# Turbulent Transitions in Optical Wave Propagation

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We report the direct observation of the onset of turbulence in propagating one-dimensional optical waves. The transition occurs as the disordered hosting material passes from being linear to one with extreme nonlinearity. As the response grows, increased wave interaction causes a modulational unstable quasihomogeneous flow to be superseded by a chaotic and spatially incoherent one. Statistical analysis of high-resolution wave behavior in the turbulent regime unveils the emergence of concomitant rogue waves. The transition, observed in a photorefractive ferroelectric crystal, introduces a new and rich experimental setting for the study of optical wave turbulence and information transport in conditions dominated by large fluctuations and extreme nonlinearity.

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Turbulence is a universal phenomenon in which a system is characterized by many out-of-equilibrium degrees of freedom [1]. Turbulent transitions attract great interest because the onset of spatiotemporal disorder profoundly changes the physical features of a system, the paradigm being the transport and drag properties of a fluid in a pipe and channel flow [2–4]. Manifestations of turbulence can also occur in waves, these including acoustic [5], spin [6], and optical waves [7]. In fact, when nonlinear interaction involves the excitation of a large number of waves, phase, and amplitude fluctuations may lead to a stochastic field described statistically using wave-turbulence theory [8]. Wave turbulence usually refers to weakly nonlinear wave systems in which the linear evolution scale can be separated from the nonlinear one. Generally, these systems are dominantly influenced by some external noise and have negligible intrinsic (internal) disorder. On the other hand, as linear and nonlinear scales are comparable, strongly nonlinear coherent structures may emerge and interplay with the incoherent wave field (strong wave turbulence). In optics the onset of strong turbulence greatly alters coherence and statistics of light, as observed for pulse trains in a ring resonator [9], semiconductor lasers with feedback [10], and, recently, in tailored Raman fiber lasers [11–14]. However, experimental studies of wave-turbulent behavior in the spatial domain, where light is not trapped and actually propagates in space, are especially challenging [15–18]. In particular, direct evidence of a fully developed turbulent transition for propagating waves has remained elusive.

In this Letter, we observe the onset of turbulence for nonlinear beam propagation in photorefractive ferroelectric crystals. In our experiments, for strong nonlinearity, one-dimensional wave dynamics sharply pass from a coherent and homogeneous state to a fully incoherent and disordered one. The transition involves the coupling of two stochastic effects: external noise associated with the initial condition and internal fluctuations of the nonlinear response. For coherent and quasihomogeneous initial states optical turbulence set in via modulational instability, with the appearance of stochastic features and shot-to-shot fluctuations. In the turbulent regime, where a large number of spatial spectral modes are found to compete and interplay, we also identified the emergence of transient optical events of extreme amplitude with long-tail statistics, known as rogue waves [19–22]. Optical rogue events may be supported by different mechanisms [23–28], optical turbulence among these [29–32]. In this respect, our results provide clear evidence of the specific role of both noise-seeded instability and wave-turbulent dynamics in generating rogue waves for spatially extended nonlinear beam propagation.

To reveal transitions to turbulence in the spatial domain we make use of an experimental setup [Fig. 1(a)] based on the unique out-of equilibrium photorefractive and electro-optic properties of nanodisordered ferroelectric crystals in the proximity of the structural phase transition [33,34], which has been shown to support a very rich nonlinear light dynamics [35–37]. A line (one-dimensional) Gaussian beam (wavelength $\lambda = 532$ nm) of waist $\omega_0 = 7\ \mu$m along the $x$ direction and quasihomogeneous along the $y$ direction is launched in a photorefractive ferroelectric crystal of potassium-lithium-tantalate-niobate, $K_{1-\alpha}Li_{\alpha}Ta_{1-\beta}Nb_{\beta}O_3$, with $\alpha = 0.04$ and $\beta = 0.38$. The sample is a zero-cut optical quality specimen with size $2.4^{(x)} \times 2.0^{(y)} \times 1.7^{(z)}$ mm ($l_x \times l_y \times l_z$) and with the structural transition occurring at the Curie temperature $T_C = 294$ K; large dielectric fluctuations generally persist also above this point so that nonlinear light dynamics is studied systematically with a high accuracy at $T = T_C + 2$ K. The input wave

 

