom in the ensemble equivalently. For an ensemble of N qubits, this corresponds to controlling a Hilbert space spanned by the Dicke states, the symmetric subspace of N spin-1/2 particles, with total spin $J = N/2$. The control and measurement of Dicke states is more tractable since the symmetric subspace grows linearly with the number of particles, whereas in a general tensor-product space, the dimension grows exponentially. This should allow us to develop new tools for the control and measurement of many-body systems. Dicke-state control has been demonstrated in ionic [11] and photonic [12] systems, and proposed for Bose-Einstein condensates [13]. A simple case of two-atom symmetric control based on the Rydberg blockade was demonstrated by Jau *et al.* to produce Bell states [8].

To achieve Dicke-state control we will employ an isomorphism between the dynamics of the Rydberg-blockaded symmetric atomic ensemble and the Jaynes-Cummings model (JCM) [14–18]. The nonlinear dynamics of the JCM have been well studied in cavity QED and provide a powerful platform for quantum control [19–22]. To see this isomorphism, we consider a collection of N atoms individually held in an array of optical dipole traps [23]. For concreteness, we consider ^{133}Cs atoms as employed in Refs. [8,24,25] and encode qubits in the clock states $|0\rangle \equiv |6S_{1/2}, F = 3, M_F = 0\rangle$ and $|1\rangle \equiv |6S_{1/2}, F = 4, M_F = 0\rangle$ separated by hyperfine energy $\hbar\omega_{\mathrm{HF}}$. We assume that the ensemble is uniformly illuminated by a 318 nm laser, coupling $|1\rangle$ to $|r\rangle \equiv |nP_{3/2}, M_J = 3/2\rangle$ in every atom with the same Rabi frequency Ω_r and detuning Δ_r (Fig. 1). Insofar as the interactions are independent of the atoms' spatial positions, in second quantization, and in the rotating fame at the laser frequency, the many-body Hamiltonian is

$$H = E_0 a_0^\dagger a_0 + (E_0 + \hbar\omega_{\mathrm{HF}}) a_1^\dagger a_1 + (E_0 + \hbar\omega_{\mathrm{HF}} - \hbar\Delta_r) a_r^\dagger a_r + \frac{\hbar\Omega_r}{2}(a_r^\dagger a_1 + a_1^\dagger a_r) + V_{\mathrm{dd}}, \tag{1}$$

where $a_i^\dagger$ creates an atom in the state $|i\rangle$ symmetrically across the ensemble, so $[a_i^\dagger, a_j] = \delta_{i,j}$, the Bose commutation relations. When the electric dipole-dipole interaction V_{dd} is sufficiently strong across the whole ensemble, the analog of the Pauli exclusion principle allows only one Rydberg atom at a time, enforcing a perfect blockade. For example, for $n = 84$, with the van der Waals coefficient $C_6/h = -610$ GHz μm^6 [24], and for $\Omega_r/2\pi = 5$ MHz, the blockade radius is $r_B \equiv (C_6/\hbar\Omega_r)^{1/6} \approx 7.04$ μm. A 3×3 square array of nine atoms in dipole traps separated by 2 μm is safely blockaded. For these atoms, the dipole blockade thus effectively fermionizes the Rydberg state, and its creation operator now obeys an anticommutation relation: $a_r \to c_r$, $\{c_r^\dagger, c_r\} = 1$. Making the Jordan-Wigner transformation from fermionic to Pauli operators, since we have one "mode," $c_r \to \sigma_-$. Taking $E_0 = 0$ and rewriting Eq. (1) in terms of these operators gives

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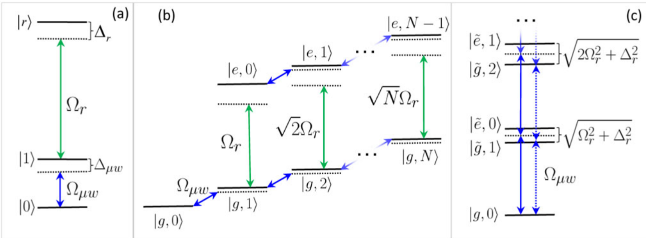