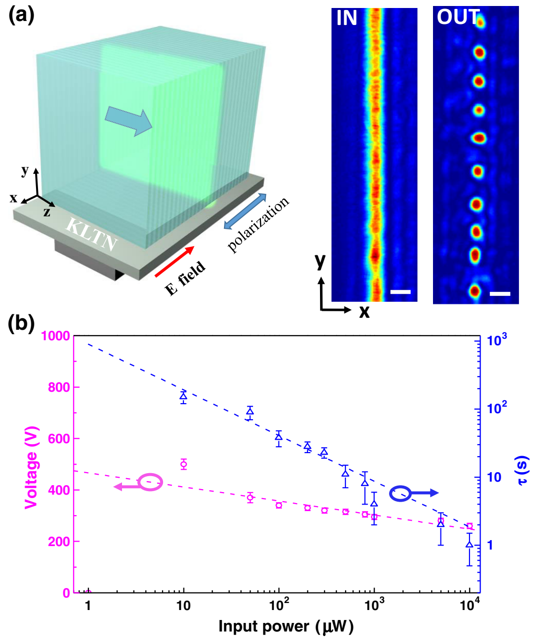


FIG. 1. Nonlinear wave propagation in unstable photorefractive ferroelectric crystals. (a) Sketch of the setup geometry adopted. Scale bars for input and output intensity distributions correspond to 20 $\mu$m. (b) Characterization of transverse breaking: intensity dependence of the process, with the minimum required voltage and the average time scale $\tau$ providing the formation of periodic structures. Lines are linear fits.

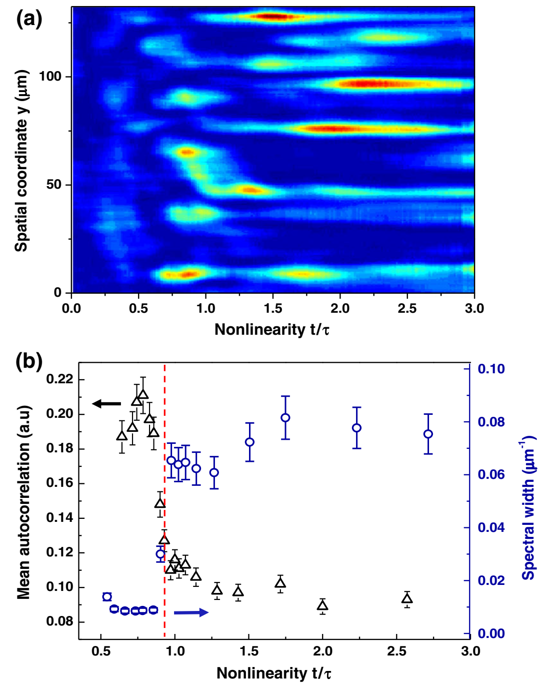


FIG. 2. Observation of the turbulent transition in one-dimensional beam dynamics. (a) Detected output spatial intensity distributions as a function of the nonlinearity expressed through the continuous dimensionless control parameter $t/\tau$. (b) Corresponding width of the spatial Fourier spectrum and mean intensity autocorrelation (see main text) increasing the nonlinearity. The red line serves as a guide at the sharp transition signaling the onset of optical turbulence.

copropagates along the $z$ axis of the crystal with a uniform background intensity and nonlinearity sets in when an external bias field is applied parallel to the polarization of the propagating wave (maximum electro-optic coupling). The spatial intensity distribution is measured at the input and then at the output of the crystal along the initially quasihomogeneous $y$ direction in different nonlinear conditions by means of a high-resolution imaging system composed by an objective lens ($NA = 0.5$) and a CCD camera at 15 Hz.

As a physical parameter to study the transition to turbulence we consider the physical time ruling light dynamics at the crystal output. In fact, the photorefractive nonlinearity has the peculiar property of being noninstantaneous and accumulates in time, since it involves a buildup of a photogenerated space-charge field [38]. In this way, observations at different times correspond to beam propagation for increasing nonlinearity up to saturation [39]. A typical time scale $\tau$ for beam dynamics is fixed through its symmetry breaking into periodic coherent structures [Fig. 1(a)], a process that inhibits stable spatial $(1+1)D$ soliton formation. In fact, this stage can be accurately identified experimentally and we first characterize it varying the accessible experimental parameters. In particular, changing the input power, we measure the threshold voltage to observe transverse breakup and filaments formation. Results are reported in Fig. 1(b) and show how, increasing the input power, an almost linear scaling is found. Moreover, fixing the bias field to $V = 500$ V and varying the input power, we measure the averaged time $\tau$, that is the effective nonlinearity at which the periodic breakup is observed. $\tau$ is found to decrease also linearly with the input power [Fig. 1(b)], in agreement with the fact that the photorefractive nonlinearity buildup rate is inversely related to the peak intensity [38]. The dimensionless continuous control parameter of the nonlinearity is thus $t/\tau$, where $t$ is the evolving time. Hereafter, we consider a laser power $P = 0.5\ \mu$W, with $\tau \approx 8$ s [Fig. 1(b)]. We estimate local variations of the refractive index up to $10^{-3}$ at $t/\tau \simeq 1$ and up to $10^{-2}$ for $t/\tau \simeq 2$.

Direct evidence of the onset of turbulence as the nonlinearity increases is reported in Fig. 2. Once the quasihomogeneous input line beam has experienced symmetry breaking via modulational instability, a sharp transition into a chaotic state with pseudorecurrent patterns occurs for $t/\tau \gtrsim 1$ [Fig. 2(a)]. Some of these filaments can have an extremely large intensity, as we discuss hereafter. This transition corresponds to the loss of spatial coherence that persists only on small scales. Measuring the width of the spatial Fourier spectrum, we found a sharp increase of almost one order of magnitude [Fig. 2(b)]. Correspondingly, the long-range autocorrelation of the intensity light distribution $I(y)$ abruptly decreases, as shown in Fig. 2(b), where we have averaged over large $r$ distances the absolute value of the quantity (autocorrelation function) $g(r) = \langle [I(y) - \langle I \rangle][(I(y+r) - \langle I \rangle] \rangle$ normalized to $g(0)$. We stress that detecting the onset of turbulence as a sharp transition signaling departure from coherence is a method that goes beyond the stability properties of the input flux. In fact, an analogous transition has been reported in conditions where the homogeneous state of the dynamics is stable [11] and unstable [14] with respect to perturbations.

We note that a behavior similar to Fig. 2 has been numerically observed studying the nonlinear stage of modulation instability in the framework of the nonlinear Schrödinger equation [40]. Here, the incoherent state generated during wave evolution is referred to as integrable turbulence [40]. However, our results depart from this scenario since the presence of a saturable nonlinearity makes wave dynamics nonintegrable. This means that the observed turbulent transition weakly depends on the input wave and can occur also without the modulational instability process. We demonstrate this repeating the experiments with an inhomogeneous coherent input wave; using a spatial light modulator the input field is modulated along the $y$ direction with a periodic component. As shown in Fig. 3(a), a transition to turbulence is observed at $t/\tau \approx 1$. In this case, beam breaking is dominated by the input spatial frequency $k_y = 0.02\ \mu\text{m}^{-1}$ and modulation instability is only weakly involved, as noise experiences small amplification on this scale. In Fig. 3(b) the input power spectrum is shown in comparison with the quasihomogeneous case. The picture can be easily extended to generically modulated input waves.

In order to study statistical and stochastic properties of the optical state before and after the transition to turbulence, we consider the quasihomogeneous input case and we collect data for approximately two hundred uncorrelated experiments in the same conditions used in Fig. 2. Each realization naturally presents a different noise configuration, which is caused by fluctuations of the input wave arising from the experimental setup [Fig. 3(b)] and by local variations of the electro-optic response. These two

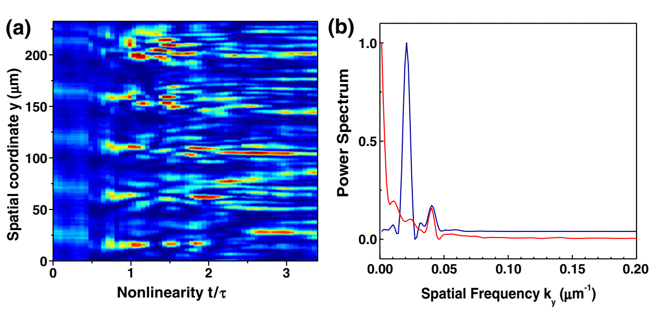


FIG. 3. Onset of optical turbulence for spatially modulated input waves. (a) Output intensity distribution increasing the nonlinearity $t/\tau$. (b) Input power spectrum for spatially modulated beams (blue line) in comparison with the quasihomogeneous case of Fig. 2 (red line).