FIG. 1. (a) Basic level structure for the three-level atom: a qubit is encoded in the ground hyperfine states, and logical $|1\rangle$ is optically coupled to a Rydberg state, while logical $|0\rangle$ is far off resonance and effectively uncoupled. (b) Bare states for N atoms, symmetrically coupled, under a perfect blockade. (c) N-atom dressed states, exhibiting the nonlinear JC ladder energy-level structure. Full Hilbert space control performs best when the microwave is tuned near resonance with the bare $|0\rangle \leftrightarrow |1\rangle$ transition (solid arrows), while dressed-ground control performs best when the microwave is tuned near the dressed-ground state transitions (dotted arrows).

$$H = H_{\text{JC}} = \hbar\omega_{\text{HF}} a_1^\dagger a_1 + \hbar\omega_0 \sigma_+\sigma_- + \hbar g(\sigma_+ a_1 + a_1^\dagger \sigma_-), \tag{2}$$

where $\hbar\omega_0 = \hbar\omega_{\text{HF}} - \hbar\Delta_r$ and $g = \Omega_r/2$. The dynamics of the many-body state is described by the familiar Jaynes-Cummings (JC) Hamiltonian [14]. Here, the presence or absence of a Rydberg excitation plays the role of the two-level atom in a conventional cavity QED setting, and the number of atoms in $|1\rangle$ takes the place of photons as the system's bosonic degree of freedom.

Under this mapping, the bare states of the JCM are symmetric superpositions of n atoms in $|1\rangle$, with the remaining $N-n$ atoms distributed between $|0\rangle$ and $|r\rangle$ as

$$|g,n\rangle \equiv a_1^{\dagger n} a_0^{\dagger N-n}|0\rangle = \{|0\rangle^{\otimes N-n}|1\rangle^{\otimes n}\}_{\text{sym}}, \tag{3}$$

$$|e,n\rangle \equiv c_r^\dagger a_1^{\dagger n} a_0^{\dagger N-n-1}|0\rangle = \{|0\rangle^{\otimes N-n-1}|1\rangle^{\otimes n}|r\rangle\}_{\text{sym}}. \tag{4}$$

We recognize these states also as Dicke states, or eigenstates of a collective spin, with $J = N/2$, $N-1/2$ for the ground and excited manifolds, respectively. We denote the Rydberg-dressed states $\{|\tilde{g},n\rangle, |\tilde{e},n-1\rangle\}$ with energies $E_{\pm,n} = -\hbar\Delta_r/2 \pm (\hbar/2)\text{sgn}(\Delta_r)\sqrt{n\Omega_r^2+\Delta_r^2}$ that represent the well-known JC ladder.

We quantify the entangling power of the JC Hamiltonian by the nonlinear shift of the dressed states

$$\kappa = \langle\tilde{g},2|H_{\text{JC}}|\tilde{g},2\rangle - 2\langle\tilde{g},1|H_{\text{JC}}|\tilde{g},1\rangle. \tag{5}$$

In the weak dressing limit, $|\Delta_r| \gg \Omega_r$, $\kappa \approx -\Omega^4/8\Delta_r^3$, and the nonlinearity of H_{JC} is fully described by the two-body coupling κ according to

$$\langle\tilde{g},n|H_{\text{JC}}|\tilde{g},n\rangle - n\langle\tilde{g},1|H_{\text{JC}}|\tilde{g},1\rangle \approx (n^2-n)\frac{\kappa}{2}. \tag{6}$$

For our atomic ensemble, on the ground manifold, the Hamiltonian is then a quadratic function of the collective spin

$$H_{\text{JC}}^{(g)} = \frac{N}{2}\hbar\omega_{\text{HF}} + \left(\hbar\omega_{\text{HF}} + \frac{\hbar\Omega_r^2}{4\Delta_r} + N\frac{\hbar\kappa}{2}\right)J_z + \frac{\hbar\kappa}{2}J_z^2. \tag{7}$$

This describes the one-atom light-shift plus entangling two-atom interactions that yield a one-axis-twisting Hamiltonian [26]. This Hamiltonian produces cat states when applied to a spin coherent state, $e^{-i\kappa T J_z^2/2}(|0\rangle + |1\rangle)^{\otimes N} = e^{-i\pi/4}(|0\rangle^{\otimes N} + i|1\rangle^{\otimes N})$, when $T = \pi/\kappa$ [27,28].

We endeavor to go beyond cat states, employing quantum control to generate arbitrary target states in the ground Dicke subspace, $|\Psi_{\text{target}}\rangle = \sum_{n=0}^N c_n|g,n\rangle$. In the context of cavity QED, such states correspond to the atom in $|g\rangle$ and a nonclassical state of the field, with up to N photons. The nonlinearity of the JCM provides numerous handles for achieving this with various degrees of control [19–22], Here, we show that the tools of optimal control can be used to generate fast state-to-state maps producing arbitrary target states.

In the setting of optimal control we consider a time-dependent Hamiltonian of the form $H(t) = H_{\text{JC}} + H_c[\phi(t)]$, where the control Hamiltonian H_c is a functional of the waveform $\phi(t)$. Through standard techniques, one can determine if the system is controllable on a Hilbert space of dimension d, meaning that there exists a $\phi(t)$ such that $H(t)$ can generate any unitary map in the group SU(d) after some time T. For our system, the total Hilbert space is $\mathcal{H} = \mathcal{H}_{J=N/2}^{(g)} \oplus \mathcal{H}_{J=(N-1)/2}^{(e)}$, corresponding to the ground or excited manifolds of the JCM with up to N excitations.

The Hilbert space and control Hamiltonians for our system bear a close resemblance to the control of the magnetic sublevels of the hyperfine spins in ground-state alkali-metal atoms, as employed in the seminal experiments of Jessen [29,30]. There, the combination of phase-modulated Larmor precession that generates SU(2) control on the spins with pairwise couplings between the sublevels of the two manifolds is sufficient for arbitrary control [31]. Taking a similar strategy here, the couplings between the two manifolds are achieved by the Rydberg laser. The arbitrary SU(2) control on each of the ground and excited manifolds corresponds to driving the system's bosonic degree of freedom in the JCM. We can achieve this because, unlike a true harmonic oscillator, the atomic system is finite dimensional. Our control Hamiltonian is thus a microwave (or two-photon Raman transition) coupling $|0\rangle$ to $|1\rangle$ in each atom. The Rabi frequency and detuning are fixed at $\Omega_{\mu\text{w}}$ and $\Delta_{\mu\text{w}} = \omega_{\mu\text{w}} - \omega_{\text{HF}}$, respectively, but the microwave's phase can vary as a function of time. Assuming the microwave illuminates the entire ensemble symmetrically, in the frame rotating at the microwave frequency, the control Hamiltonian is

$$H_c(t) = \frac{\hbar\Omega_{\mu w}}{2}[\cos\phi(t)J_x + \sin\phi(t)J_y] - \hbar\omega_{\mu w}\left(J_z + \frac{N}{2}\right), \tag{8}$$

where $\phi(t)$ is the time-dependent phase. $H_c(t)$ generates arbitrary SU(2) rotations of the ground and excited manifolds. Analogous to Ref. [31], $H(t) = H_{JC} + H_c(t)$ renders the system fully controllable, i.e., one can generate an arbitrary unitary map on the full Hilbert space (see the Supplemental Material [32]).

Insofar as our system is controllable, we know there is always some (nonunique) waveform $\phi(t)$ that will generate any $|\Psi_{\rm target}\rangle$ in the Dicke subspace from an initial fiducial state. We consider here control waveforms consisting of sequences of s "phase steps" of length Δt, for a total run time of $T = s\Delta t$, as in Ref. [29]. The range of possible control waveforms can be parametrized by an s-dimensional vector $\vec{\phi}$. We take as our fiducial state $|\Psi_0\rangle = |g, 0\rangle$. Turning on the Rydberg laser dresses the remaining eigenstates in the JC ladder, and control is performed in the dressed basis. The target state of the control is thus $|\tilde{\Psi}_{\rm target}\rangle = \sum_n c_n|\tilde{g}, n\rangle$. After the control sequence, one would adiabatically undress the atom to achieve the target state in the bare ground Dicke subspace.