stochastic effects are coupled, since local intensity fluctuations are amplified by the giant response of the material and inhomogeneity in the nonlinearity strongly affects light dynamics. We underline that fast material fluctuations are crucial in observing the onset of turbulence; the transition is found to disappear as the crystal is heated to a few degrees above the operational temperature. Moreover, since disorder in the material is not fixed on the time scale of the experiment and it is furthermore modified by the wave, Anderson localization effects cannot occur in our case [41,42]. An ensemble spectral analysis at moderate nonlinearity preceding the transition reveals the modulational unstable regime. The well-defined peak in Fig. 4(d) shows that the typical spatial frequency experiencing maximum gain is $\bar{k}_y = 0.05\ \mu\text{m}^{-1}$. However, during the single-shot dynamics, higher or lower frequencies can also emerge easily and compete with the characteristic one. In fact, modulational instabilities are generally known to possess a strong dependence on the specific noise realization, with properties varying from shot to shot [43]. In Figs. 4(a) and 4(b) we show single-shot measurements, each as an example characterizing a particular type of fluctuation. We note that as the frequency $\bar{k}_y$ is mainly excited, localized structures have a weakly varying peak intensity and there is equipartition of power across the generated mode [see also inset in Fig. 1(a)]. On the other hand, broad and double-frequency amplification results into a coherent pattern presenting large intensity fluctuations. Completely different is the scenario in the turbulent regime. Intensity distributions vary stochastically from shot to shot, as shown in Fig. 4(c) for several independent realizations acquired at $t/\tau \simeq 2.5$, where we expect the nonlinearity to be fully saturated. Waves are characterized by random phases in analogy with optical realizations of wave-turbulence theory [16], although from the statistics discussed hereafter we realize that some correlations between modes actually exist. In Fig. 4(e) we report the ensemble power spectrum; it is extremely broad and without specific resonances, with the peak associated to the amplification of $\bar{k}_y$ before the transition that results fully relaxed towards lower spatial frequencies. The spectrum is well fitted at low frequencies by a power law behavior $\propto k_y^{-\gamma}$, with the scaling exponent $\gamma = 0.15 \pm 0.01$. Therefore, we observe evidence of an inverse cascade as the nonlinearity increases, since the majority of the wave action is now located at low transverse wave numbers. However, this flux of wave action towards large scales should be distinguished from the one occurring in wave-turbulence theory. In weak turbulence, an inverse cascade occurs for random waves at weak nonlinearity under forcing at intermediate scales [16,17]; here, it occurs in highly nonlinear conditions and after the modulational instability stage.

In the disordered regime, part of which is shown in Fig. 4(c), we also note the appearance of several bright localized spots. Statistical analysis shows that they are