We seek a $\vec{\phi}$ such that the fidelity of the output with the target state, $\mathcal{F}(\vec{\phi}) = |\langle\tilde{\Psi}_{\rm target}|U(\vec{\phi}, T)|\Psi_0\rangle|^2$, is sufficiently high. We find $\vec{\phi}$ with the well-known GRAPE gradient ascent algorithm [34]. The results are illustrated in Fig. 2 for a six-atom ensemble. The choice of optimal parameters such as laser or microwave power and detuning will depend on fundamental sources of error such as decoherence as well as practical experimental concerns. In particular, it is desirable to minimize the runtime and complexity of our protocol, so we typically seek the minimum T and s needed for high-fidelity control. It takes $2d - 2$ real numbers to specify a d-dimensional target pure state, which puts a lower bound on s. For N atoms including both ground and Rydberg symmetric states, this gives $s \geq 4N$. In practice, we find that this inequality can often be saturated. More heuristically, we can predict that the "quantum speed limit" is set by $T \gtrsim \pi/\kappa$, the minimum time required to generate a cat state from a separable state based on the one-axis twisting Hamiltonian, as discussed above. Whether this bound can be saturated depends on the choice of experimental parameters.

To achieve optimal fidelities, we wish to perform control in the shortest possible time compared with our system's decoherence time. Decoherence due to photon scattering, occurring at a rate γ, is of particular concern, so maximizing κ/γ is an important goal. Since κ scales as Ω_r^4/Δ_r^3 in the weak dressing regime, it is highly sensitive to the power and detuning of the Rydberg laser. By contrast, γ scales as Ω_r^2/Δ_r^2, so $\kappa/\gamma \propto \Omega_r^2/\Delta_r$ increases with increased laser power and decreased detuning. Based on this, increasing our dressing strength is a winning strategy in the fight against decoherence, and has the added benefit of reducing the total run time. This suggests that maximum laser power, at or near resonance, is the best choice of parameters.

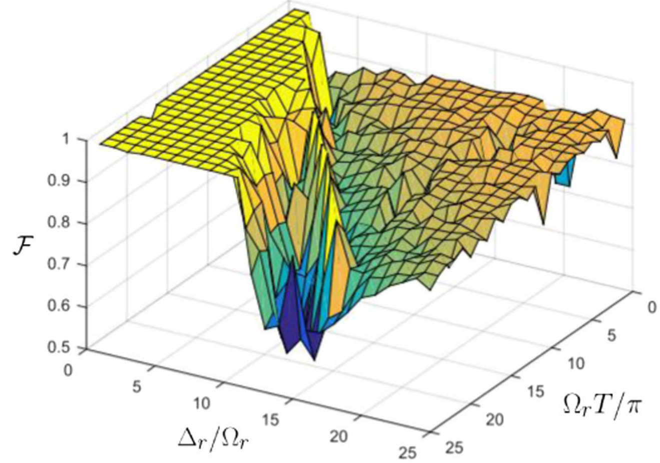

FIG. 2. Simulated control fidelities to produce a six-atom cat state in the dressed basis, starting from $|g, 0\rangle$, as a function of Rydberg laser detuning Δ_r and run time T, using $s = 25$ phase steps. For any Δ_r, there is a minimum control time above which the fidelity is arbitrarily close to 1. As Δ_r increases, κ decreases and the minimum control time gets longer. Infidelities shown here are due solely to the quantum speed limit; decoherence is not included in the simulations.

No matter how short the control time is in principle, we must still have s phase steps, and quickly switching a microwave's phase is not trivial. With a resonant laser power that yields a Rabi frequency of a few megahertz and $\sim$10 atoms, the required Δt per phase step can easily shrink to tens of nanoseconds or less. The demands on the microwave switch time are even more strict, since the phase must change quickly enough to preserve the piecewise-constant approximation of $\phi(t)$. The number of steps in the control waveform, then, is a primary limiting factor in the speed and feasibility of this protocol. Once the control time is limited by experimental restrictions on Δt rather than by κ, increasing κ is no longer beneficial; stronger dressing will only increase γ and other sources of error without any offsetting benefit of control speed. On the other hand, as long as κ is the limiting factor, increased dressing strength is advantageous as per the reasoning above. The optimal parameter regime, therefore, is highly dependent on the particulars of the experiment: Δ_r should be large enough to make the two speed limits match, if possible, but no higher.

Since phase switching requirements limit the speed of our protocol, control could be significantly accelerated by reducing the number of phase steps needed. This can be achieved if we can adiabatically eliminate the dressed-excited states, and restrict the dynamics to the dressed-ground manifold, the Dicke subspace of interest. In this case the dimension of the control Hilbert space is cut in half, and we need only $2N$ parameters to specify a target in it. We clearly see that this is possible in the far off resonance limit. Then, the JC Hamiltonian on the ground manifold takes the form of a quadratic light shift, Eq. (6). This, together with the SU(2) control generated by the microwaves renders the qudit J fully controllable on $\mathrm{SU}(2J+1) = \mathrm{SU}(N+1)$.

More generally, we return to Eq. (8), describing the effect of coupling induced by the microwave or Raman transition. In the bare basis the microwave couples $|0\rangle$ to $|1\rangle$ without acting on $|r\rangle$, so $\langle e, m|H_{\mu w}|g, n\rangle = 0$. By contrast, the dressed Rydberg states $|\tilde{e}, n\rangle$ have some $|g, n\rangle$ character, so the microwave coupling between the dressed-ground and Rydberg states is nonzero. In the weak dressing regime we can approximate the magnitude of this coupling as $\langle \tilde{e}, m|H_{\mu w}|\tilde{g}, n\rangle \approx \sin(\theta_n/2)\langle g, m|H_{\mu w}|g, n\rangle \approx (\sqrt{n}\Omega_r/2|\Delta_r|)\langle g, m|H_{\mu w}|g, n\rangle$. The effective Rabi rate is suppressed by an order of Ω_r/Δ_r for dressed ground-excited coupling compared to dressed ground-ground couplings. Excitation is also suppressed by microwave detuning. The saturation parameter is on the order of $\Omega_{\mu w}^2/\Delta_r^2$. Combining these suppressing factors, we find that the microwave will approximately preserve the dressed-ground population as long as $\sqrt{N}\Omega_r\Omega_{\mu w}^2/\Delta_r^3 \ll 1$. Under this condition, we find that dressed-ground control can be performed in $2N$ phase steps, as expected. Because this condition requires a large Δ_r, it goes hand in hand with a small κ, so ground manifold control is much slower than full Hilbert space control if the phase steps are allowed to be arbitrarily short. Whether the trade-off between κ and s is worthwhile will depend on the N and the minimum Δt in a given experiment.

Dressed-ground and full Hilbert space control are optimized with qualitatively different choices of microwave and laser parameters. When we control the whole Hilbert space, the system traverses both ground and excited states to get to its destination, so all states must be coupled strongly to each other. Since the light shift provides a gap between the ground and Rydberg manifolds and is of order $\sqrt{\Omega_r^2 + \Delta_r^2}$, $\Omega_{\mu w}$ needs to be at least that large to strongly drive both transitions at once. Both to relax this condition and to maximize the interaction strength, Δ_r should be kept small compared to Ω_r. The microwave resonance should also be tuned approximately halfway between the ground and Rydberg states in the rotating frame ($\Delta_{\mu w} \approx \Delta_r/2$), so that all manifolds are roughly equally coupled [see Fig. 1(c)]. If these conditions are not met, population transfer between the ground and Rydberg manifolds will be slow compared to intramanifold transfer, and control can be bottlenecked by the population getting "stuck" in the Rydberg manifold for extended periods.