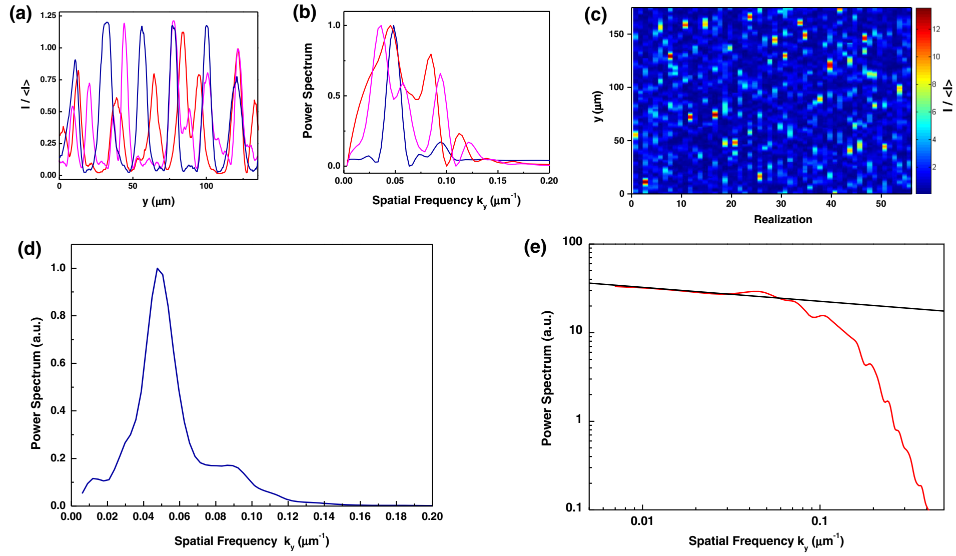


FIG. 4. Spectral properties of the optical state before and after the turbulent transition. (a),(b) Intensity and spectral sample distributions of single-shot measurements at moderate nonlinearity ($t/\tau \approx 1$) showing the excitation of the spatial frequency $\bar{k_y} = 0.05\ \mu\text{m}^{-1}$ (blue line), spectrally broad noise amplification (red line), and simultaneous development of well-defined low and high frequency modes (magenta line). (c) Single-shot disordered intensity distributions detected in the turbulent regime at $t/\tau \simeq 2.5$. (d) Ensemble spectrum of modulational instability before the transition ($t/\tau \approx 1$). (e) Measured wave-turbulent power spectrum (red line) fitted on large spatial scales with the scaling behavior $\propto k_y^{-\gamma}$, $\gamma = 0.15 \pm 0.01$ (black line).

rogue waves. We first consider the probability distribution function (PDF) of peak-intensity values of localized structures emerging from instabilities before the turbulent transition. We analyze more than $10^3$ events, so as to populate the histogram reported in Fig. 5(a). This PDF contains a high-intensity peak embedded into a broad distribution. The peak, at $I/\langle I\rangle \simeq 1.2$, is deterministic and closely related to the peak in the gain spectrum of Fig. 4(d), as it arises from structures belonging to the maximally amplified frequency $\bar{k_y}$ [see Fig. 4(a)]. Random fluctuations in this stage populate the rest of the distribution, with tails compatible with a Gaussian decay, implying that extreme events occur here with low probability. This allows us to conclude that in our system giant perturbations not arise in coherent structures generated by stochastic fluctuations in instability. On the contrary, as reported in Fig. 5(b), the PDF measured deep into the turbulent regime at $t/\tau \simeq 2.5$ presents the long-tail anomalous behavior defining rogue wave phenomena. In fact, for large intensities it deviates from the Gaussian distribution expected for incoherent fields, that implies a decay according to $P(I) = \exp(-I/\langle I\rangle)/\langle I\rangle$ [44]. We note that our setup is able to detect with high resolution the formation dynamics of each rogue wave. We found that extreme events suddenly disappear as $t/\tau$ further varies, so that no traces are found after their passing. Moreover, from our data, inelastic interactions between less intense structures in the wake of extreme events are not so evident. This fact may involve the presence of a different saturation-dependent process in rogue wave appearance.

To conclude, we have observed the onset of turbulence in light flow through a nonlinear medium. The transition in one-dimensional beam dynamics occurs through nonlinear wave interaction and leads from a modulational unstable to a chaotic state where optical coherence is lost and several spatial modes are simultaneously excited. We have found that the stochastic nature of the process manifests itself in a different intensity distribution function before and after the turbulent transition, where concomitant rogue events have also been detected. These observations demonstrate how optical beam propagation in stochastic and nonlinear conditions can form a tool to generate and investigate wave-turbulent phenomena. Further developments include the study of two-dimensional space phenomena, as well as

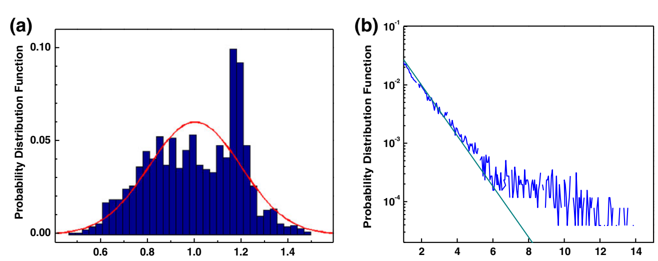


FIG. 5. Spatial rogue wave generation in the turbulent regime. (a) Peak-intensity PDF of localized structures emerging from instability at $t/\tau \approx 1$, experimental counts (blue bars), and the Gaussian trend of the distribution tails (red line). (b) Measured long-tail statistics in optical turbulence at $t/\tau \approx 2.5$ (blue line) and consistent Gaussian exponential scaling for comparison.

the building of a nonlinear wave model that, taking into account fluctuations in the nonlinear response of the medium, can properly describe the observed turbulent regime. Our results may open important routes to control and exploit disordered light transport in extreme nonlinear conditions and pave the way to the development of optical devices that exploit the huge sensitivity of chaotic states. At the same time, they represent an important step in the understanding of anomalous wave events in spatially extended and nonintegrable systems.

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