On the other hand, dressed-ground control relies on the assumption of adiabatic elimination of the Rydberg manifold, so the parameters should be chosen to minimize the coupling between the ground and excited manifolds. $\Omega_{\mu w}$ should be small compared to the light shift gap, and a large Δ_r makes this easier to accomplish. Likewise, the microwave should be tuned near resonance with the transitions between dressed-ground states to allow strong dressed ground-ground coupling with minimal dressed ground-excited coupling. The microwave Rabi frequency also should be large compared to κ to ensure that off resonant driving to the excited dressed states is negligible over the entire control time. Thus, $\Omega_{\mu w}$ should scale inversely with Δ_r. If these conditions are not met, a significant population can leak into the dressed-excited manifold, where $2N$ free parameters are no longer enough to bring it back to the dressed-ground manifold. Dressed-ground and full Hilbert space control thus provide two complementary methods that function well in different regimes. The two methods produce waveforms, each optimal for its respective parameter regime, that reach the same destination in Hilbert space but take qualitatively different paths to get there. This is illustrated in Fig. 3, which shows how both types of control can be used to produce a seven-atom cat state. In both cases we employ the JCM (2), plus microwave control. Fast full Hilbert-space control is achieved in 1 μs with phase steps

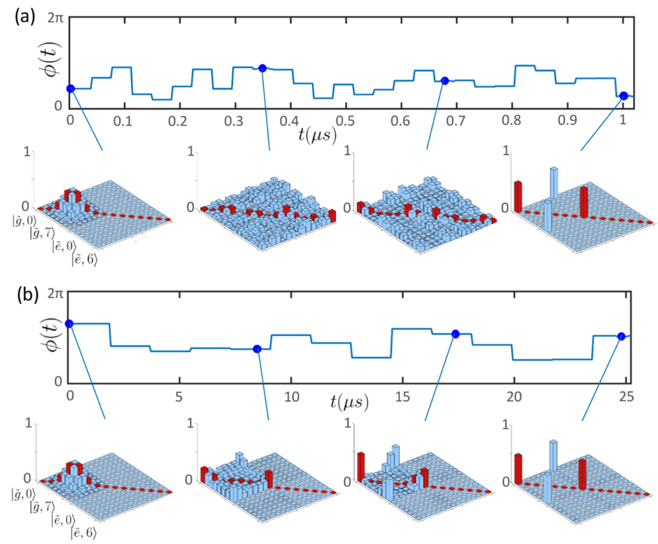

FIG. 3. Mapping the seven-atom spin coherent state $(|0\rangle + |1\rangle)^{\otimes 7}/\sqrt{2^7}$ to the cat state $(|0\rangle^{\otimes 7} + |1\rangle^{\otimes 7})/\sqrt{2}$. The line plots show the microwave phase $\phi(t)$ found via optimal control, and the bar charts show the real part of the 15×15 density matrix at various snapshots in time. (a) Full Hilbert space control: $\Omega_r/2\pi = 5$ MHz, $\Delta_r/2\pi = 2.5$ MHz, $\Omega_{\mu w}/2\pi = 12.5$ MHz, and $\Delta_{\mu w}/2\pi = 1.25$ MHz. All 15 dressed states are traversed during control, so $4N = 28$ phase steps are needed. (b) Dressed-ground control: $\Omega_r/2\pi = 5$ MHz, $\Delta_r/2\pi = 15$ MHz, $\Omega_{\mu w}/2\pi = 100$ kHz, and $\Delta_{\mu w}/2\pi = -400$ kHz. The population remains in the eight dressed-ground states, so only $2N = 14$ phase steps are needed, but weaker Rydberg coupling reduces κ by more than an order of magnitude with a commensurate increase in run time.

of 35.7 ns; dressed-ground state control requires 25 μs but with phase steps of 1.79 μs.

Finally, all of the analysis in this work assumed a perfectly known model Hamiltonian and neglected decoherence and experimental noise. Of particular importance, our JCM requires a perfect blockade. For the parameters considered here, this limits us to consider ensembles with ~10 atoms, though employing Rydberg states at higher principle quantum numbers, or by packing atoms closer together in an optical lattice, one can reach ~100 atoms with a perfect blockade as seen in recent experiments [35]. Fundamental decoherence occurs via spontaneous decay of the Rydberg state. While the atomic lifetimes are long compared to the control times considered here, experiments show shorter coherence times that are still unexplained [35]. Reducing this decoherence will be essential to achieve high fidelity control. Technical noise includes sensitivity to background electric fields, which can be managed with a proper experimental approach [24,25]. Other technical challenges include uncertainties in the parameters of the Hamiltonian including the Rabi frequencies and detunings. These may be mitigated with the techniques of robust control, which have been an essential tool to achieve the high fidelity control of qudits [30].

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